Feedback

Consider the system:

$$\begin{cases} y(t) = G(q^{-1})u(t) + H(q^{-1})e(t) \\ u(t) = -F(q^{-1})y(t) + L(q^{-1})v(t) \end{cases}$$

where

- The input $u(t)$ is determined through feedback.
- F and L are called regulators.
- The signal $v(t)$ can be the reference signal or noise entering the regulator.

What Happens in a Closed Loop Experiment?

- The input $u(t)$ depends on past $y(t)$ (and hence on past $e(t)$).

- The aim of control is to apply a $u(t)$ which minimizes the deviation between $y(d)$ and a reference signal $v(t)$. Good control often requires a $u(t)$ of bounded energy.

- SI requires PE, hence substantial energy of $u(t)$.

- The frequency content of $u(t)$ is limited by the true system.

An example

System:

$$\begin{cases} y(t) + ay(t-1) = bu(t-1) + e(t), \; E[e^2(t)] = \lambda^2 \\ u(t) = -fy(t) \end{cases}$$

Model structure:

$$y(t) + \hat{a}y(t-1) = \hat{b}u(t-1) + \epsilon(t)$$

Estimate by PEM

$$\begin{cases} \hat{a} = a + f\gamma \\ \hat{b} = b - \gamma \end{cases}$$

where γ is any scalar. There is no unique solution, hence the parameters are not estimated consistently.

Closed-loop behavior

Open-loop system:

$$\begin{cases} y(t) = G(q^{-1})u(t) + H(q^{-1})e(t) \\ u(t) = -F(q^{-1})y(t) + L(q^{-1})v(t). \end{cases}$$

Closed loop system:

$$\begin{cases} y(t) = (I + GF)^{-1}GLv(t) + He(t) \\ u(t) = I - F1 + GF^{-1}GLv(t) - FI + GF^{-1}He(t). \end{cases}$$