General Model Structure (SISO)

$$y(t) = G\left(q^{-1}, \theta\right) u(t) + H\left(q^{-1}, \theta\right) e(t)$$

- where

$$G\left(q^{-1}, \theta\right) = \frac{A\left(q^{-1}\right)}{B\left(q^{-1}\right)} = \frac{b_1 q^{-n_k} + b_2 q^{-n_k-1} + \cdots + b_{n_b} q^{-n_k-n_b}}{1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}}$$

- and

$$H\left(q^{-1}, \theta\right) = \frac{C\left(q^{-1}\right)}{D\left(q^{-1}\right)} = \frac{1 + c_1 q^{-1} + \cdots + c_{n_c} q^{-n_c}}{1 + d_1 q^{-1} + \cdots + d_{n_d} q^{-n_d}}$$

- and $e(t)$ is white noise with variance λ^2 and

$$\theta = \left(a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}, c_1, \ldots, c_{n_c}, d_1, \ldots, d_{n_d},\right)^T$$

- Often $\lambda^2 = \lambda^2(\theta)$.

Assumptions

- Time delay $n_k \geq 1 \rightarrow G(0, \theta) = 0$ (often also $G(0, \theta) = 0$).
- $G^{-1}(q^{-1}, \theta)$ and $H^{-1}(q^{-1}, \theta)$ are asymptotically stable (...). Often also $H(q^{-1}, \theta)$ needs to be asymptotically stable.

General Model Structures (Ct'd)

Commonly used simplified models

- ARMAX

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t).$$

Here $A(q^{-1})$ describes the dynamics. Both inputs and noise are governed by the same dynamics.

- ARX

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t).$$

- FIR

$$y(t) = B(q^{-1})u(t) + e(t).$$

- OE

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + e(t).$$