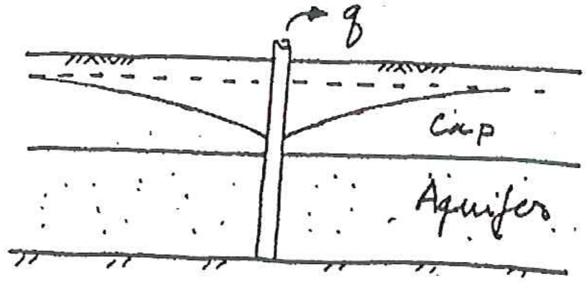
Lecture 7

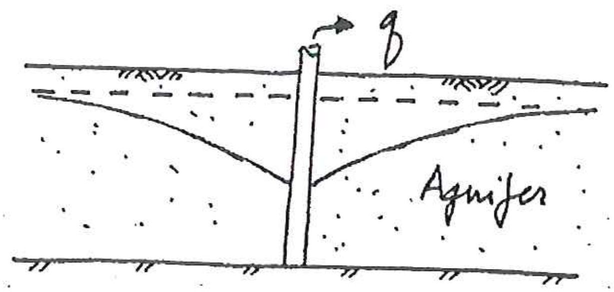
Flow Toward Wells

Flow Toward Wells

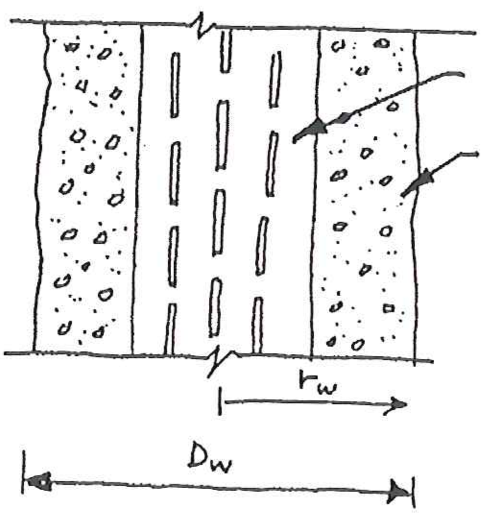
Artesian condition - piezometric surface stays above top of aquifer



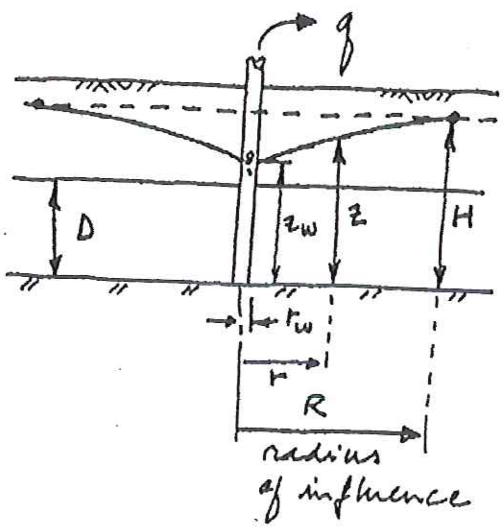
Gravity flow condition - piezometric surface is within aquifer.



Well Diameter



Arterian Well Flow



Darcy's Law: $q = k\,i\,A$

$$q = k \frac{dz}{dr} \cdot 2\pi r \cdot D$$

$$\frac{q}{2\pi k D} \frac{dr}{r} = dz$$

$$\frac{q}{2\pi k D} \ln r = z + \text{const.}$$

Boundary Conditions

(A) Observation wells - good data

Tell us, for example when $r = r_1$, $z = z_1$

and when $r = r_2$, $z = z_2$

$$\frac{q}{2\pi k D} \ln r_1 = z_1 + \text{const.}$$

$$\frac{q}{2\pi k D} \ln r_2 = z_2 + \text{const.}$$

$$\frac{q}{2\pi k D} (\ln r_2 - \ln r_1) = z_2 - z_1 \quad \Big\} \text{ Basic relationship}$$

$$K = \frac{q}{2\pi D (z_2 - z_1)} \ln \frac{r_2}{r_1} \quad \Big\} \text{ To calculate } k \text{ from pump test.}$$

Boundary Conditions

(B) "Radius of Influence" - approximate

We assume when $r = R$, $Z = H$

and when $r = r_W$, $Z = Z_W$

R is usually assumed to be 1000 ft to 3000 ft

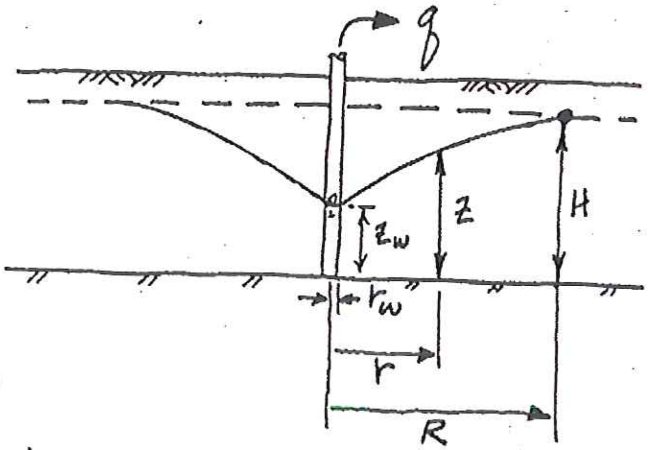
$$\frac{q}{2\pi KD} \ln r_W = Z_W + \text{const.}$$

$$\frac{q}{2\pi KD} \ln R = H + \text{const}$$

$$\frac{q}{2\pi KD} (\ln R - \ln r_W) = H - Z_W \quad \Big\} \text{ Basic relationship}$$

$$q = \frac{2\pi KD(H - Z_W)}{\ln(R/r_W)} \quad \Big\} \text{ To calculate flow for given drawdown}$$

Gravity Well Flow



We need assumptions

Dupuit assumptions:

① $i = \frac{dz}{dr}$

② i = constant on vertical line

Niether strictly True

Good approximations

Darcy's Law: $q = k\,i\,A$

$$q = k \frac{dz}{dr} \cdot 2\pi r z$$

Gravity Well Flow (continued)

$$q = k \frac{dz}{dr} 2\pi r z$$

$$\frac{q}{2\pi k} \frac{dr}{r} = z\,dz$$

$$\frac{q}{2\pi k} \ln r = \frac{z^2}{2} + \text{const.}$$

$$\frac{q}{\pi k} \ln r = z^2 + \text{const.}$$

Boundary Conditions

(A) Observation Wells $\quad r = r_1, \; z = z_1$
$\quad r = r_2, \; z = z_2$

$$\frac{q}{\pi k} \ln r_1 = z_1^2 + \text{const.}$$

$$\frac{q}{\pi k} \ln r_2 = z_2^2 + \text{const.}$$

$$\frac{q}{\pi k} (\ln r_2 - \ln r_1) = z_2^2 - z_1^2 \quad \Big\} \text{ Basic Eqn.}$$

$$k = \frac{q}{\pi (z_2^2 - z_1^2)} \ln \frac{r_2}{r_1} \quad \Big\} \text{ To calculate } k \text{ from pump test.}$$

(B) Radius of Influence $\quad r = R, \; z = H$
$\quad r = r_w, \; z = z_w$

$$\frac{q}{\pi k} \ln r_w = z_w^2 + \text{const.}$$

$$\frac{q}{\pi k} \ln R = H^2 + \text{const.}$$

$$\frac{q}{\pi k} (\ln R - \ln r_w) = H^2 - z_w^2 \quad \Big\} \text{ Basic Eqn.}$$

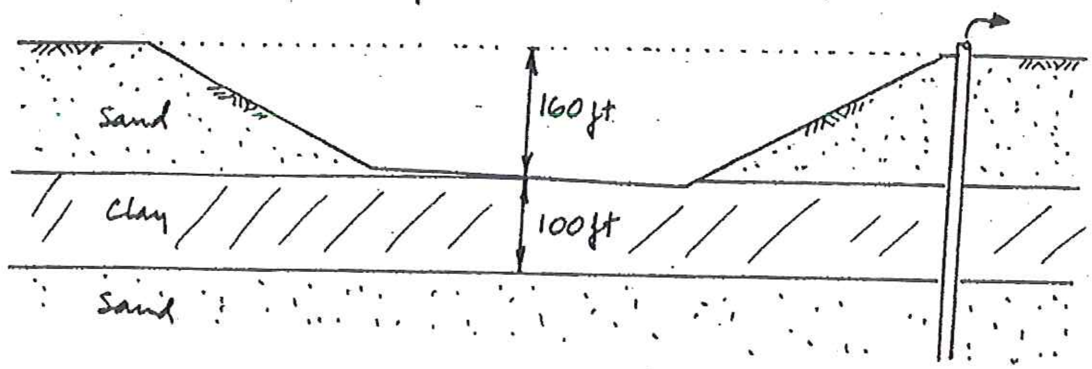
$$q = \frac{\pi k (H^2 - z_w)^2}{\ln (R/r_w)} \quad \Big\} \text{ To calculate flow for given drawdown}$$

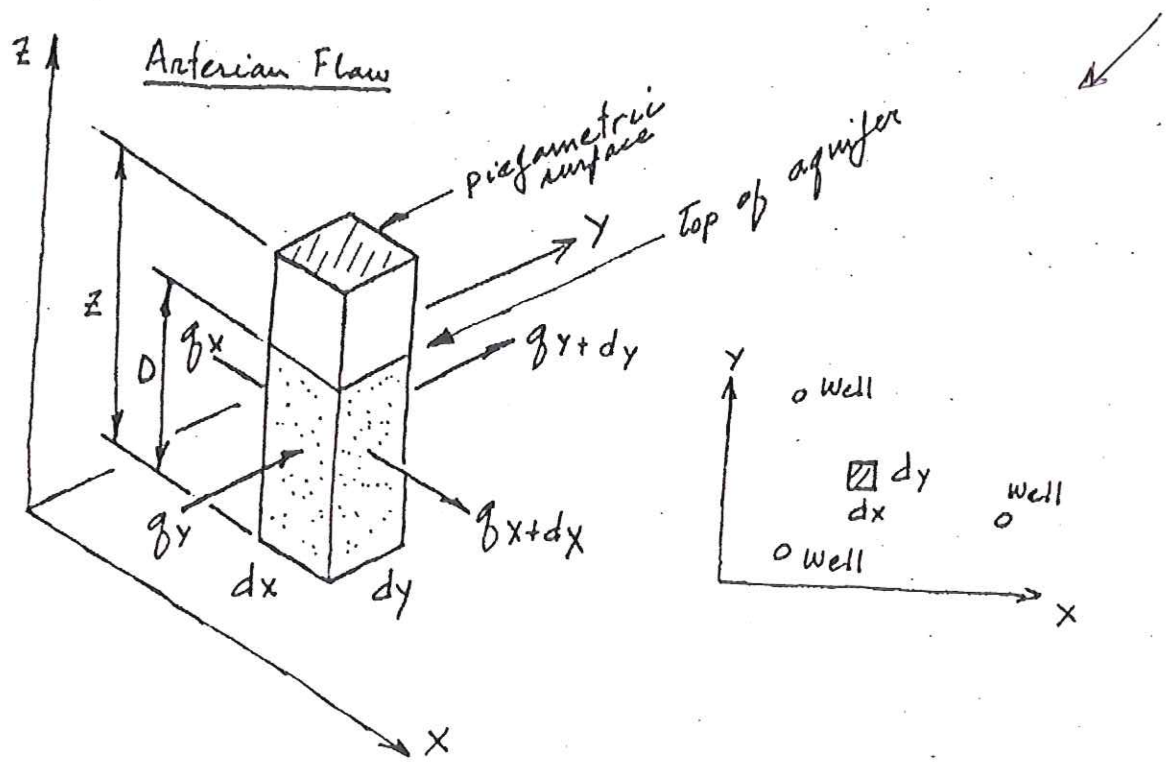
Acuifer Agui. Aquifer Acuifer

Multiple Well Systems

Often use more than one well or well point for dewatering or pressure relief.

Example - Buena Vista Pumping Plant excavation
5 deep wells





Flow is horizontal (no assumptions required)

$$i_x = \frac{\partial z}{\partial x} \qquad i_y = \frac{\partial z}{\partial y}$$

Darcy's Law: $q = k i A$

$$q_x = k \frac{\partial z}{\partial x} D\, dy$$

$$q_{x+dx} = k\left(\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial x} dx\right) D \cdot dy$$

$$q_y = k \frac{\partial z}{\partial y} \cdot D \cdot dx$$

$$q_{y+dy} = k\left(\frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial y^2} dy\right) D \cdot dx$$

$$q_{x+dx} - q_x + q_{y+dy} - q_y = 0 \qquad \text{Steady Flow}$$

$$k D \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) dx\, dy = 0$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \quad \Bigg\} \quad \text{Governing Eqn. is LaPlace Eqn. in } z.$$

<u>Rule of Superposition</u>: To determine z for multiple wells, add expressions for z due to each well, and add a constant to satisfy the boundary conditions.

Solution for one well

$$z = H - \frac{q}{2\pi K D} \ln \frac{R}{r}$$

Solution for Two wells

$$z = 2H - \frac{q_1}{2\pi K D} \ln \frac{R}{r_1} - \frac{q_2}{2\pi K D} \ln \frac{R}{r_2} + \text{const.}$$

Solution for n wells

$$z = nH - \sum_{i=1}^{n} \left[\frac{q_i}{2\pi K D} \ln \frac{R}{r_i} \right] + \text{const.}$$

Boundary Condition

When $r_1 = r_2 = \cdots r_i = \cdots r_n = R,\ z = H \Big\}$ approximately True

$$H = nH + \sum_{i=1}^{n} \left[\frac{q_i}{2\pi K D} \ln\left(\frac{R}{R}\right) \right] + \text{const.}$$

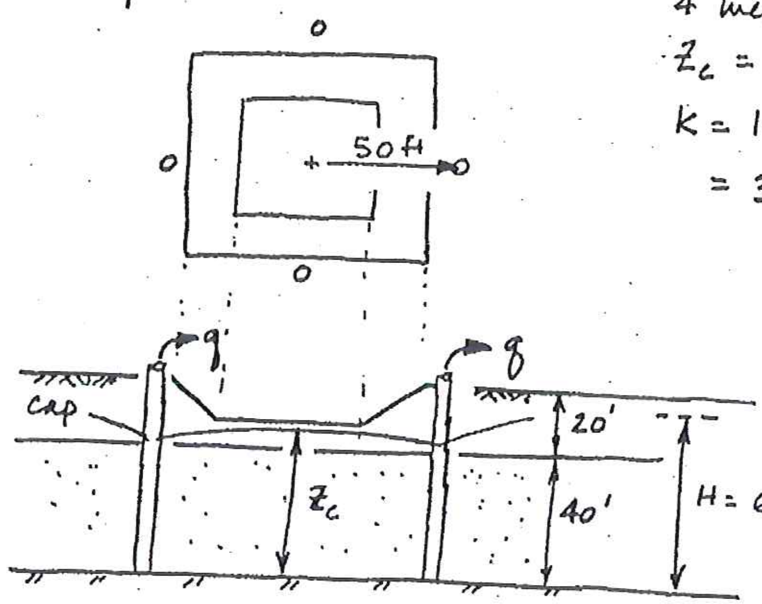
$$H = nH + 0 + \text{const.}$$

$$\text{const.} = (1-n)H$$

$$z = H - \sum_{i=1}^{n} \left[\frac{q_i}{2\pi K D} \ln \frac{R}{r_i} \right]$$

Multiple Well Artesian Flow

<u>Example</u>



4 wells

$z_c = 45$ ft (we want)

$K = 10^{-3}$ cm/sec

$= 3 \times 10^{-5}$ ft/sec

$$z = H - \sum_{i=1}^{n} \left[\frac{q_i}{2\pi K D} \ln \frac{R}{r_i} \right]$$

$$45 = 60 - \frac{\Sigma q_i}{2\pi (3\times10^{-5})(40)} \ln \frac{2000}{50}$$ ← assumed R

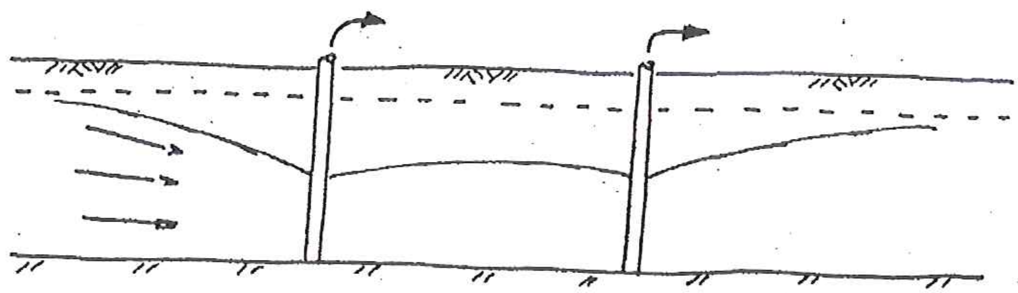
$$45 = 60 - 490 \, \Sigma q_i$$

$$\Sigma q_i = \frac{15}{490} = 0.031 \text{ ft}^3/\text{sec}$$

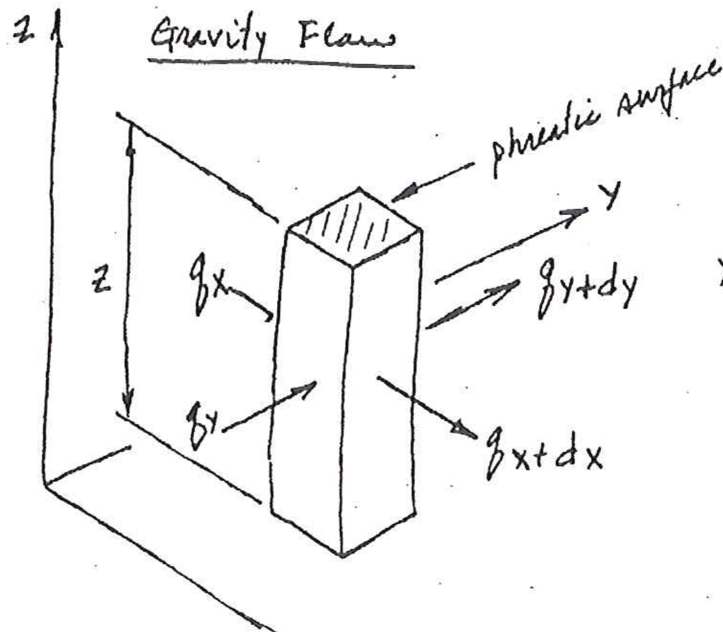
$$\Sigma q_i = 14 \text{ gpm} = 20{,}000 \text{ gpd}$$

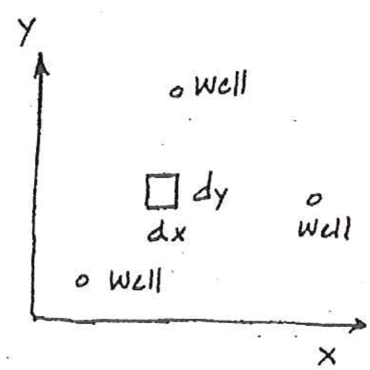
Multiple Well System - Gravity Flow

The situation is a little different for gravity flow, and that leads to a different rule of superposition.



Flow is not horizontal - we use the Dupuit assumptions to develop the solution.





Governing Eqn.

The derivation is longer than for artesian flow

The result is that the governing equation is the LaPlace Eqn. in z^2:

$$\frac{\partial^2 (z^2)}{\partial x^2} + \frac{\partial^2 (z^2)}{\partial y^2} = 0$$

Rule of Superposition: To determine z for multiple wells, add expressions for z^2 due to each well, and add a constant to satisfy the boundary conditions.

Solution for one well:

$$z^2 = H^2 - \frac{q}{\pi k} \ln \frac{R}{r}$$

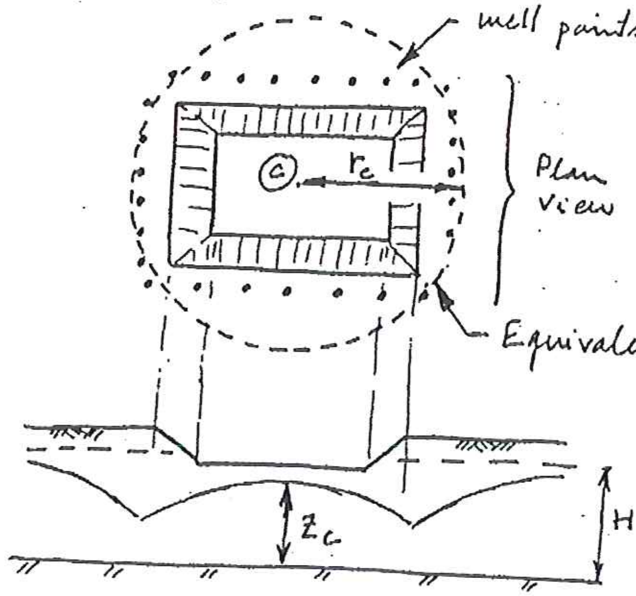
Solution for n wells:

$$z^2 = n H^2 - \sum_{i=1}^{n} \left[\frac{q_i}{\pi k} \ln \frac{R}{r_i} \right] + \text{const.}$$

$$\text{const.} = (1-n) H^2$$

$$z^2 = H^2 - \sum_{i=1}^{n} \left[\frac{q_i}{\pi k} \ln \frac{R}{r_i} \right]$$

Ring of Wells



With well points There may be a lot of terms to sum.

$$z^2 = H^2 - \sum_{i=1}^{n} \frac{q_i}{\pi k} \ln \frac{R}{r_i}$$

If we replace the actual layout with a circular array, every well is the same distance from the center.

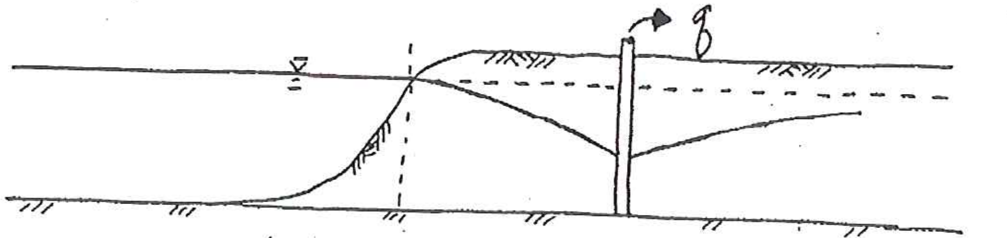
At c, $r = r_c$ for every well in the circular array.

$$z_c^2 = H^2 - \sum_{i=1}^{n} \frac{q_i}{\pi k} \ln \frac{R}{r_c}$$

$$z_c^2 = H^2 - \frac{1}{\pi k} \ln \frac{R}{r_c} \sum_{i=1}^{n} q_i$$

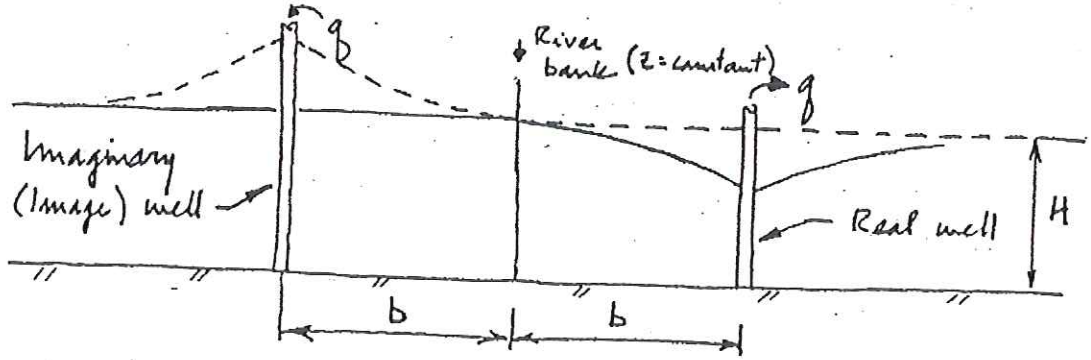
$$z^2 = H^2 - \frac{Q}{\pi k} \ln \frac{R}{r_c}$$ } Q = total flow from all wells

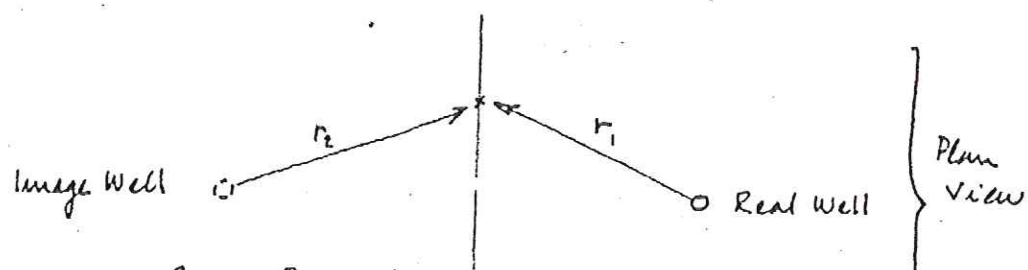
<u>Well Adjacent to a River</u> (Line Source)



The piezometric level at the river bank never changes (it is a boundary condition).

We can represent this "line source" by using an imaginary well, equally distant on the opposite side of the river bank from the real well, pumping water in.

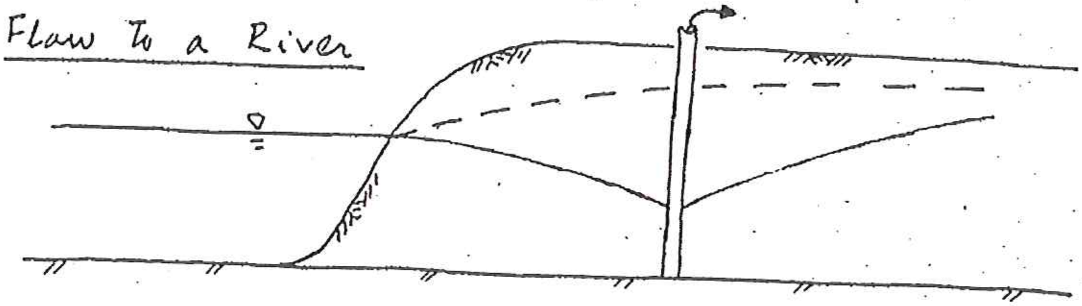




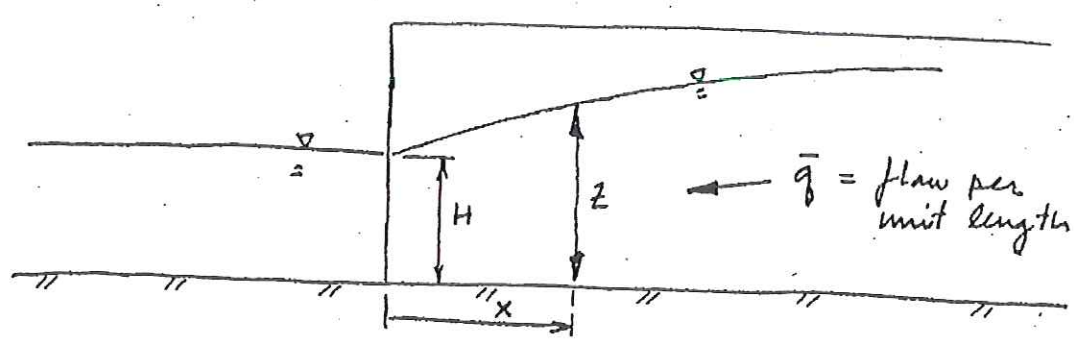
$$z^2 = H^2 - \frac{q}{\pi k} \ln \frac{R}{r_1} + \frac{q}{\pi k} \ln \frac{R}{r_2}$$

$$z^2 = H^2 - \frac{q}{\pi k} (\ln R - \ln r_1 - \ln R + \ln r_2) = H^2 - \frac{q}{\pi k} \ln \frac{r_2}{r_1}$$

Flow To a River



Groundwater Flow:



Dupuit assumptions: ① $i = \frac{dz}{dx}$

② i = constant on vertical line

Darcy's Law: $q = kiA$

$$\bar{q} = k\frac{dz}{dx}\cdot z \cdot 1$$

$$\bar{q}\,dx = kz\,dz$$

$$\bar{q}x = \frac{kz^2}{2} + \text{const.}$$

when $x = 0$, $z = H$

$$0 = \frac{kH^2}{2} + \text{const}$$

$$\text{const} = -\frac{kH^2}{2}$$

$$\bar{q}x = \frac{kz^2}{2} - \frac{kH^2}{2}$$

$$\frac{2\bar{q}x}{k} = z^2 - H^2$$

$$z^2 = H^2 + \frac{2\bar{q}}{k}\cdot x$$

Superimposing Well Flow and Groundwater Flow

Well flow (line source) $z^2 = H^2 - \frac{q}{\pi k} \ln \frac{r_2}{r_1}$

Groundwater flow $z^2 = H^2 + \frac{2\bar{q}}{k} \cdot x$

Combined flow $z^2 = 2H^2 - \frac{q}{\pi k} \ln \frac{r_2}{r_1} + \frac{2\bar{q}}{k} \cdot x$ + const.

when $x = 0$, $r_2 = r_1$, $z^2 = H^2$

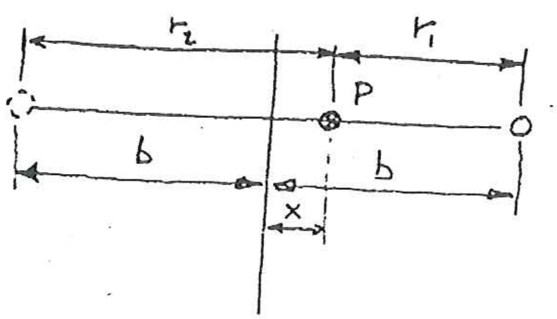
$$H^2 = 2H^2 - 0 + 0 + \text{const.}$$

$$\text{const.} = -H^2$$

$$z^2 = H^2 - \frac{q}{\pi k} \ln \frac{r_2}{r_1} + \frac{2\bar{q}}{k} x$$

How much could we pump without pumping water from the river into our well?
We could pump until the phreatic surface became horizontal at the river bank.

On a line perpendicular to the river bank:



$r_1 = b - x$

$r_2 = b + x$

For points on this line, like point P

$$z^2 = H^2 + \frac{2\bar{q}}{K} \cdot x - \frac{q}{\pi K} \ln\left(\frac{b+x}{b-x}\right)$$

we can find the slope by differentiating w.r.t x

$$2z\frac{dz}{dx} = 0 + \frac{2\bar{q}}{K} - \frac{q}{\pi K}\frac{d}{dx}\left[\ln(b+x) - \ln(b-x)\right]$$

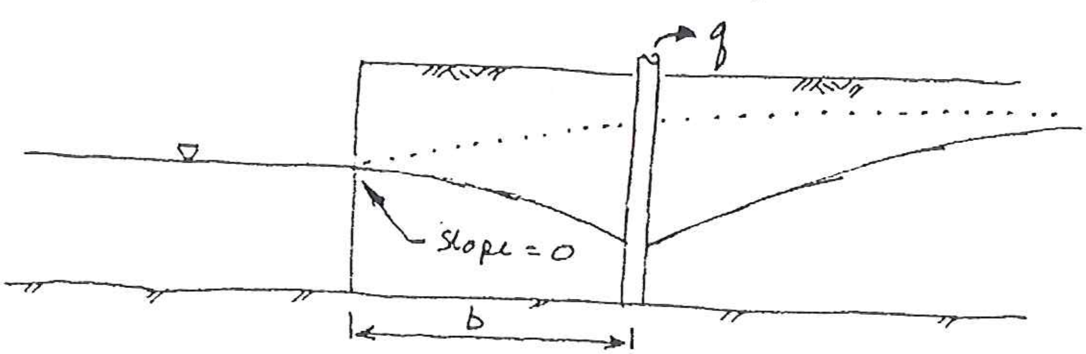
$$2z\frac{dz}{dx} = \frac{2\bar{q}}{K} - \frac{q}{\pi K}\left[\frac{1}{b+x} - \frac{1}{b-x}\right]$$

$$2z\frac{dz}{dx} = \frac{2\bar{q}}{K} - \frac{q}{\pi K}\left[\frac{2b}{b^2-x^2}\right]$$

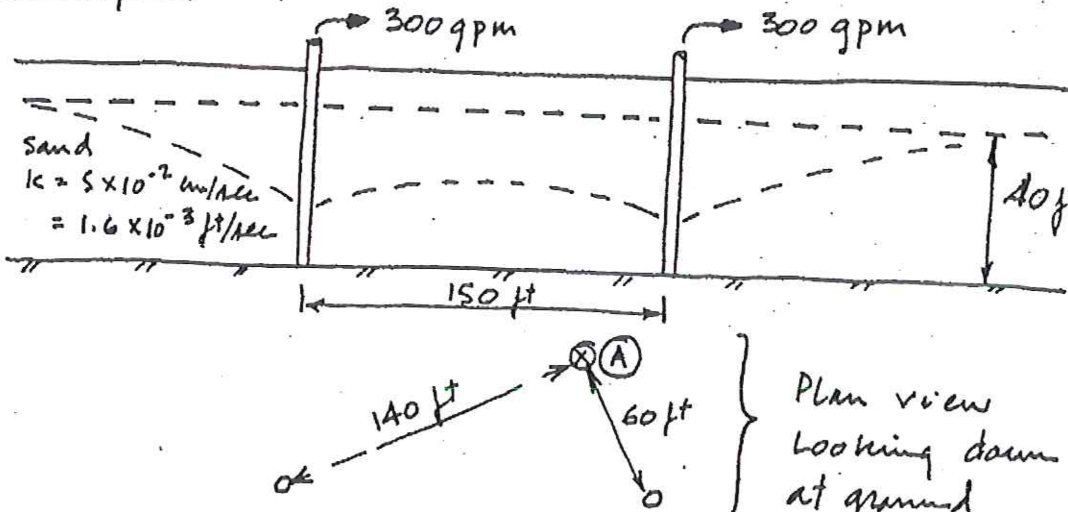
$dz/dx = 0$ @ $x = 0$, so $0 = \frac{2\bar{q}}{K} - \frac{q}{\pi K}\left[\frac{2b}{b^2}\right]$

$$\frac{q}{\pi \cancel{K}}\left(\frac{2}{b}\right) = \frac{2\bar{q}}{\cancel{K}}$$

$q = \pi b \bar{q}$ is the maximum amount of water we can pump without getting river water



Multiple Well - Gravity Flow Example



Calculate Z at (A)

$$Z^2 = H^2 - \sum_{i=1}^{n} \frac{q_i}{\pi k} \ln \frac{R}{r_i}$$

assumed R

$$Z_A^2 = (40)^2 - \frac{(300/450)}{\pi(1.6\times10^{-3})}\left[\ln\frac{2000}{140} + \ln\frac{2000}{60}\right]$$

$$Z_A^2 = 1600 - (133)(2.66 + 3.51) = 782 \text{ ft}^2$$

$$Z_A = 28.0 \text{ ft}$$

(with $R = 3000$ ft, $Z_A = 25.9$ ft)