1/[P3] Suspensión Magnética

$$M\ddot{z} = Mg - K_m \frac{i^2}{z^2} \Rightarrow \ddot{z} = g - \frac{K_m}{M}\frac{i^2}{z^2}$$

$$L\frac{di}{dt} + Ri = V \Rightarrow \frac{di}{dt} = \frac{1}{L}\left[V - R \cdot i\right]$$

Luego el vector de estado esta dado por.

$$\vec{x} = \begin{bmatrix} z \\ \dot{z} \\ i \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{K_m}{M}\frac{x_3^2}{x_1^2} \\ -\frac{R \cdot i}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ V \end{bmatrix}$$

haciendo $\dot{\vec{x}} = 0$ se obtiene el punto de operación:

$$\left.\begin{aligned} & x_2 = 0 \\ & g - K_m \frac{x_3^2}{x_1^2} = 0 \\ & \frac{V - Ri}{L} = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} & x_2 = 0 \\ & x_3 = \frac{V}{R} \cong 0{,}5439\ [A] \\ & x_1 = \sqrt{\frac{K_m}{Mg}}\, x_3 = \sqrt{\frac{K_m}{Mg}} \cdot \frac{V}{R} = 0{,}1 \end{aligned}$$

linealizando se tiene:

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \\ \Delta \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2 k_m x_3^2}{M x_1^3} & 0 & -\frac{2 k_m x_3}{M x_1^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Delta V \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \\ \Delta \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 196,1 & 0 & -36,08 \\ 0 & 0 & -920 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Delta V \end{bmatrix}$$

$$\Delta y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix}$$

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B$$

$$G(s) = \frac{-36080}{(s+920)(s^2 - 196,1)} = z^3$$

$$G_c(s) = (k_p + k_d s)$$

$$1 + G_c(s) G(s) = 0$$

$$s^3 + 920 \cdot s^2 - 36080 (k_I + 0,0054) s - 36080 (k_p + 5)$$

a partir de los requerimientos

$$\%ov = 5\% = 0{,}05 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0{,}6901 > 0{,}69$$

$$\Rightarrow t_s = \frac{4{,}5\,\xi}{\omega_n} \Rightarrow \omega_n = 3{,}1055$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\Rightarrow s_{1,2} = -2{,}1431 \pm j\,2{,}2475$$

$$(s-a)(s-s_1)(s-s_2) = 0$$

$$\Rightarrow s^3 + (4{,}286 - a)s^2 + (9{,}644 - 4{,}286a)s - 9{,}644a = 0$$

Igualando

$$(4{,}286 - a) = 920$$

$$9{,}644 - 4{,}286a = -36080\,(K_I + 0{,}0054)$$

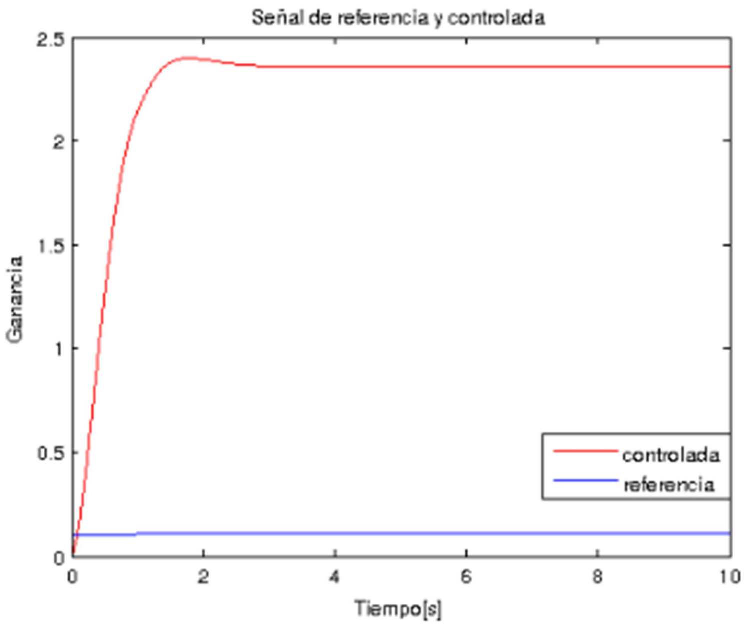
$$9{,}644a = -36080\,(K_p + 5)$$

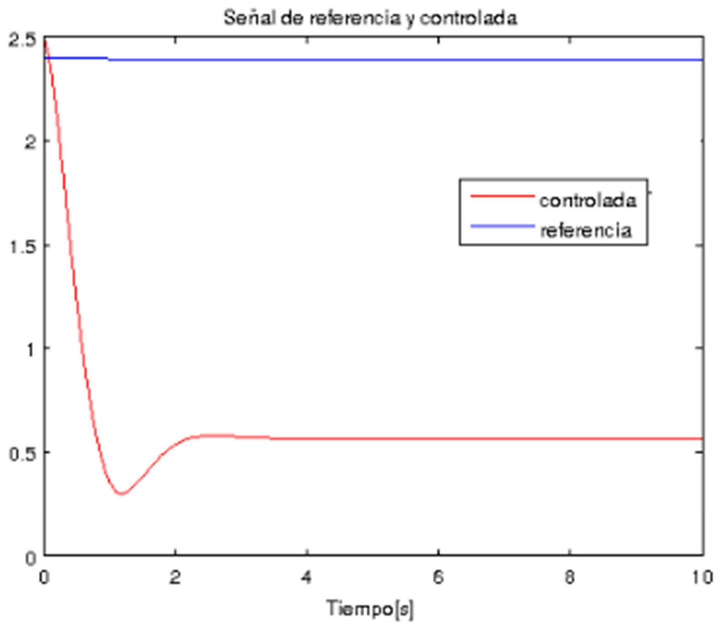
$$\Rightarrow a = 915{,}7140$$

$$K_p = -4{,}7552$$

$$K_d = -0{,}1144$$

$$\Rightarrow \boxed{G_c(s) = -4{,}7552 - 0{,}1144 \cdot s}$$


Señal de referencia y controlada
Ganancia
Tiempo[s]
controlada
referencia
0
0.5
1
1.5
2
2.5
0
2
4
6
8
10


Señal de referencia y controlada
Tiempo[s]
controlada
referencia
0
0.5
1
1.5
2
2.5
0
2
4
6
8
10

Controlador PI

$$G_c(s) = k_p + \frac{k_I}{s}$$

$$1 + G_c(s)\,G(s) = 0$$

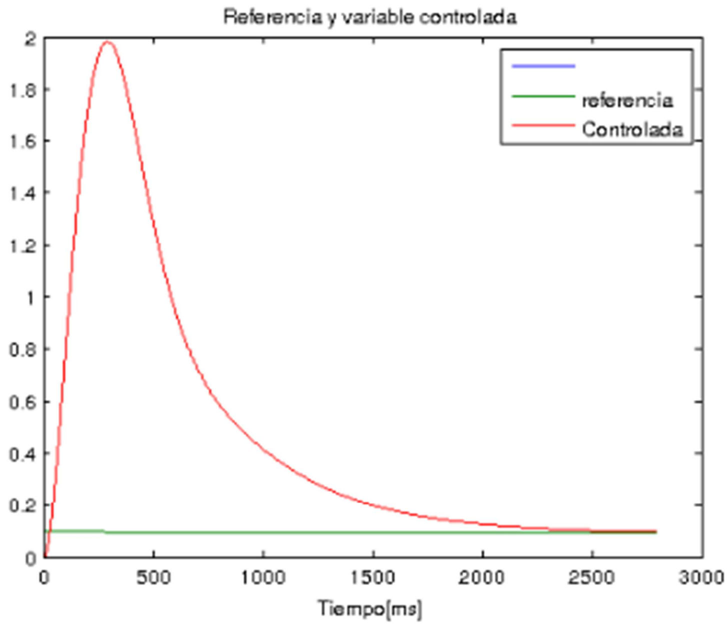
$$s^4 + 920 s^3 - 196.1\, s^2 - 36080\,(k_p + 5)\,s - 36080\, k_I = 0$$

a partir de los requerimientos se tiene

$$s^4 + (-a - b + 4{,}2862)\,s^3$$
$$+ \left(a(b - 4{,}2862) - 4{,}2862\,b + 9.6441\right) s^2$$
$$+ \left(a(4{,}2862\,b - 9{,}6441) - 9.6441\,b\right) s + 9.6441\,ab = 0$$

$$\Rightarrow \begin{cases} k_p = -4{,}7541 \\ k_I = 1{,}1041 \end{cases}$$

Resultado de simular el controlador PI

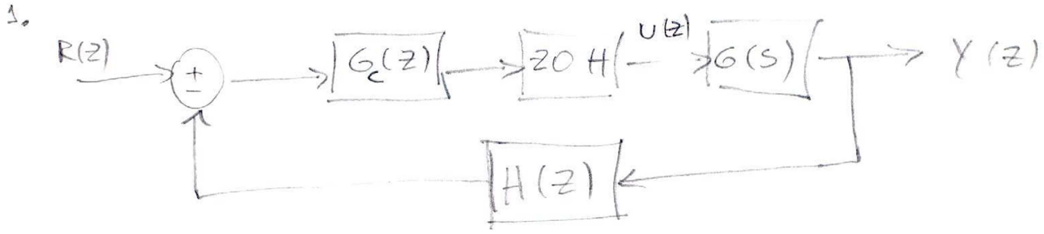


P2

$$H(s) = \frac{\omega(s)}{V(s)} = \frac{2}{s^2 + 12 \cdot s + 20}$$

$T_s = 1$ [seg]

1.



V. manipulada (U(z)): Voltaje aplicado al motor

V. controlada (Y(z)): Velocidad angular del motor

sensor (H(z)): sensor de velocidad angular

ZOH: Retenedor de orden cero

$G_c(z)$: controlador discreto

2. Primero se pasa la función de transferencia del espacio s a z.

$$G(z) = \mathcal{Z}_1 \left\{ \frac{1 - e^{-Ts}}{s} \cdot \frac{2}{s^2 + 12 \cdot s + 20} \right\}$$

$$= 2\,(1 - z^{-1})\, \mathcal{Z}_1 \left\{ \frac{1}{s(s^2 + 12s + 20)} \right\}$$

$$= (1 - z^{-1})\, \mathcal{Z}_1 \left\{ \frac{0{,}025}{s+10} - \frac{0{,}125}{s+2} + \frac{0{,}1}{s} \right\}$$

$$= (1 - z^{-1}) \left[\frac{0{,}025\, z}{(z - 0{,}000045)} - \frac{0{,}125 \cdot z}{(z - 0{,}1353)} + \frac{0{,}1\, z}{z - 1} \right]$$

$$\Rightarrow G(z) = \frac{0{,}0831z + 0{,}0034}{(z^2 - 0{,}1354z + 6{,}144\times10^{-6})}$$

3. Calcular controlador PI.

$t_s = 2$ seg

$MOV = 2\% = 0{,}02 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0{,}7797$

como $\xi > 0{,}69$

$t_s = \frac{4{,}5\,\xi}{\omega_n} \Rightarrow \omega_n = 1{,}7543$

a partir de lo anterior se tiene que la ecuación característica es:

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\Rightarrow s_{1,2} = -1{,}3678 \pm j\,1{,}0985$

luego

$z_{1,2} = e^{T_s(s_{1,2})} = 0{,}1159 \pm j\,0{,}2268.$

$z^2 - 0{,}2318z + 0{,}640$

El controlador PI es de la forma:

$$G_c(z) = \left(K_P + \frac{K_I}{1-z^{-1}}\right)$$

a partir de la ecuación

$1 + G_c(z)\,G(z) = 0$

$$1 + \left(K_P + \frac{K_I}{1-z^{-1}}\right)\left(\frac{0{,}0831z + 0{,}0034}{z^2 - 0{,}1354z + 6{,}144\times10^{-6}}\right) = 0$$

$$\Rightarrow z^3 + 0{,}0831\left(K_I + K_p - 13{,}66\right)z^2 + 0{,}0034\left(K_I - 23{,}44_{12}\left(K_p - 1{,}6989\right)\right)z$$

$$- 0{,}0034\left(K_p + 0{,}0018\right) = 0$$

Luego $(z - z_1)(z - z_2)(z - a) = 0$

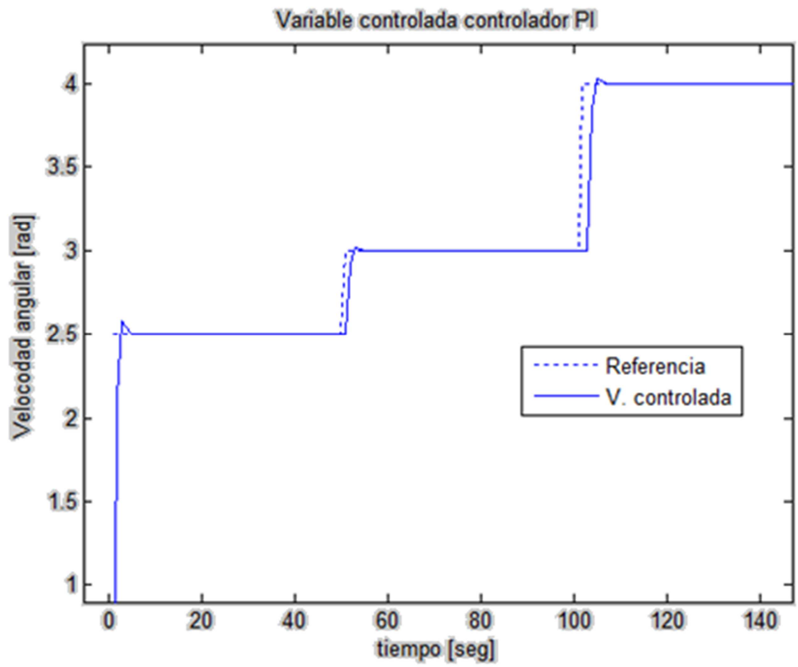
$$\Rightarrow z^3 + (-a - 0{,}2318)z^2 + (0{,}2318a + 0{,}0649)z - 0{,}0649a = 0$$

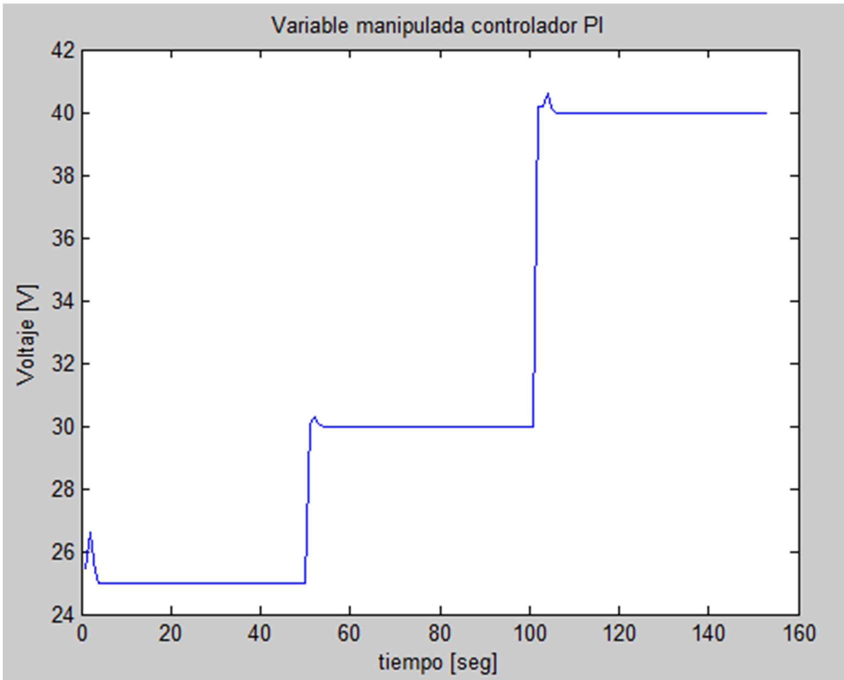
$$\Rightarrow \boxed{\begin{aligned} &0{,}0831\left(K_I + K_p - 13{,}66\right) = -a - 0{,}2318 \\ &0{,}0034\left(K_I - 23{,}44\left(K_p - 1{,}699\right)\right) = 0{,}2318a + 0{,}0649 \\ &+0{,}0034\left(K_p + 0{,}0018\right) = +0{,}0649a \end{aligned}}$$

$$\Rightarrow \left|\begin{aligned} a &= 0{,}0579 \\ K_I &= 9{,}0708 \\ K_p &= 1{,}1032 \end{aligned}\right.$$

$$\Rightarrow \boxed{G_c(z) = \left(1{,}1032 + \frac{9{,}0708}{1 - z^{-1}}\right)}$$

$e_{ss} = 0$


Variable controlada controlador PI
Velocodad angular [rad]
tiempo [seg]
Referencia
V. controlada
4
3.5
3
2.5
2
1.5
1
0
20
40
60
80
100
120
140


Variable manipulada controlador PI
Voltaje [V]
tiempo [seg]
42
40
38
36
34
32
30
28
26
24
0
20
40
60
80
100
120
140
160

El controlador PD es de la forma

$G_c(z) = \left(K_p + K_d(1 - z^{-1})\right)$

Luego se tiene: $1 + G_c(z)\,G(z) = 0$

$$1 + \left(K_p + K_d(1 - z^{-1})\right)\left(\frac{0{,}0831\,z + 0{,}0034}{z^2 - 0{,}1354\,z + 6{,}144 \times 10^{-6}}\right) = 0$$

$$\Rightarrow z^3 + 0{,}0831\,(K_d + K_p - 1{,}6294)\,z^2$$
$$- 0{,}0797\,(K_d - 0{,}0427\,(K_p + 0{,}0018))\,z - 0{,}0034\,K_d = 0$$

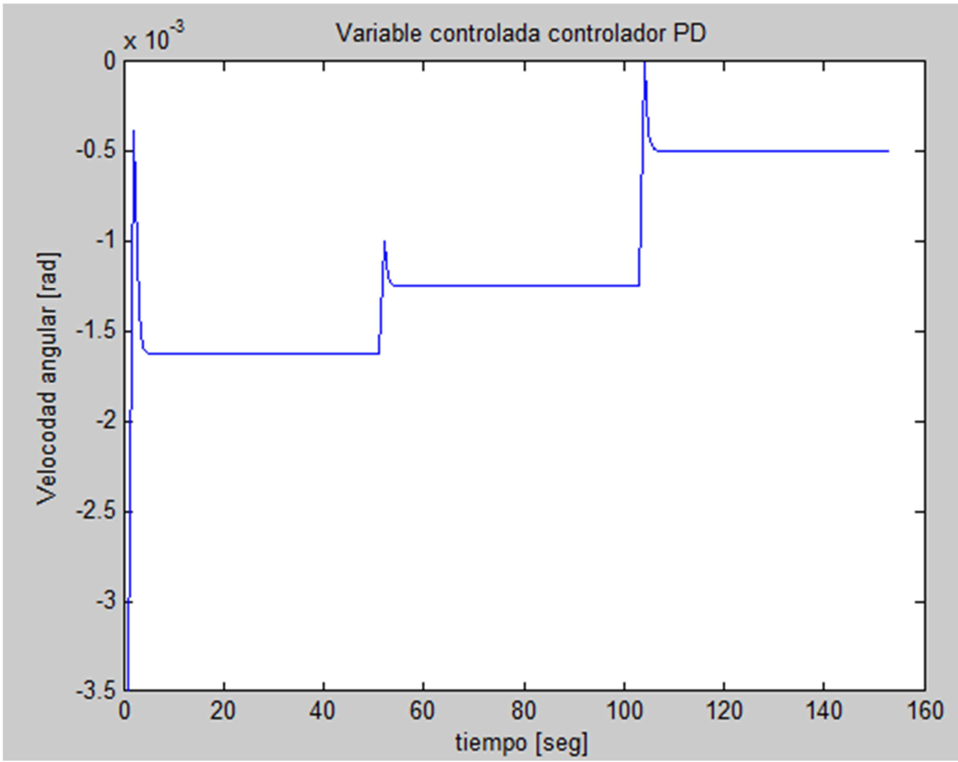
Igualando con la ecuación

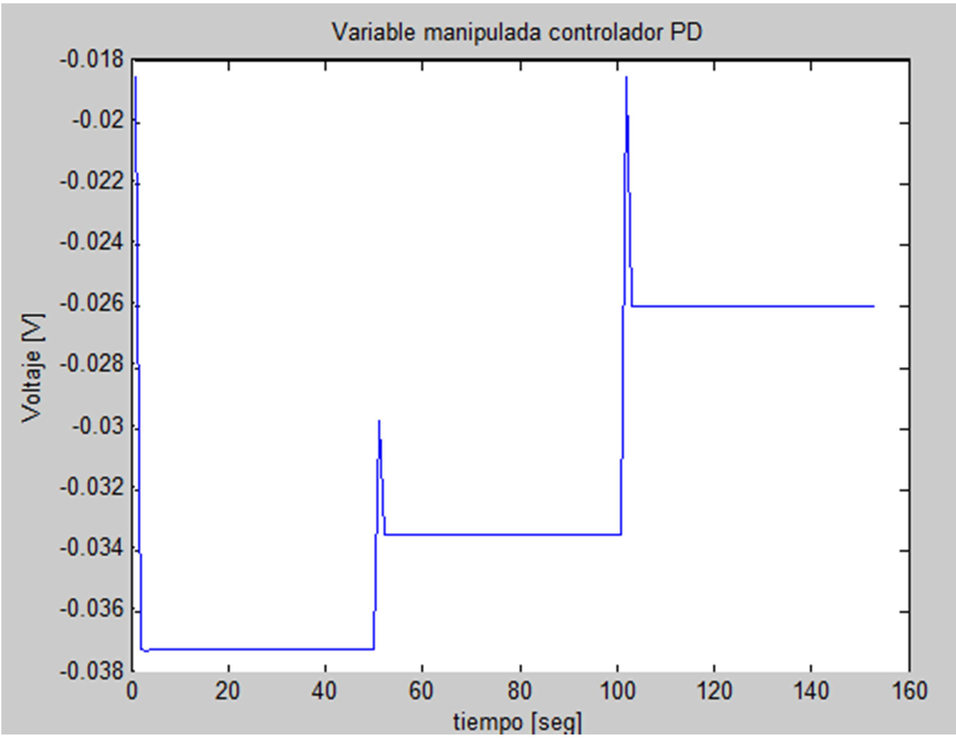
$(z - z_1)(z - z_2)(z - a) = 0$

$$0{,}0831\,(K_d + K_p - 1{,}629) = -a - 0{,}2318$$
$$-0{,}0797\left[K_d - 0{,}0427\,(K_p + 0{,}0018)\right] = 0{,}2318\,a + 0{,}0649$$
$$+\ 0{,}0034\,K_d = 0{,}0649\,a$$

$$\Rightarrow \boxed{\begin{aligned} a &= -0{,}0370 \\ K_d &= -0{,}7068 \\ K_p &= -0{,}0075 \end{aligned}} \Rightarrow G_c(z) = -0{,}0079 - 0{,}7069\,(1 - z^{-1})$$

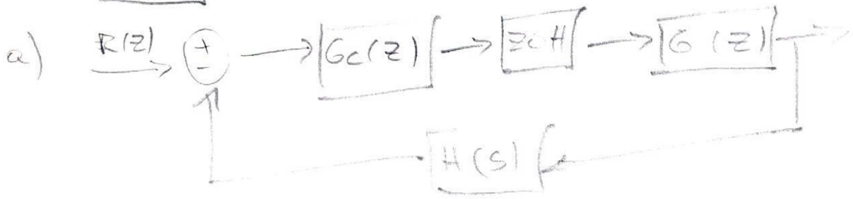
$$e_{ss} = \lim_{z \to 1}\,(1 - z^{-1})\,E(z) = \frac{1}{1 + 0{,}1 \cdot 0{,}0079} = \boxed{1{,}0008}$$


Variable controlada controlador PD
x 10^-3
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
Velocodad angular [rad]
0
20
40
60
80
100
120
140
160
tiempo [seg]


Variable manipulada controlador PD
-0.018
-0.02
-0.022
-0.024
-0.026
-0.028
-0.03
-0.032
-0.034
-0.036
-0.038
Voltaje [V]
0
20
40
60
80
100
120
140
160
tiempo [seg]

$\boxed{P3}$

a)



V. manipulada: flujo de agua.

V. controlada: nivel del estanque de agua

$$H(z) = \frac{3.82\times 10^{-5} z^{-1}}{1 - z^{-1}} = \frac{3.82 \times 10^{-5} z}{z - 1}$$

b) $mov = 25\% = 0{,}25 \Rightarrow \xi = 0{,}4037 < 0{,}69$

$\Rightarrow t_s = 100 = \frac{3.2}{w_n \xi} \Rightarrow w_n = 0{,}0793$

luego $s^2 + 2\xi w_n \cdot s + w_n^2 = 0$

$\Rightarrow s_{1,2} = -0{,}032 \pm j\, 0{,}0725$

$z_{1,2} = e^{T s_{1,2}}$

$\Rightarrow z_{1,2} = 0{,}966 \pm j\, 0{,}702$

$\Rightarrow (z - z_1)(z - z_2) = 0$

$\Rightarrow z^2 - 1{,}9318 z + 0{,}9380 = 0$ (1)

a partir de

$1 + G_c(z)\, G(z) = 0$

donde $G_c(z) = K_P + \frac{K_I}{(1 - z^{-1})}$

$$1 + \frac{\left[(k_p + k_I) z - k_p \right]}{z-1} \cdot \frac{3{,}82 \times 10^{-5} z}{(z-1)} = 0$$

$$\Rightarrow z^2 - \frac{(2 + 3{,}82 \times 10^{-5} k_p) z}{1 + 3{,}82 \times 10^{-5} (k_p + k_I)} + \frac{1}{(1 + 3{,}82 \times 10^{-5} (k_p + k_I))} = 0 \ (2$$

Igualando (1) y (2) se tiene:

$k_p = 1560{,}0755$

$k_I = 170{,}2408$

$$\Rightarrow G_c(z) = 1560{,}0755 + \frac{170{,}2408}{(1 - z^{-1})}$$

c) se tiene que $T = 1$ seg

además se tiene que la relación entre un controlador continuo y uno discreto es:

$$G_c(s) = k_p + \frac{k_p}{T_i s} + k_p T_d s$$

$$G_c(z) = k_{Pd} + \frac{k_{Id}}{1 - z^{-1}} + k_{Dd} (1 - z^{-1})$$

donde
$$k_{Pd} = k_p - \frac{k_p T}{2 T_i}$$

$$k_{Id} = \frac{k_p T}{T_i}$$

$$k_{Dd} = \frac{k_p T_d}{T}$$

De la parte b) se tiene:

$K_{pd} = 1560,0755$

$K_{Id} = 170,2408$

$T = 1$

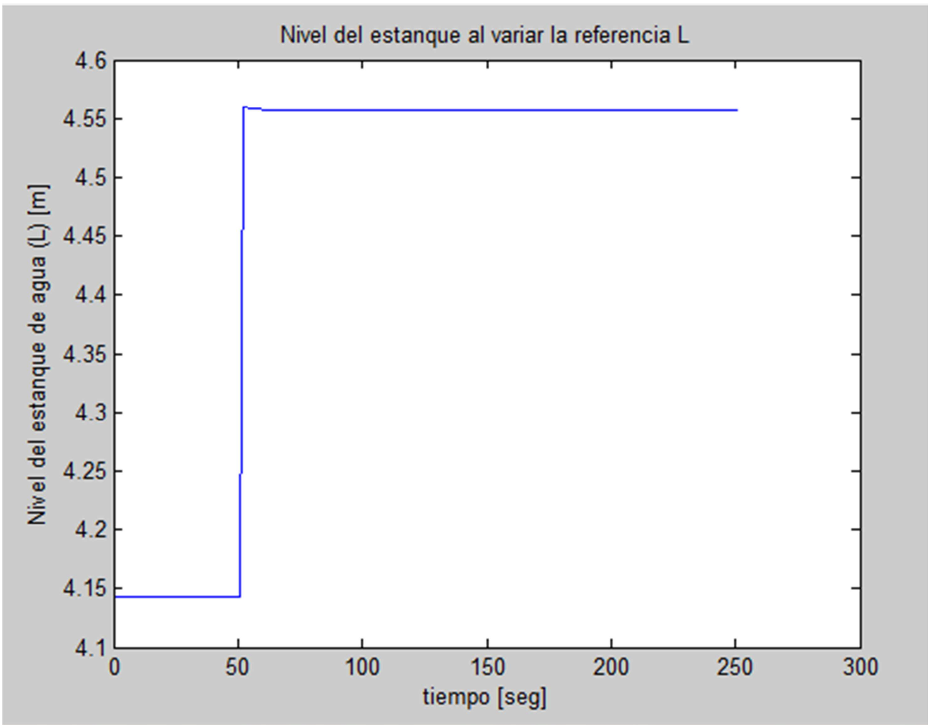
a partir de lo anterior se tiene

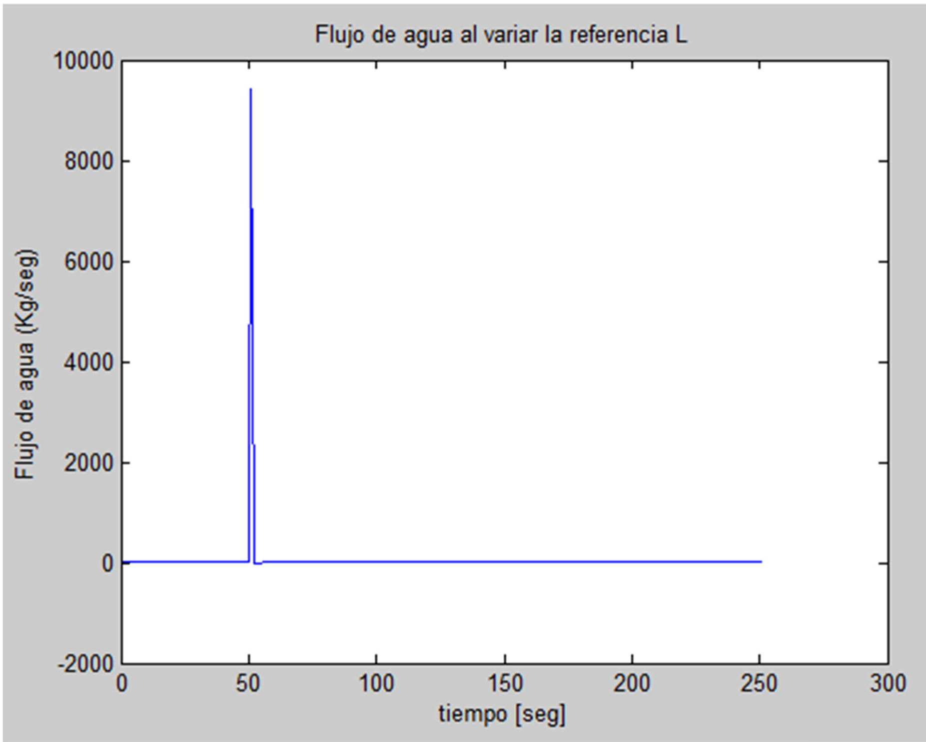
$K_p = 1645,8259$

$T_i = 9,6676$

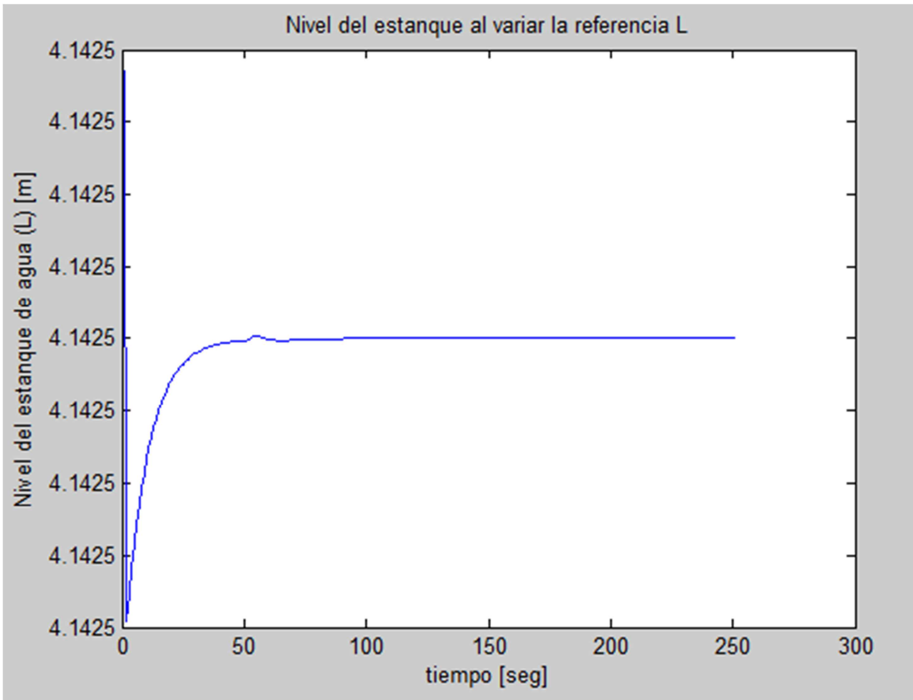
$$\Rightarrow G_c(s) = 1645,8259 + \frac{1645,8259}{9,6676} \cdot \frac{1}{s}$$

$$\Rightarrow \boxed{G_c(s) = 1645,8259 + \frac{170,2414}{s}}$$

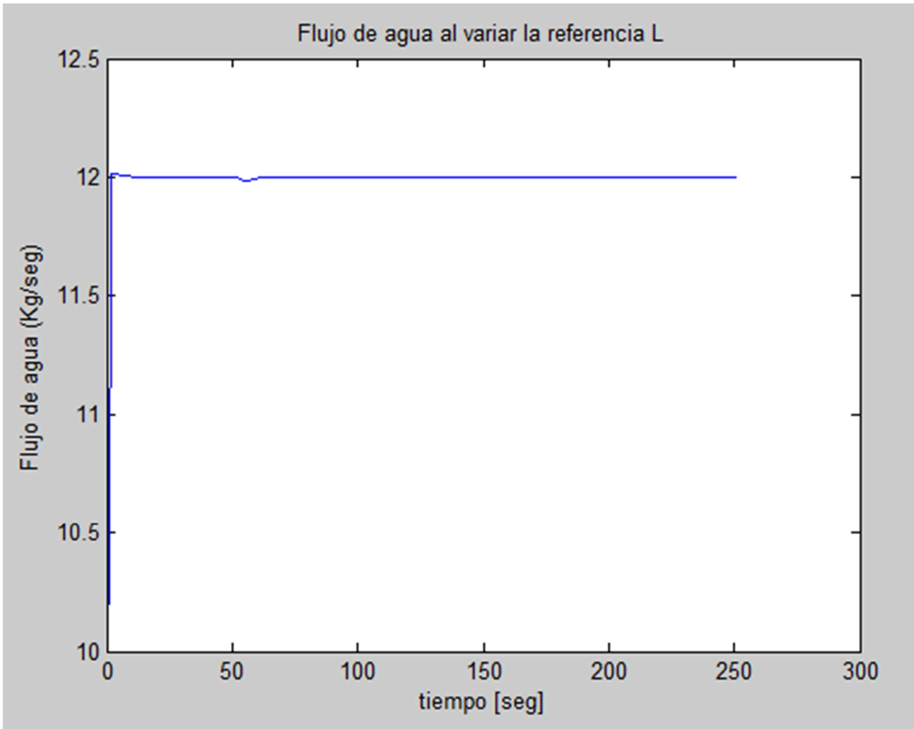




Flujo de agua al variar la referencia L
10000
8000
6000
4000
2000
0
-2000
0
50
100
150
200
250
300
tiempo [seg]
Flujo de agua (Kg/seg)



Nivel del estanque al variar la referencia L
4.1425
4.1425
4.1425
4.1425
4.1425
4.1425
4.1425
4.1425
4.1425
0
50
100
150
200
250
300
tiempo [seg]
Nivel del estanque de agua (L) [m]


Flujo de agua al variar la referencia L
Flujo de agua (Kg/seg)
tiempo [seg]
12.5
12
11.5
11
10.5
10
0
50
100
150
200
250
300