CHAPTER 17

SINGLE PILES: DYNAMIC ANALYSIS, LOAD TESTS

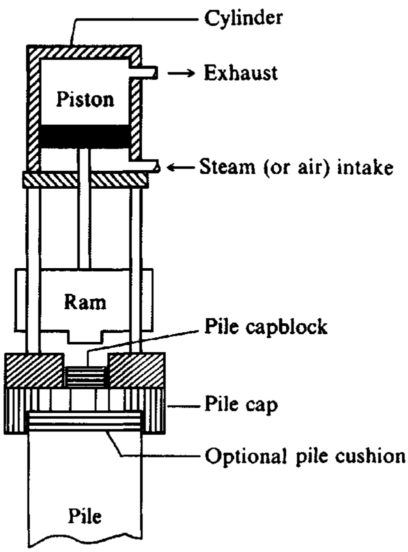
17-1 DYNAMIC ANALYSIS

Estimating the ultimate capacity of a pile while it is being driven into the ground at the site has resulted in numerous equations being presented to the engineering profession. Unfortunately, none of the equations is consistently reliable or reliable over an extended range of pile capacity. Because of this, the best means for predicting pile capacity by dynamic means consists in driving a pile, recording the driving history, and load testing the pile. It would be reasonable to assume that other piles with a similar driving history at that site would develop approximately the same load capacity. This chapter will examine some of the driving equations, the load test, and some of the numerous reasons why dynamic pile prediction is so poor. Some of the field problems associated with pile driving such as splicing, redriving, and heave will also be briefly examined. A brief introduction to the *wave equation* method of dynamic analysis will also be presented.

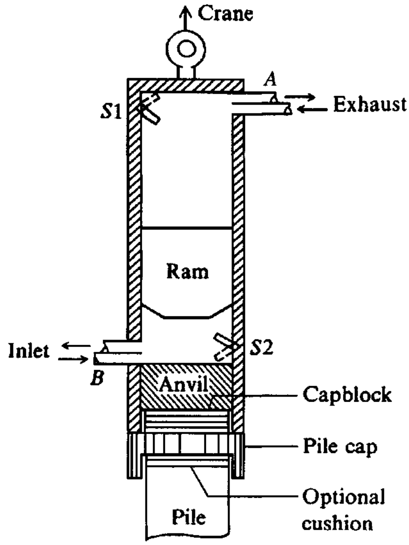
Probably one of the best sources of practical considerations in pile driving is given by Hal Hunt, *Design and Installation of Driven Pile Foundations,* published by the Associated Pile and Fitting Corp., Clifton, NJ, 1979 (217 pages).

17-2 PILE DRIVING

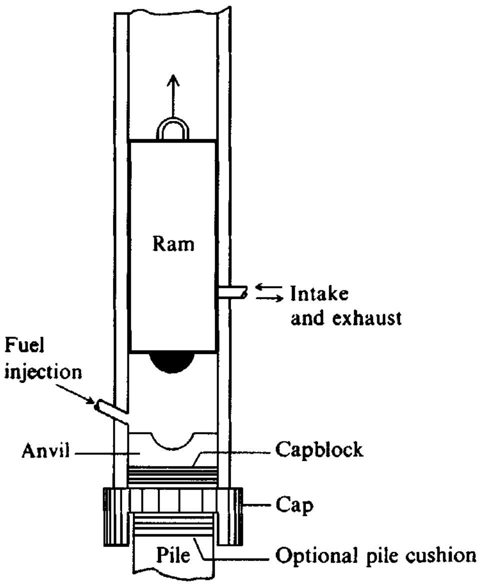
Piles are inserted into the ground using a pile hammer resting on or clamped to the top of the pile cap, which is, in turn, connected to the pile. The pile may contain a capblock between the cap and hammer as shown in Fig. 17-1. The cap usually rests on the pile and may be of, or contain, adequate geometry to effect a reasonably close fit. A pile cushion is sometimes used between the cap and pile (particularly concrete piles) to make the hammer impulses produce a more uniform driving pressure across the pile cross section. The pile and hammer are aligned vertically using leads suspended by a crane-type device except for the vibratory



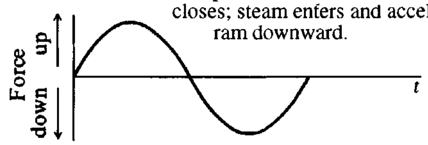
(*a*) Single-acting hammer. At bottom of stroke, intake opens with steam pressure raising ram. At top of lift steam is shut off and intake becomes exhaust, allowing ram to fall.

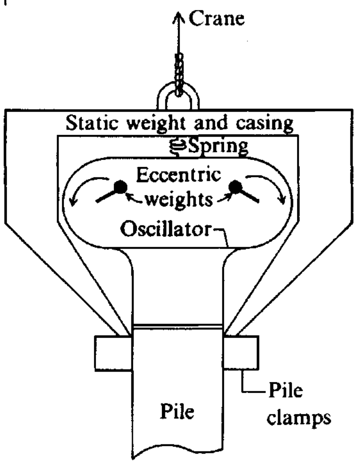


(*b*) Double-acting hammer. Ram in down position trips $S2$, which opens inlet and closes exhaust valves at B and shuts inlet and opens exhaust at A; hammer then rises from steam pressure at B. Ram in up position trips $S1$, which shuts inlet B and opens exhaust; valve A exhaust closes; steam enters and accelerates ram downward.



(*c*) Diesel hammer. Crane initially lifts ram. Ram is released and falls; at select point fuel is injected. Ram collides with anvil, igniting fuel. Resulting explosion drives pile and lifts ram for next cycle.





(*d*) Vibratory hammer. External power source (electric motor or electric-driven hydraulic pump) rotates eccentric weights in relative directions shown. Horizontal force components cancel—vertical force components add.

Figure 17-1 Schematics of several pile hammers.

hammers, which normally do not use leads. Piles may also be inserted by jetting or partial augering.

Leads provide free travel of the hammer as the pile penetrates the soil and are on the order of 6 m longer than the pile to provide adequate space for the hammer and other appurtenances.

Mandrels are used to assist in driving pipe piles. These devices fit inside the pipe and rest on the baseplate when the pipe is closed-end; they become the pile point for open-end piles. The mandrel becomes the driving element, which basically drags the pipe down with it during driving so that the thin pipe shell is not damaged.

Spuds are sometimes used in pile-driving operations to penetrate hard strata or seat the pile in rock. The spud may be a separate driving device or simply a massive point attached to the pile, especially for **HP** piles seated into rock. Seating a driven pile into sloping rock is a difficult task as the pile may tend to follow the rock slope. This tendency may not be readily detected without a load test. Special driving points may be required to assist in seating the point adequately into the rock slope.

Pile hammers are the devices used to impart sufficient energy to the pile so that it penetrates the soil. Several pile hammers are described in the following paragraphs.

Drop Hammers

Drop hammers are still occasionally used for small, relatively inaccessible jobs. The drop hammer consists of a metal weight fitted with a lifting hook and guides for traveling down the leads (or guides) with reasonable freedom and alignment. The hook is connected to a cable, which fits over a sheave block and is connected to a hoisting drum. The weight is lifted and tripped, freely falling to a collision with the pile. The impact drives the pile into the ground. Principal disadvantages are the slow rate of blows and length of leads required during the early driving to obtain a sufficient height of fall to drive the pile.

Single-Acting Hammers

Single-acting hammers are idealized in Fig. 17-1*a*. Steam or air pressure is used to lift the ram to the necessary height. The ram then drops by gravity onto the anvil, which transmits the impact energy to the capblock, thence to the pile. The hammer is characterized by a relatively slow rate of blows. The hammer length must be such as to obtain a reasonable impact velocity (h or height of ram fall), or else the driving energy will be small. The blow rate is considerably higher than that of the drop hammer. In general the ratio of ram weight to pile weight including appurtenances should be on the order of 0.5 to 1.0. Table A-2 in the Appendix gives typical lengths and other useful data.

Double-Acting Hammers

These hammers (Fig. 17-1*b*) use steam both to lift the ram and to accelerate it downward. Differential-acting hammers are quite similar except that more control over the steam (or air) is exerted to maintain an essentially constant pressure (nonexpansion) on the accelerating side of the ram piston. This increase in pressure results in a greater energy output per blow than with the conventional double-acting hammer. The blow rate and energy output are usually higher for double-acting or differential hammers (at least for the same ram weight), but steam consumption is also higher than for the single-acting hammer. The length may be a meter or more shorter for the double-acting hammer than for the single-acting hammer with length ranges on the order of 2 to 4.5 m. The ratio of ram weight to pile weight should be between 0.50 and 1.

When compressed air instead of steam is used with single- or double-acting hammers, there is the additional problem of the system icing up at temperatures close to freezing.

Diesel Hammers

Diesel hammers (Fig. 17-1*c*) consist of a cylinder or casing, ram, anvil block, and simple fuel injection system. To start the operation, the ram is raised in the field as fuel is injected near the anvil block, then the ram is released. As the ram falls, the air and fuel compress and become hot because of the compression; when the ram is near the anvil, the heat is sufficient to ignite the air-fuel mixture. The resulting explosion (1) advances the pile and (2) lifts the ram. If the pile advance is very great as in soft soils, the ram is not lifted by the explosion sufficiently to ignite the air-fuel mixture on the next cycle, requiring that the ram be again manually lifted. It is thus evident that the hammer works most efficiently in hard soils or where the penetration is quite low (point-bearing piles when rock or hardpan is encountered) because maximum ram lift will be obtained.

Diesel hammers are highly mobile, have low fuel consumption (on the order of 4 to 16 L/hr), are lighter than steam hammers, and operate efficiently in temperatures as low as 0°C. There is no need for a steam or air supply generation unit and the resulting hoses. The diesel hammer has a length varying from about 3.5 to 8.2 m (4.5 to 6 m average). The ratio of ram weight to pile weight should be on the order of 0.25 to 1.0.

Jetting or Preaugering

A water jet is sometimes used to assist in inserting the pile into the ground. That is, a high-pressure stream of water is applied at the pile point to displace the soil. This method may be used to loosen sand or small gravel where for some reason the pile must penetrate to a greater depth in the material than necessary for point bearing. Care must be exercised that the jetting does not lower the point-bearing value. Some additional driving after the jet is halted should ensure seating the point on firm soil.

Preaugering is also sometimes used where a firm upper stratum overlies a compressible stratum, which in turn overlies firmer material into which it is desired to seat the pile point. Preaugering will reduce the driving effort through the upper firm material.

For both jetting and preaugering, considerable engineering judgment is required to model the dynamic pile capacity equations (and static equations) to the field system.

Pile Extraction

Piles may be pulled for inspection for driving damage. Sudden increases of penetration rate may be an indication of broken or badly bent piles. Pile *extractors* are devices specifically fabricated for pulling piles. Double-acting steam hammers may be turned upside down and suitably attached to the pile for the driving impulse and to a hoisting device (crane) to apply a pull at least equal to the weight of the hammer and pile. The hammer impacts loosen and lift the pile, and the crane provides a constant pull to hoist it from the hole. The lower broken part of a wooden pile (metal piles seldom break) is usually left in place, but may cause further driving problems.

Vibratory Drivers

Since about 1949 vibratory drivers have been used to insert piles. The principle of the vibratory driver is two counterrotating eccentric weights (Fig. 17-1*d*). The frequency (ranging from 0 to about 20 Hz) is readily computed using equations given in Chap. 20. The driver provides two vertical impulses of as much as 700^+ kN at amplitudes of 6 to 50 mm each revolution—one up and one down. The downward pulse acts with the pile weight to increase the apparent gravity force. The pile insertion (also for terraprobing) is accomplished by

1. The push-pull of the counterrotating weights— push (+pile weight) > pull upward
2. The conversion of the soil in the immediate vicinity of the pile to a viscous fluid

Best results using vibratory driving are obtained in cohesionless deposits. Results are fairly good in silty and clayey deposits. Impulse hammers are used in heavy clays or soils with appreciable numbers of boulders.

Three principal advantages of the vibratory driver (where soils are compatible) are these:

1. Reduced driving vibrations—the vibrations are not eliminated but they are less than using impact drivers.
2. Reduced noise.
3. Great speed of penetration—penetration rates of 50^+ mm/s are possible.

At present the ultimate pile capacity P_u for vibration-driven piles can only be estimated using static pile methods, although Davisson (1970) developed an equation that purports to estimate the capacity of the patented Bodine Resonant Driver (BRD) used principally by Raymond Concrete Pile company. Other vibratory drivers currently used include the patented vibro driver of the L. B. Foster company and a hydraulic-powered device available from McKiernan-Terry Corporation. The BRD equation (but not for tip on rock) is

$$P_u = \frac{A(\text{hp}) + Br_p}{r_p + \Omega \times S_L} \quad \text{(lb or kN)} \tag{17-1}$$

A = 550 ft · lb/s (Fps); 0.746 kJ/s (SI)
B = hammer weight, 22 000 lb in Fps; 98 kN in SI for Bodine hammers
r_p = final rate of penetration, m/s or ft/s
Ω = frequency, Hz
S_L = loss factor, ft/cycle or m/cycle (see table following)
hp = horsepower delivered to the pile

	Loss factor for:	
	Closed-end pipe	**HP piles**
Soil at pipe tip	**m/cycle × 10^{-3} (ft/cycle)**	
Loose silt, sand, or gravel	0.244 (0.0008)	−0.213 (−0.0007)
Medium dense sand or sand and gravel	0.762 (0.0025)	0.762 (0.0025)
Dense sand or sand and gravel	2.438 (0.008)	2.134 (0.007)

Example 17-1. Use the BRD equation to estimate the dynamic pile capacity on p. 12 of *Foundation Facts* [the page following the Davisson (1970) reference]:

hp = 414   Final penetration r_p = 240 s/ft = 787.4 s/m = 0.001 27 m/s

Closed-end pipe pile 325 × 4.54 mm wall approximately 30.5 m long and filled with concrete after driving. Soil is dense coarse sand and gravel (based on SPT blow count); thus, $S_L = 2.44 \times 10^{-3}$ m/cycle from table, $\Omega = 126$ Hz.

Substituting, and with Bodine driver, we find

$$P_u = \frac{0.746(414) + 98(0.001\,27)}{0.001\,27 + 126(0.002\,44)} = 1000 \text{ kN}$$

The load test (pipe filled with concrete) indicated $P_u = 2450$ kN. The pile insertion was terminated nearly on rock for which no S_L was given, and one may debate if that action affects the foregoing results. In pile driving, however, piles are often driven until the point reaches approximate refusal—this practice will always affect the final penetration rate used in Eq. (17-1). It is expected that the computed capacity of friction piles compared to load tests might be in closer agreement.

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17-3 THE RATIONAL PILE FORMULA

Dynamic formulas have been widely used to predict pile capacity. Some means is needed in the field to determine when a pile has reached a satisfactory bearing value other than by simply driving it to some predetermined depth. Driving the pile to a predetermined depth may or may not obtain the required bearing value because of normal soil variations both laterally and vertically.

It is generally accepted that the dynamic formulas do not provide very reliable predictions. Predictions tend to improve by using a load test in conjunction with the equation to adjust the input variables. Predictions by persons with experience in a given area and using certain equipment and with a good knowledge of the input variables of weights, etc., are often considerably better than many of the predictions found in the literature where authors use the reported results of other writers in statistical types of analyses.

The basic dynamic pile capacity formula, termed the *rational pile formula,* will be derived in the following material. Nearly all the dynamic pile formulas currently used are based on this equation—generally by simplifying certain terms. The rational pile formula depends upon impulse-momentum principles.

For the derivation of the rational pile formulas, refer to Fig. 17-2 and the following list of symbols. Applicable symbols from this list are used also with the several pile formulas of the next section and in Table 17-1. The units for the symbol are in parentheses; e.g., (FTL) is the product of variables with units of force, time, and length.

A = pile cross-sectional area (L^2)
E = modulus of elasticity (FL^{-2})
e_h = hammer efficiency
E_h = manufacturer's hammer-energy rating (FL)
g = acceleration of gravity (LT^{-2})
h = height of all of ram (L)

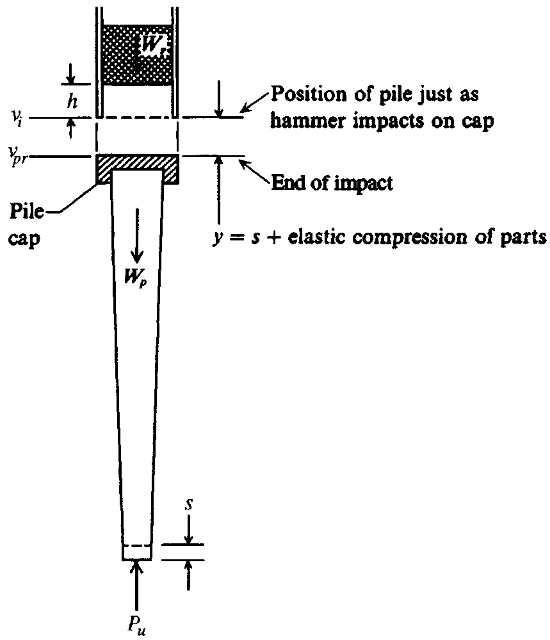

Figure 17-2 Significance of certain terms used in the dynamic pile-driving equations.

- I = amount of impulse causing compression or change in momentum (FT)
- k_1 = elastic compression of capblock and pile cap and is a form of P_uL/AE (L)
- k_2 = elastic compression of pile and is of a form P_uL/AE (L)
- k_3 = elastic compression of soil, also termed *quake* for wave equation analysis (L)
- L = pile length (L)
- m = mass (weight/g) (FT^2L^{-1})
- M_r = ram momentum = $m_r v_i$ (FT)
- n = coefficient of restitution
- nI = amount of impulse causing restitution (FT)
- P_u = ultimate pile capacity (F)
- s = amount of point penetration per blow (L)
- v_{bc} = velocity of pile and ram at end of compression period (LT^{-1})
- v_i = velocity of ram at the moment of impact (LT^{-1})
- v_{pr} = velocity of pile at the end of period of restitution (LT^{-1})
- v_{rr} = velocity of ram at the end of the period of restitution (LT^{-1})
- W_p = weight of pile including weight of pile cap, all or part of the soil "plug," driving shoe, and capblock (also includes anvil for double-acting steam hammers) (F)
- W_r = weight of ram (for double-acting hammers include weight of casing) (F)

At impact, the ram momentum is

$$M_r = \frac{W_r v_i}{g}$$

TABLE 17-1

Several dynamic pile formulas (use any consistent set of units)

Many (of the more progressive) building codes no longer specify the pile-driving equation(s) to use to estimate pile capacity. A suitable equation is left to the designer (who may have to justify it to the local building official). Several other dynamic formulae are given in Young (1981).

Canadian National Building Code (use SF = 3) as used in Table 17-5 but C_3 simplified to that shown here

$$P_u = \frac{e_h E_h C_1}{s + C_2 C_3} \qquad C_1 = \frac{W_r + n^2(0.5W_p)}{W_r + W_p}$$

$$C_2 = \frac{3P_u}{2A} \qquad C_3 = \frac{L}{E} + C_4$$

$$C_4 = 0.0001 \text{ in.}^3/\text{k (Fps)}$$
$$= 3.7 \times 10^{-10}\text{m}^3/\text{kN (SI)}$$

Note that product of $C_2 C_3$ gives units of s.

Danish formula [Olson and Flaate (1967)] (use SF = 3 to 6)

$$P_u = \frac{e_h E_h}{s + C_1} \qquad C_1 = \sqrt{\frac{e_h E_h L}{2AE}} \quad \text{(units of } s\text{)}$$

Eytelwein formula (use SF = 6) [Chellis (1961)]

$$P_u = \frac{e_h E_h}{s + C(W_p/W_r)} \qquad C = 2.5 \text{ mm} = 0.1 \text{ in.}$$

Gates formula [Gates (1957)] (use SF = 3)

$$P_u = a\sqrt{e_h E_h}(b - \log s)$$

P_u = kips or kN E_h = kips · ft or kN · m

	s	a	b
Fps	in.	27	1.0
SI	mm	104.5	2.4

e_h = 0.75 for drop and 0.85 for all other hammers

Janbu [see Olson and Flaate (1967), Mansur and Hunter (1970)] (use SF = 3 to 6)

$$P_u = \frac{e_h E_h}{k_u s} \qquad C_d = 0.75 + 0.15\frac{W_p}{W_r}$$

$$K_u = C_d\left(1 + \sqrt{1 + \frac{\lambda}{C_d}}\right) \qquad \lambda = \frac{e_h E_h L}{AEs^2}$$

Use consistent units to compute P_u. There is some disagreement of using e_h since it appears to be in C_d; however, a better statistical fit tends to be obtained by using e_h as shown.

TABLE 17-1
Several dynamic pile formulas (use any consistent set of units) *(continued)*

Modified ENR [ENR (1965)] formula (use SF = 6)

$$P_u = \left[\frac{1.25 e_h E_h}{s + C}\right]\left[\frac{W_r + n^2 W_p}{W_r + W_p}\right] \qquad C = 2.5 \text{ mm} = 0.1 \text{ in.}$$

AASHTO [(1990)[1]; Sec. 3.6.2 p. 251] $P_u \leq 1$ and SF = 6; primarily for timber piles]

$$P_u = \frac{2h(W_r + A_r p)}{s + C} \qquad C = 2.5 \text{ mm} = 0.1 \text{ in.}$$

For double-acting steam hammers take A_r = ram cross-sectional area and p = steam (or air) pressure; for single-acting and gravity, $A_r p = 0$. Use consistent units. Take $e_h \cong 1.0$. The above or other formulas may be used for steel and concrete piles. Set s = penetration of last 10 to 20 blows for steam hammers.

Navy-McKay formula (use SF = 6)

$$P_u = \frac{e_h E_h}{s(1 + 0.3C_1)} \qquad C_1 = \frac{W_p}{W_r}$$

Pacific Coast Uniform Building Code (PCUBC) (from Uniform Building Code,[2] Chap. 28) (use SF = 4)

$$P_u = \frac{e_h E_h C_1}{s + C_2} \qquad C_1 = \frac{W_r + kW_p}{W_r + W_p}$$

k = 0.25 for steel piles
= 0.10 for all other piles

$$C_2 = \frac{P_u L}{AE} \quad \text{(units of } s\text{)}$$

In general start with C_2 = 0.0 and compute value of P_u; reduce value by 25 percent; compute C_2 and a new value of P_u. Use this value of P_u to compute a new C_2, etc. until P_u used $\cong$ P_u computed.

[1]AASHTO (1990) allows any Department of Transportation–approved pile formula in addition to this one.
[2]Not in 1976 and later UBC editions; it can still be used, just not in code.

At the end of the compression period the ram momentum is

$$M_r = \frac{W_r v_i}{g} - I$$

with a velocity of

$$v_{bc} = \left(\frac{W_r v_i}{g} - I\right)\frac{g}{W_r} \tag{a}$$

If we assume at this instant the pile momentum $M_p = I$, the pile velocity is

$$v_{bc} = \frac{g}{W_p} I \tag{b}$$

Next, if we assume that the pile and ram have not separated at the end of the compression period, the instantaneous velocities of the pile and ram are equal; therefore, combining equations (*a*) and (*b*), we have

$$I = v_i \frac{W_r W_p}{g(W_r + W_p)} \tag{c}$$

At the end of the period of restitution, the momentum of the pile is

$$I + nI = \frac{W_p}{g} v_{pr} \qquad (d)$$

and substituting Eq. (*c*) for I and solving for the pile velocity, we see that

$$v_{pr} = \frac{W_r + nW_r}{W_r + W_p} v_i \qquad (e)$$

At the end of the period of restitution, the momentum of the ram is

$$\frac{W_r v_i}{g} - I - nI = \frac{W_r v_{rr}}{g} \qquad (f)$$

Substituting for I and solving for v_{rr}, we obtain

$$v_{rr} = \frac{W_r - nW_p}{W_r + W_p} v_i \qquad (g)$$

The total energy available in the pile and ram at the end of the period of restitution is

$$(\tfrac{1}{2} m v_{pr}^2)_{\text{pile}} + (\tfrac{1}{2} m v_{rr}^2)_{\text{ram}}$$

and substituting (*e*) for v_{pr} and (*g*) for v_{rr} and with some simplification one obtains

$$\frac{W_r}{2g} v_{rr}^2 + \frac{W_p}{2g} v_{pr}^2 = e_h W_r h \frac{W_r + n^2 W_p}{W_r + W_p} \qquad (h)$$

If the system were 100 percent efficient, the ultimate load P_u multiplied by the point displacement s should be

$$P_u s = e_h W_r h$$

The instant pile top displacement is $s + k_1 + k_2 + k_3$, of which only s is permanent, and the actual input energy to the pile system is

$$e_h W_r h = P_u(s + k_1 + k_2 + k_3) = P_u(s + C)$$

Replacing the equivalent energy term with the equivalent from equation (*h*), we find

$$P_u = \frac{e_h W_r h}{s + C} \frac{W_r + n^2 W_p}{W_r + W_p} \qquad (i)$$

Cummings (1940) correctly points out that Eq. (*h*) already includes the effects of the losses associated with k_i; however, the form of Eq. (*i*) is generally accepted and used.

The term k_2 can be taken as the elastic compression of the pile $P_u L/AE$ with the corresponding strain energy of $P_u^2 L/2AE$.

Rewriting Eq. (*i*) and factoring out $\frac{1}{2}$ from all the k terms for strain energy, the Hiley (1930)[1] equation is obtained:

$$P_u = \left[\frac{e_h W_r h}{s + \frac{1}{2}(k_1 + k_2 + k_3)}\right]\left[\frac{W_r + n^2 W_p}{W_r + W_p}\right] \qquad (17\text{-}2)$$

[1]Cummings (1940) indicates that Redtenbacher (ca. 1859) may be the originator of this equation.

For double-acting or differential steam hammers, Chellis (1941, 1961) suggested the following form of the Hiley equation:

$$P_u = \left[\frac{e_h E_h}{s + \frac{1}{2}(k_1 + k_2 + k_3)}\right]\left[\frac{W + n^2 W_p}{W + W_p}\right] \qquad (17\text{-}3)$$

According to Chellis, the manufacturer's energy rating of E_h is based on an equivalent hammer weight term W and height of ram fall h as follows:

$$E_h = Wh = (W_r + \text{weight of casing})h$$

Inspection of the derivation of the Hiley equation indicates the energy loss fraction should be modified to W as shown in Eq. (17-3) also.

A careful inspection of the Hiley equation or Eq. (i), together with a separation of terms, results in

$$\text{Energy in} = \text{work} + \text{impact loss} + \text{cap loss} + \text{pile loss} + \text{soil loss}$$

$$e_h W_r h = P_u s + e_h W h \frac{W_p(1 - n^2)}{W_p + W_r} + P_u k_1 + P_u k_2 + P_u k_3$$

Best results from the dynamic formula as a pile capacity prediction tool are obtained when a careful and separate assessment is made of the several loss factors.

There may be some question of the correctness of computing the strain energy k_2 based on a gradually applied P_u as $P_u^2 L/2AE$ when an impulse-type load is actually applied for which the strain energy is $P_u^2 L/AE$. Use of the given equation form seems to give an adequate estimate of the ultimate pile capacity; however, we might note that the k_2 term would not produce a great difference in P_u whether used as k_2 or the more correct value of $k_2/2$.

It is necessary to use consistent units in Eqs. (17-2) and (17-3) so that the value of P_u is obtained in the force units contained in W_r. For example, if h = ft and s = in., it is necessary to multiply by 12; if h = m and s = mm, it is necessary to multiply by 1000 to obtain the correct value of P_u.

17-4 OTHER DYNAMIC FORMULAS AND GENERAL CONSIDERATIONS

All of the dynamic pile-driving formulas except the Gates formula shown in Table 17-1 are derived from Eq. (17-2) or (17-3) by using various assumptions. The assumptions usually reflect the author's personal experiences and/or attempts to simplify the equation for practical use. Since interpretation of user experience is highly subjective and coupled with wide variability of soils and hammer conditions, the dynamic formulas do not have very good correlation with field experience—especially when used by others in different geographical areas or for statistical comparisons. Statistical comparisons are especially difficult owing to the scarcity of realistic input into the equations of hammer efficiencies, and weights of hammer and driving equipment such as caps, capblocks, and driving points and any soil "plug." For example, Chellis (1961) suggested that pile tips founded on rock or relatively impenetrable material should use a value for pile weight of $W_p/2$. This can make some, even considerable, difference in the loss factor. Also, where is the breakpoint for the factor 2? It would appear that for medium dense materials a factor of 0.75 might be used, gradually increasing to 1.00 for friction piles. Likewise, if the user does not adjust the Hiley equation to include correctly the ram and/or applicable portions of casing and anvil weights, considerable discrepancies can result. Finally, the equations are heavily dependent on hammer efficiency, which must be estimated and which can change during driving operations on the same job.

If we define the impact term in the Hiley equation as

$$C_1 = \frac{W_r + n^2 W_p}{W_r + W_p}$$

and rearrange it to

$$C_1 = \frac{1 + n^2 W_r/W_p}{1 + W_r/W_p}$$

and take $n^2 W_r/W_p \cong 0$, we obtain

$$C_1 = \frac{1}{1 + W_r/W_p}$$

which becomes the starting point for the several formula factors.

The *Engineering News* (commonly, but incorrectly termed the ENR) formula was published in the *Engineering News* ca. 1888 (which merged with McGraw-Hill in 1917 to become the *Engineering News-Record*) and was developed for wood piles using a drop hammer with an approximate safety factor (SF) of 6. The formula has been modified for different driving equipment and is probably the most used of the several "dynamic" pile formulas. It was obtained by lumping all the elastic compression into a single factor $C = \mathbf{25}$ mm (1 in.) with $C_1 = \mathbf{1}$ to obtain for drop hammers (length units of s and h must be the same)

$$P_u = \frac{e_h W_r h}{s + 25} \tag{17-4}$$

and for steam hammers with $C = \mathbf{2.54}$ mm (0.1 in.) obtain

$$P_u = \frac{e_h W_r h}{s + 2.54} \tag{17-5}$$

Equations (17-4) and (17-5) will be called the ENR formulas.[2] A more recent ENR modification (and approximately as used in Table 17-5) is

$$P_u = \left(\frac{e_h W_r h}{s + C}\right)\left(\frac{W_r + n^2 W_p}{W_r + W_p}\right) \tag{17-6}$$

Values of k_1 for use in Eq. (17-2) or (17-3) are presented in Table 17-2. Values of hammer efficiency depend on the condition of the hammer and capblock and possibly the soil (especially for diesel hammers). In the absence of known values the following may be taken as representative of hammers in reasonably good operating condition:

Type	Efficiency e_h
Drop hammers	0.75–1.00
Single-acting hammers	0.75–0.85
Double-acting or differential	0.85
Diesel hammers	0.85–1.00

[2]The author will refer to these formulas as the ENR since this is its commonly used designation in nearly all of the technical literature on pile driving.

TABLE 17-2
Values for k_1—temporary elastic compression of pile head and cap*
For driving stresses larger than 14 MPa use k_1 in last column

Pile material	Driving stresses P/A on pile head or cap, MPa (ksi)			
	3.5 (0.5)	7.0 (1.0)	10.5 (1.5)	14 (2.0)
	k_1, mm (in.)			
Steel piling or pipe				
Directly on head	0	0	0	0
Directly on head of timber pile	1.0 (0.05)	2.0 (0.10)	3.0 (0.15)	5.0 (0.20)
Precast concrete pile with 75–100 mm packing inside cap	3.0 (0.12)	6.0 (0.25)	9.0 (0.37)	12.5 (0.50)
Steel-covered cap containing wood packing for steel **HP** or pipe piling	1.0 (0.04)	2.0 (0.05)	3.0 (0.12)	4.0 (0.16)
5-mm fiber disk between two 10-mm steel plates	0.5 (0.02)	1.0 (0.04)	1.5 (0.06)	2.0 (0.08)

*After Chellis (1961).

Chellis (1961) suggested increasing the efficiency 10 percent when using Eq. (17-2) or (17-3) to compute the driving stresses. Since the reliability of the equations is already with considerable scatter both (+) and (−), it does not appear necessary to make this adjustment.

Table 17-3 presents representative values of the coefficient of restitution n. Again the actual value will depend upon the type and condition of the capblock material and whether a pile cushion is used with concrete piles.

The term k_2 is computed as P_uL/AE, and one may arbitrarily take the k_3 term (quake) as

$$k_3 = 0.0 \text{ for hard soil (rock, very dense sand, and gravels)}$$
$$= 2.5 \text{ to } 5 \text{ mm } (0.1 \text{ to } 0.2 \text{ in.})$$

Equation (17-2) and following must be adjusted when piles are driven on a batter. It will be necessary to compute the axial pile component of W_rh and further reduce this for the friction lost due to the normal component of the pile hammer on the leads or guide. A reasonable estimate of the friction coefficient f between hammer and leads may be taken as

$$f = \tan\theta = 0.10$$

TABLE 17-3
Representative values of coefficient of restitution for use in the dynamic pile-driving equations*

Material	n
Broomed wood	0
Wood piles (nondeteriorated end)	0.25
Compact wood cushion on steel pile	0.32
Compact wood cushion over steel pile	0.40
Steel-on-steel anvil on either steel or concrete pile	0.50
Cast-iron hammer on concrete pile without cap	0.40

*After ASCE (1941).

For small wood piles on the order of 100 to 150 mm used to support small buildings on soil with a water table at or very near the ground surface Yttrup et al. (1989) suggest using

$$P_u = \frac{0.4Wh}{s} \tag{17-7}$$

in kN when W = kN; h, s = m. This formula is applicable for drop hammers mounted on small conventional tractors.

PLUG WEIGHT. Open-end pipe piles always cut a soil plug. The plug usually does not fill the pipe when observed from above since it is much compressed both from vibration and from side friction on the interior walls. The plug weight can be estimated as

$$W_{\text{plug}} = \gamma' \times V_{\text{pipe}} \tag{17-8}$$

where V_{pipe} = internal pipe volume. This weight may be critical when the pile is nearly driven to the required depth since it is a maximum at that time.

HP piles will also have a plug of unknown dimensions; however, it would not be a great error to assume the plug length L_{plug} is one-half the embedded length of the pile (when blow counts are taken for pile capacity or for penetration resistance). The plug weight (refer also to Fig. 16-11*c*) in this case is

$$W_{\text{plug}} = 0.50L_{\text{pile}} \times b_f \times d \times \gamma' \tag{17-8a}$$

Equation (17-8*a*) includes the web t_w and flange thickness t_f in the soil volume but the plug length is an estimate, so the computation as shown is adequate.

Use effective unit weight γ' for the soil, as the water will have a flotation effect for both the soil and the pile.

The "pile" weight should be the actual weight W_p plus plug, or

$$W_p = W_p + W_{\text{plug}} \tag{17-9}$$

for use in any of the equations given that uses a pile weight term W_p.

The plug weight was not included in the past because few persons ever checked the derivation of the equations to see how the pile weight term was treated. Do not include the plug weight unless the equation you are using includes the pile weight in a term similar to the second term in the Hiley equation.

Example 17-2. Estimate the allowable pile capacity of test pile No. 1 reported by Mansur and Hunter (1970, Tables 2, 4, 5, and 6) by the *ENR, Janbu, Gates,* and *Hiley* equations (see Table 17-1) and Eq. (17-3). The data have been converted to SI for this edition. (The example in Fps is in the previous edition.)

Other data:

Hammer = Vulcan 140C	W_r = 62.3 kN (Table A-2 of Appendix)	
Hammer E_h = 48.8 kN · m	e_h = 0.78 (efficiency table, this section)	
Pile = 305 mm pipe	A = 11 045 mm^2 (incl. instrumentation)	
Pile L_p = 16.76 m	E = 200 000 MPa	γ_{st} = 77.0 kN/m^3

Pile set s = 305/16 = 19 mm/blow (given in reference)

Pile cap + capblock = 7.61 kN

Pile driven closed end—no plug

Load test: P_u = **1245.4** kN

Solution.

a. By the *ENR* equation [Eq. (17-5)] and using SF = 6:
Make a direct substitution:

$$P_{ult} = \frac{e_h W_r h}{s + 2.54} = \frac{0.78 \times 48.8 \times 1000}{19 + 2.54} = \mathbf{1245}\text{ kN}$$

$$P_a = \frac{1245}{6} = \mathbf{295}\text{ kN}$$

b. By the *Janbu* equation (see Table 17-1) and average SF = 4.5:

$$\text{Weight of pile (no plug)} = A_p \times \gamma_{st} \times L_p$$

$$= \frac{11\,045}{10^6} \times 77.0 \times 16.76 = \mathbf{21.86}\text{ kN}$$

$$AE = 11\,045 \times 0.200 = 2209\text{ MN} \qquad \text{(the } 10^6 \text{ terms cancel)}$$

$$C_d = 0.75 + 0.15 \times \frac{W_p}{W_r} = 0.75 + 0.15 \times \frac{21.86}{63.3} = 0.80$$

$$\lambda = \frac{e_h E_h L}{AEs^2} = \frac{0.78 \times 48.8 \times 16.76}{2.209 \times 19^2} = 0.80 \qquad \text{(the } 10^6 \text{ terms cancel)}$$

$$k_u \doteq C_d\left(1 + \sqrt{1 + \frac{\lambda}{C_d}}\right) = 0.80\left(1 + \sqrt{1 + \frac{0.80}{0.80}}\right) = 1.93$$

Making the necessary substitutions, we find

$$P_u = \frac{e_h E_h}{k_u s} = \frac{0.78 \times 48.8}{1.93 \times 0.019} = \mathbf{1038}\text{ kN}$$

$$P_a = \frac{1038}{4.5} = \mathbf{231}\text{ kN}$$

c. By the *Gates* equation (see Table 17-1) with SF = 3:

$$P_u = a\sqrt{e_h E_h}(b - \log s) = 104.5\sqrt{e_h E_h}(2.4 - \log s)$$

Making substitutions, we obtain

$$P_u = 104.5\sqrt{0.78 \times 48.8}\,(2.4 - \log 19) = \mathbf{754}\text{ kN}$$

$$P_a = \frac{754}{3} = \mathbf{251}\text{ kN}$$

d. By the *Hiley* equation [Eq. (17-3)] with SF = 4:

$$P_u = \left[\frac{e_h E_h}{s + \frac{1}{2}(k_1 + k_2 + k_3)}\right]\left[\frac{W + n^2 W_p}{W + W_p}\right] \qquad (17\text{-}3)$$

W = weight of hammer = 125 kN (see Table A-2 of Appendix)
Let us estimate k_1:

$$f_p = \frac{P}{A_p} = \frac{125 \times 10^3}{11\,045} = 11.3\text{ MPa}$$

From Table 17-2 we have

k_1	f_p
3.0	10.5
5.0	14.0

Interpolating, we obtain $k_1 = 3.5$ mm.
The term $k_3 = 2.5$ mm [given in text following Eq. (17-6)]. Then we obtain k_s by trial. As a first trial, assume $P_u = 900$ kN:

$$k_2 = \frac{P_u L}{AE} = \frac{900 \times 16.76}{2209} = 6.8 \text{ mm} \qquad \text{(Note: The } 10^n \text{ terms cancel as used.)}$$

$$s = 19 \text{ mm (set per blow and given)} \qquad n = 0.5 \text{ (Table 17-3)}$$

Substituting values into Eq. (17-3) (1000 converts kN · m to kN · mm), we obtain

$$P_u = \left[\frac{0.78 \times 48.8 \times 1000}{19 + \frac{1}{2}(3.5 + 6.8 + 2.5)}\right]\left[\frac{125 + 0.5^2 \times 21.86}{125 + 21.86}\right]$$

$$= \frac{38\,064}{25.4} \times 0.888 = 1331 \text{ kN} \qquad \text{(rounded)}$$

Since we used $P_u = 900$ kN and computed 1331 kN, we must revise k_s to something between 900 and 1331. Try $P_u = 1260$ and by proportion obtain $k_2 = 6.8 \times 1260/900 = 9.5$ mm; again, substituting, we have

$$P_u = \frac{0.78 \times 48.8 \times 1000}{19 + \frac{1}{2}(3.5 + 9.5 + 2.5)} \times 0.888 = 1264 \text{ kN} \approx 1260 \text{ kN used} \qquad \text{(O. K.)}$$

Use $P_u = \mathbf{1260}$ kN
$P_a = 1260/4 = \mathbf{315}$ kN

Summary.

Method	**P_u, kN**	**P_a, kN**
ENR	1245	295
Janbu	1038	231
Gates	754	251
Hiley	1260	315
Measured	**1245**	

The Gates value of P_a for design would be recommended. It was developed for this range of pile capacities. It does not, however, give the best load test value. Both the ENR and Hiley equations give better values for this case. The ENR and Gates equations have the advantage of simplicity. From this spread of P_u it is evident that one should always use more than one equation to see if there are large differences. The agreement of the ENR and Hiley equations may be as much coincidence as equation accuracy.

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Example 17-3. Estimate the ultimate pile capacity P_u of test pile No. 6 (**HP** pile) from the Mansur and Hunter (1970) reference. Use the *ENR*, *Janbu*, and *PCUBC* equations. The original Fps data were soft-converted to SI by the author. Given:

HP360 × 109(14 × 73) (see Table A-1 of Appendix for pile section data)
Capblock = 5.4 kN (1220 lb) Pile length L = 12.18 m (40 ft)
Hammer: Vulcan 80C $E_h = 33.12$ kN · m $\gamma' = 9.8$ kN/m^3
$W_r = 35.58$ kN (see Table A-2 of Appendix)
Pile weight without plug = 109 × 9.807 × 12.18/1000 = 13.01 kN
Pile weight + capblock = W_p = 13.01 + 5.4 = 18.4 kN
Pile weight *with plug* = 18.4 + 0.5 × 12.18 × 0.346 × 0.371 × 9.8 = 26.2 kN
AE = 3 313 000 kN Take $e_h = 0.84$
Set = 17 blows/ft → 18 mm/blow Load test: **1245** kN

Solution.

a. By the *ENR* equation (Eq. 17-5), we can directly substitute C = 0.1 in. = 2.5 mm = 0.0025 m, s = 18 mm = 0.018 m, to find

$$P_u = \frac{e_h E_h}{s + C} = \frac{0.84 \times 33.12}{0.018 + 0.0025} = \mathbf{1357} \text{ kN} > 1245$$

b. By the *Janbu* equation in Table 17-1 (but we will not use plug), we find

$$C_d = 0.75 + 0.15\frac{W_p}{W_r} = 0.75 + 0.15\frac{18.5}{35.58} = \mathbf{0.83}$$

$$\lambda = \frac{e_h E_h L}{AEs^2} = \frac{0.84 \times 33.12 \times 12.18}{3.313 \times 10^6 \times 0.018^2} = \mathbf{0.316}$$

$$k_u = C_d\left(1 + \sqrt{1 + \frac{\lambda}{C_d}}\right) = 0.83\left(1 + \sqrt{1 + \frac{0.316}{0.83}}\right) = 1.805$$

$$P_u = \frac{e_h E_h}{k_u s} = \frac{0.84 \times 33.12}{1.805 \times 0.018} = \mathbf{856} \text{ kN} < 1245 \text{ measured}$$

c. By the *PCUBC* formula of Table 17-1, and using a pile plug, based on computation methods (*a*) and (*b*), $P_u \approx 900$ kN.

Also use k = 0.25 (from Table 17-1) to find

$$C_2 = \frac{P_u L}{AE} = \frac{900 \times 12.18}{3.313 \times 10^6} = \mathbf{0.00331} \text{ m}$$

$$P_u = \left(\frac{e_h E_h}{s + C_2}\right)\left(\frac{W_r + kW_p}{W_r + W_p}\right) = \left(\frac{0.84 \times 33.12}{0.018 + 0.00331}\right)\left(\frac{35.58 + 0.25 \times 26.2}{35.58 + 26.2}\right)$$

$$= 1305.5 \times 0.682 = \mathbf{890} \text{ kN} < 1245$$

Since the 900 kN assumed is sufficiently close to the 890 kN computed, we will use P_u = 890 kN.

Summary.

	P_u, kN
ENR	1357
Janbu	856
PCUBC	890
Measured	1245

The use of a soil plug for the *PCUBC* formula reduces the computed value from about 960 to 890 but appears (when compared with the other methods) to give a more reasonable value—or at least as good a value as not considering the plug.

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17-5 RELIABILITY OF DYNAMIC PILE-DRIVING FORMULAS

Many attempts have been made to improve the reliability of the dynamic formulas. A most comprehensive pile-testing program was undertaken under the direction of the Michigan State Highway Commission (1965). In this program 88 piles were driven and tested as shown in Table 17-4 using the following hammers in the driving operations:

Vulcan No. 1, 50C and 80C
McKiernan-Terry DE30 and DE40
Raymond 15-M
Link-Belt 312 and 520
Delmag D12 and D22

From using the various dynamic formulas based on pile-load tests this study found that the true safety factors are as indicated in Table 17-5. The table indicates reasonable values for the Gates formula in the 0- to 1800-kN load range (range in which the formula was derived). The modified *Engineering News-Record* [Eq. (17-6)] formula is reasonably valid over the entire range of load tests. It was proposed from these tests that the modified *Engineering News-Record* formula as given in Eq. (17-6) be further modified as shown in Table 17-1. This study also brought to light that the amount of energy actually input to the pile for penetration is considerably different from the manufacturer's rating. The actual energy input was heavily

TABLE 17-4

Summary of piles driven in the Michigan State Highway Commission (1965) test program

Pile type	Dimensions, mm	Weight kN/m	Manufactured by	Approx. length range, m	Number driven
HP sections CBP124 (**HP** 12 × 53)	305 flange	0.773	US Steel	13.4–26.8	48
305mm OD pipe piles (mandrel-driven)	6.35 wall 5.84 wall 4.55 wall	0.458 0.433 0.330	Armco	13.4–54.3	16 6 11
Monotube piles, fluted tapered, F 12-7 (9.1 m taper section) and an N 12-7 entension	305 nominal	F 0.286 N 0.358	Union Metal Manufacturing Co.	16.8–24.4	5
Step-taper shell with 2.4 m sections	241 OD tip	Varies	Raymond International	17.7–20.4	2

TABLE 17-5
Summary of safety factor range for equations used in the Michigan Pile Test Program

	Upper and lower limits of SF = P_u/P_d* Range of P_u, kips		
Formula	**0 to 900**	**900 to 1800**	**1800 to 3100**
Engineering News	1.1–2.4	0.9–2.1	1.2–2.7
Hiley	1.1–4.2	3.0–6.5	4.0–9.6
Pacific Coast Uniform Building Code	2.7–5.3	4.3–9.7	8.8–16.5
Redtenbacher	1.7–3.6	2.8–6.5	6.0–10.9
Eytelwein	1.0–2.4	1.0–3.8	2.2–4.1
Navy-McKay	0.8–3.0	0.2–2.5	0.2–3.0
Rankine	0.9–1.7	1.3–2.7	2.3–5.1
Canadian National Building Code	3.2–6.0	5.1–11.1	10.1–19.9
Modified *Engineering News*	1.7–4.4	1.6–5.2	2.7–5.3
Gates	1.8–3.0	2.5–4.6	3.8–7.3
Rabe	1.0–4.8	2.4–7.0	3.2–8.0

*P_u = ultimate test load.
P_d = *design* capacity, using the safety factor recommended for the equation (values range from 2 to 6, depending on the formula).

dependent on hammer base, capblock, pile cap, and pile cap-pile interfacing. Energy input/E_h was found to range from about 26 to 65 percent—averaging less than 50.

Olson and Flaate (1967) performed a statistical analysis on some 93 other piles and concluded that the Hiley equation [Eq. (17-3)] and the Janbu and Gates formulas (Table 17-1) produced the least deviations and highest statistical correlations. This analysis was based largely on data reported in the literature; thus, some considerable estimating of pile weight, average penetration, pile cap weight, capblock weight, and condition (for n and use of a cushion for concrete piles) was required. The hammer condition, which would be particularly critical in obtaining either e_h or E_h, was generally not known.

An earlier statistical analysis of 30 piles of timber, steel, and concrete was presented by ASCE (1946, p. 28) from a previous discussion of a progress report [ASCE (1941)], which prompted Peck (1942) to propose a pile formula of P_u = 810 kN (91 tons). For the reported data it was statistically as good as any of the several dynamic equations used for computing the pile capacity.

A major problem with using statistical analyses primarily based on piles reported in technical literature is that although one can obtain a large data base it is not of much value. The reason is that there are not sufficient data given for the reader to make a reliable judgment of significant parameters to consider. Where the person making the analysis uses a self-generated data base (as in the case of Gates) results are generally more reliable.

17-6 THE WAVE EQUATION

The wave equation is based on using the stress wave from the hammer impact in finite-element analysis. This method was first put into practical form by Smith (1962) and later by others. A more detailed discussion of the principles and a reasonably sophisticated computer program are readily available [Bowles (1974*a*) or B-27] and will not be repeated here.

The wave equation has particular application for piling contractors in determining pile drivability with available equipment in advance of project bidding. It may also be used to estimate pile-driving stresses but does not have much application in prediction of pile capacity.

According to a pile practice survey reported by Focht and O'Neill (1985) the wave equation was used by about 30 percent of the practitioners at the time of the survey with most usage in the United States and Canada. The survey did not include contractors, so their usage is unknown. This lag between state-of-art and the state of practice is typical and results, in this case, partly from requiring both a computer and a computer program [although the latter may not be a valid reason, since this textbook included a program in 1968 as well as in Bowles (1974)]. Programs by others have been available for purchase for some time as well.

Uses of the Wave Equation

The wave equation is usually used to investigate the following problems:

1. *Pile capacity.* A plot of P_u versus set is made and the load test plotted on the curve to obtain the correct curve.
2. *Equipment compatibility.* Solutions are not obtained when the hammer is too big or too small for the pile.
3. *Driving stresses.* Plots of stress versus set can be made to ensure that the pile is not overstressed.

For the discussion to follow, refer to the list of symbols:

A = cross-sectional area of pile
C_m = relative displacement between two adjacent pile elements
D''_m = element displacement two time intervals back
D'_m = element displacement in preceding time interval DT (previous DT)
D_m = current element displacement
D_{sm} = plastic ground displacement
DT = time interval (Δt on Fig. 17-3*c*)
E_p = modulus of elasticity of pile material
F_m = element force = $C_m K_m$
F_{am} = unbalanced force in element causing acceleration ($F = ma$)
g = gravitation constant
J_i = damping constant; use J_s for side value, J_p = point value
K_m = element springs = AE/L for pile segments
K'_m = soil springs = R/quake
L_i = length of pile element (usually constant)
R_m = side or point resistance including damping effects
R'_m = amount of pile resistance (fraction of R_u) estimated to be carried by each element including the point j; for 100 percent of R_u on point j, the values of R_3 through R_{11} of Fig. 17-3*b* are zero, and $R_{12} = R_u$. Usually R_m of the first pile element is taken as zero for any assumed distribution of side and point resistance.
R_u = assumed ultimate pile capacity (same as P_u used previously)

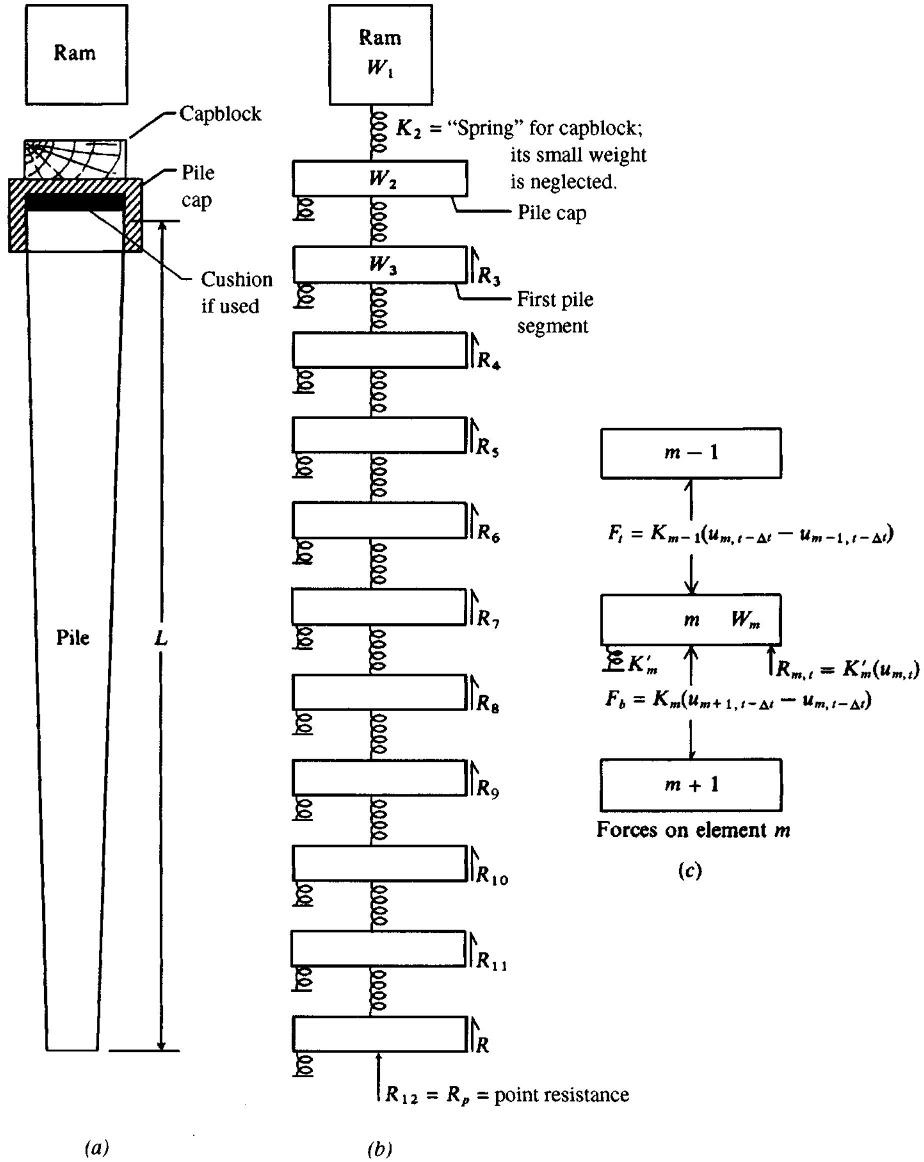


Figure 17-3 Formulation of pile into a dynamic model to solve the wave equation [*After Smith (1962)*].

t = current instant in time = number of iterations × DT
v_m = velocity of element m at DT
v'_m = velocity of element m at DT − 1
W_m = weight of pile segment m

A pile is formed into a set of discrete elements as shown in Fig. 17-3. The system is then considered in a series of separate time intervals DT chosen sufficiently small that the stress wave should just travel from one element into the next lower element during DT. Practically, this time choice is not possible, and DT is taken as a value that usually works, as in the following table:

Element material	Length, m	Trial DT, s
Steel	2.4–3.1	0.000 25
Wood	2.4–3.1	0.000 25
Concrete	2.4–3.1	0.000 33

For shorter lengths, DT should be made correspondingly smaller. The actual time DT can be approximately computed as

$$\mathrm{DT} = C\sqrt{\frac{W_m L_i}{A E_p g}}, \ \mathrm{s}$$

where C is 0.5 to 0.75; L_i = element length; $g = 9.807\ \mathrm{m/s^2}$ (in SI).

The finite-element form of the differential equation used in the wave analysis is

$$D_m = 2D'_m - D''_m + \frac{F_{am} g}{W_m}(\mathrm{DT})^2 \tag{17-10}$$

It is not necessary to solve this equation directly, however, since the items of interest for each assumed value of ultimate pile capacity P_u are these:

1. Forces in each pile segment
2. Displacement (or set) of the pile point

The instantaneous element displacement is computed alternatively as

$$D_m = D'_m + v_m(\mathrm{DT}) \tag{a}$$

With the instantaneous element displacements, the relative compression or tension movement can be computed between any two adjacent elements as

$$C_m = D_m - D_{m+1} \tag{b}$$

The force in segment m is

$$F_m = C_m\left(\frac{AE}{L}\right)_m = C_m K_m \tag{c}$$

The soil springs are computed as

$$K'_m = \frac{R'_m}{\text{quake}} \tag{d}$$

The side or point resistance term is obtained using damping with the side or point value of J and K' as appropriate:

$$R_m = (D_m - D_{sm})K'_m(1 + Jv_m) \tag{e}$$

The accelerating force in segment m is obtained by summing forces on the element to obtain

$$F_{am} = F_{m-1} - F_m - R_m \tag{f}$$

The element velocity is computed as

$$v_m = v'_m + \frac{F_{am}g}{W_m}(\text{DT}) \tag{g}$$

The wave equation requires the following computation steps:

1. Compute the displacements of each element in turn using Eq. (*a*) and consistent units. At DT = 1 there is only a displacement in element $m = 1$; at DT = 2 there are two displacements; at DT = 3 there are three displacements; DT = m computes displacements in all m pile elements.
2. Compute the plastic ground displacements D_{sm}. Values will be obtained only when $D_m >$ quake or elastic ground displacement, i.e.,

$$D_{sm} = Q - D_m \qquad (\text{but } D_{sm} > 0)$$

   This step requires two subroutines—one for the point element and one using a loop for all the other pile elements.
3. Compute side and point resistance R_m (use p instead of m for point) using Eq. (*e*). Use J_s = side damping for all except the point element; use J_p = point damping for point element. This requires one equation in a DO loop and a separate point equation.
4. Compute the spring compression in each element C_m using Eq. (*b*).
5. Compute the forces in each element using C_m and the spring constant AE/L as Eq. (*c*). Forces in the capblock and pile cap are computed separately using subroutines because these elements are not usually carrying tension and because of restitution with the dissimilar materials in the capblock and cap cushion (if used).
6. Compute the velocity of each element using Eq. (*g*).
7. Set the computed D_m and v_m into storage and reidentify as one time interval back (i.e., become D'_m and v'_m so new values can be computed for D_m and v_m for the current (new) DT).
8. Repeat as necessary (generally not less than 40 and not more than 100 iterations unless a poor value of DT is chosen or the pile-hammer compatibility is poor) until
   a. All the velocities become negative, and
   b. The point-set value becomes smaller than on previous cycles(s).

The wave equation analysis requires input data as follows:

a. Height of ram fall and ram weight P (obtain from tables such as A-2). The height is either given or back-computed as $h = E_h/W_r$. This is needed to obtain the velocity of the pile cap at DT = 1 (instant of impact), which is computed as

$$v_1 = \sqrt{e_h(2gh)}$$

b. Weight of pile cap, capblock, pile segments, driving shoe, and modulus of elasticity of pile material.

c. Values of capblock and pile cushion spring constants. Table 17-6 gives values of modulus of elasticity E for several materials used for these elements for computing the spring as $K = AE/L$. Use Table 17-3 for coefficient of restitution.

TABLE 17-6
Secant modulus of elasticity values for several capblock and pile-cushion materials*

(Approximate $A = 12$ in. or 30 cm square and $L = A$ unless other data are available to compute spring constant of AE/L.)

Material	E, ksi	E, MPa
Micarta	450	3100
Hardwood, oak	45	310
Asbestos disks	45	310
Plywood, fir	35	240
Pine	25	170
Softwood, gum	30	205

*Data from Smith (1962) and Hirsch et al. (1970).

d. Soil properties:
Quake (same as k_3 used earlier)
Point damping J_p (PJ in computer program)
Side damping J_s (SJ in computer program)—usually about $J_p/3$
Sovinic et al. (1985) performed a number of load tests on pipe piles driven open-ended and concluded that the soil plug reduces the point and side damping values on the order of $J_p/5$ and $J_s/5$. Although they did not test any **HP** piles, it would be reasonable to apply a reduction for those as well—but not nearly so large. Smith (1962) initially did not consider soil plugs; he used an **HP**310 × 79 (**HP**12 × 53) pile as an example, but most of the pipe piles considered were apparently driven closed-end—some were mandrel-driven. It is quite possible, however, that the original Smith **HP** pile example was for illustration of the method and not one where there was a load test to compare with the computed capacity by the wave equation analysis.
Typical values (no plug) for quake and for both Q and J_p (use $J_s \cong J_p/3$) are as follows:

	Quake		Damping constant J_p*	
Soil	**in.**	**mm**	**s/ft**	**s/m**
Sand	0.05–0.20	1.0–5.0	0.10–0.20	0.33–0.66
Clay	0.05–0.30	1.0–8.0	0.40–1.00	1.30–3.30
Rock	> 0.20	> 5.0		

* Reduce damping constants when there is a soil plug.

e. Estimate of percentage of the ultimate load P_u carried by the pile point (0 to 100 percent). In general, no pile carries 100 percent of the load on the point, and one should not use more than 80 to 95 percent on the point. Placing 100 percent of the load on the point produces a discontinuity in computations, since side load from skin resistance will include damping as shown in Eq. (*f*), with no side resistance $K'_m = 0.0$.

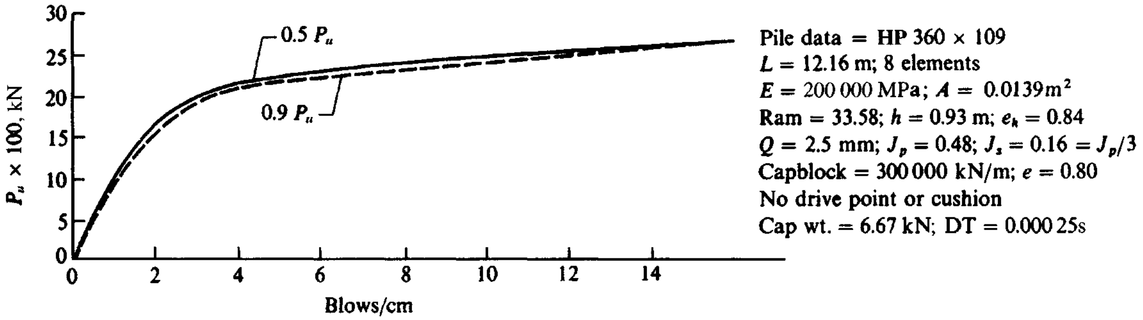
Plots of P_u versus blows per centimeter (cm) (or inch) are made by assuming several values of P_u and using the wave equation computer program to obtain the set. The blows per centimeter N is obtained as

$$N = \frac{1}{s}$$

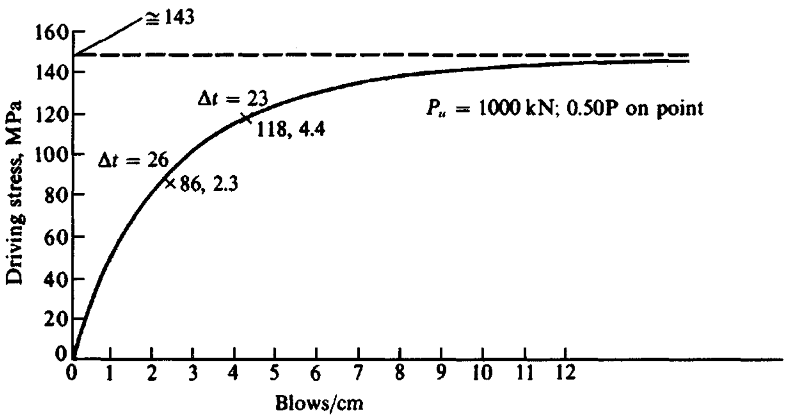
For any curve the percentage of P_u assumed to be carried by the pile point is held constant as, say, 25, 50, 75, 95 percent.

Plots of $1/s$ (or N) versus driving stress are obtained for any given P_u by obtaining from the computer output the maximum element force and the corresponding point set for some value of DT. Several other values of maximum element force (not necessarily in the same element) and set at other DTs are also selected so that enough points are obtained to draw a curve. This curve is somewhat erratic, owing to the mathematical model, and must be "faired" through the origin, since it is usually not possible to obtain $1/s$ values as low as 0.5, 1.0 and 1.5 or 2.0. In the region of large $1/s$ it is evident that the curve will approach some asymptotic value of driving stress. Curves of P_u versus blows per centimeter and driving stress versus blows per centimeter are shown in Fig. 17-4.

Figure 17-4 Output from the wave equation used to plot curves of $R_u = P_u$ versus 1/set and driving stresses versus 1/set for field use and using the pile data shown on the figure. *It is necessary to use cm units so that the blow/cm values are > 1, i.e., 1/2.5 = 0.4 but 1/0.25 = 4 and can be plotted.*



(*a*) Plot of P_u (assumed values) versus blows/cm (or 1/set, cm) for several assumed point values.



(*b*) Plot of driving stress versus blows/cm (or 1/set, cm) for the assumed value of P_u = 1000 kN at Δt values selected from the computer printout for that P_u.

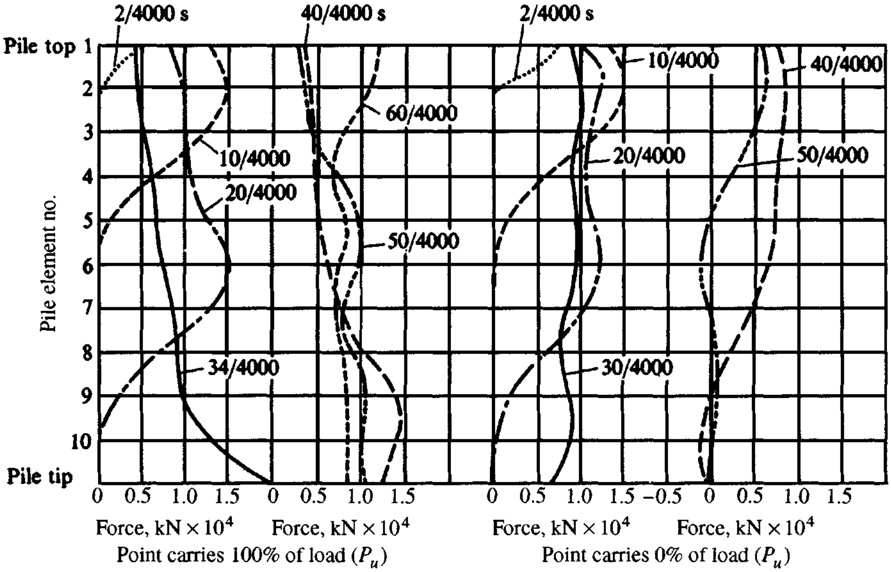


Figure 17-5 Plot of forces computed on pile elements by the wave equation using a **HP**310 × 79 pile. The plot is shown for selected time intervals. The purpose of the figure is to show in a somewhat quantitative manner the force distribution down the pile at the selected time intervals shown. The plot had to be reduced for text usage and is too small to obtain actual force values.

Traces of several stress waves down a pile are shown in Fig. 17-5 for a pile with the following data:

HP310 × 79 [**HP**12 × 53 as used by Smith (1962)]; $L_p = 30$ m;
Use 10 pile elements of 3 m each; Driving point = 0.44 kN;
Pile cap = 3.10 kN; pile cross section = 0.010 m^2;
Wt./m of pile = 0.774 kN/m (steel $\rho = 7850$ kg/m^3); $E = 200\,000$ Mpa;
Hammer: Ram = 22.2 kN; height of fall $h = 0.91$ m; $e_h = 0.80$;
$J_p = 0.50$ s/m; $J_s = 0.16$ s/m; $\Delta t = 0.00025$ s;
Capblock $n = 0.5$; capblock spring $K = 350\,000$ kN/m;
Point load = 100 and 0 percent; estimated pile load $R_u = 900$ kN

The program FADWAVE (B-27) has several output options: one is just the set and last set of computed pile element forces; a second option is that shown in Fig. 17-6; and a third option (not shown but used to plot Fig. 17-5) outputs the data of Fig. 17-6 plus the pile forces for each time increment DT (in program). The time values shown were selected, rounded, and plotted as shown in Fig. 17-5.

The output sheet of Fig. 17-6 echoes the input data (given above) and for each DT gives the set, point displacement D, maximum force in the pile F, and the element in which it occurs. For example, at time increment DT = 16 when the first point displacement occurs, the force in element 5 is 1240.3 kN. The point does not have any set until DT = 32, when it is 0.407 mm, with a point displacement $D = 2.907$ mm. The maximum pile force at this DT is in element 4 and is 964.3 kN. The maximum set = **10.417** mm and is the average of the last 6 DT computations (if you add the set values for DT = 57 through 62 and divide by 6 you

```
   TRIAL PILE HP310X79 X 30 M FOR FIG. 17-5 OF 5/E--SI

                          NAME OF DATA FILE FOR THIS EXECUTION:  FIG175A.DTA

  ++GENERAL INPUT DATA:                    NCHECK = 1
           NO OF PILE SEGMENTS =  10
           LENGTH OF PILE ELEM =  3.000 M
  NO OF ELEMENTS INCL RAM & CAP =  12
          PILE MODULUS OF ELAST =  200000. MPA

                    WT/M  OF PILE =     .7740 KN      PILE X-SECT =     .0100 M **2
           ELEM WTS, KN  :   RAM =  22.200               PILE CAP =  3.1000
          WT BOT ELEM + DRIVE PT =  2.7620           WT DRIVE PT =    .4400
                  HT OF RAM FALL =   .910 M           HAMMER EFF =    .80
             SIDE DAMP CONST,SJ =  .160   POINT DAMP CONST,PJ =  .500 S/M

  SPRING CONSTANT, KN/M:   CAPBLOCK =  350000.0   PILE CUSHION =          .0
                         1ST PILE SEG =  666666.6   2ND PILE SEG =  666666.6
         COEFF OF RESTIT:  CAPBLOCK =  .500         PILE CUSHION = 1.000
                            TIME INTERVAL, DT =  .0002500 SEC

    I       RU(I), KN     +++ ASSUMED ULT PILE RESIST RUTOT =    900.00 KN
    4           .000
    5        112.500
    6        112.500
    7        112.500
    8        112.500
    9        112.500
   10        112.500
   11        112.500
   12        112.500
   13           .000  ( % POINT =  .000)
    ++ SUM OF ABOVE RU(I) SHOULD =     900.00 KN

  NO OF ITERATIONS =  62     INPUT QUAKE =  2.500 MM
       AVERAGE SET = 10.417 MM   NO OF VALUES USED IN AVERAGE SET = 6

 DT=   1         2          3          4          5          6          7          8          9          10         11
SET=   .00000    .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000
 D=   .00000    .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000
 F=       .0      43.6      157.5      341.9      572.0      808.6     1013.9     1163.4     1250.3     1274.5     1260.0
 ELEM NO 13          3          3          3          3          3          3          3          3          3          4

 DT=  12         13         14         15         16         17         18         19         20         21         22
SET=   .00000    .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000
 D=   .00000    .00000     .00000     .00000     .00001     .00006     .00023     .00075     .00224     .00601     .01472
 F=   1331.0    1341.6     1302.1     1258.6     1240.3     1185.5     1171.5     1132.6     1097.5     1073.4     1075.7
 ELEM NO  4          4          4          5          5          5          6          6          7          7          4

 DT=  23         24         25         26         27         28         29         30         31         32         33
SET=   .00000    .00000     .00000     .00000     .00000     .00000     .00000     .00000     .00000     .40762    1.19905
 D=   .03326    .06968     .13620     .24950     .42996     .69938    1.07688    1.57391    2.18949    2.90762    3.69905
 F=   1074.0    1055.8     1029.8     1006.9      994.0      990.9      991.9      989.7      980.3      964.3      946.2
 ELEM NO  4          4          4          4          4          4          4          4          4          4          4

 DT=  34         35         36         37         38         39         40         41         42         43         44
SET=  2.02634   2.85062    3.63777    4.36279    5.01162    5.58071    6.07526    6.50681    6.89071    7.24339    7.57950
 D=  4.52634   5.35062    6.13777    6.86279    7.51162    8.08071    8.57526    9.00681    9.39071    9.74339   10.07950
 F=    931.1     921.3      915.8      910.7      902.7      890.3      875.0      859.4      845.5      833.0      819.0
 ELEM NO  4          4          4          4          4          4          4          4          4          4          4

 DT=  45         46         47         48         49         50         51         52         53         54         55
SET=  7.90916   8.23601    8.55677    8.86294    9.14414    9.39206    9.60339    9.78066    9.93055   10.06052   10.17556
 D= 10.40916  10.73601   11.05677   11.36294   11.64414   11.89206   12.10339   12.28066   12.43054   12.56052   12.67556
 F=    799.7     771.3      731.5      683.9      649.5      601.6      536.8      453.4      351.1      231.7      110.2
 ELEM NO  4          4          4          3          3          3          3          3          3          3          7

 DT=  56         57         58         59         60         61         62
SET= 10.27620  10.35895   10.41856   10.45084   10.45474   10.43253   10.38835
 D= 12.77620  12.85895   12.91856   12.95084   12.95474   12.93253   12.88835
 F=    126.3     121.9      133.9      136.1      116.2      195.5      279.9
 ELEM NO  7          7          6          6          6          4          4

   THE FORCES IN PILE SEGEMENTS ARE (3 = 1ST PILE SEGMENT)
 ELEM #  MAX ELEM FORCE    DT     LAST COMP FORCE, KN      LAST V(M,2), S/M
     2          1336.3      7                    .0               -.981
     3          1274.5     10                    .0               -.551
     4          1341.6     13                 279.9               -.054
     5          1258.6     15                 167.4               -.260
     6          1173.0     17                  51.3               -.466
     7          1097.5     20                  12.2               -.358
     8          1020.5     22                 -20.3               -.229
     9           943.6     24                   3.6               -.259
    10           873.2     27                 -21.8               -.286
    11           782.8     29                 -96.7               -.213
    12           538.6     29                 -82.2               -.209
    13              .0     62                    .0               -.250
```

Figure 17-6 Wave equation output (using program FADWAVE) for the **HP**310 × 79 given in TITLE line.

should obtain 10.417). An average is used based on the difference between the maximum set (occurs at DT = 60), and the program checks adjacent values and finds those within 0.12 mm of that value. All of these values are summed and divided by the number. Sometimes there are only three or four values—here there were six. The last six values are averaged for the set since these are so close that it is difficult to determine exactly what the set should be.

This large set (10.417 mm) occurs because the point is assumed not to carry any load. For the case of the point carrying 100 percent of the load the set = **4.881** mm. These are the two limiting cases—for the point carrying 20 to 80 percent of the load the point set would be somewhere between 4.881 and 10.417 mm.

To plot Fig. 17-4*a* one would need to obtain the set from several assumed values of R_u (900, 1200, 1500, ...) and for each execution obtain the blow/cm (as 1/1.0417 = 0.959). Since there is no such thing as a fraction of a blow, this should be rounded to 1 (an integer). The value would be 1/10.417 = 0.09 using mm; for the point load case we obtain 1/0.4881 = 2.04, which can be plotted as 2.0 (but not 1/4.881 = 0.20). Thus, it is necessary to plot these curves using 1/set with set in cm and not mm.

To plot the curve of Fig. 17-4*b* we must extract the set and corresponding F from calculations such as Fig. 17-6. We can use the list of maximum element forces versus DT to find worst cases, but there must be a "set" for the cases selected. For example, the maximum force in element 2 occurs at DT = 7 but at this time the set = 0. The first set of 0.407 mm = 0.0407 cm occurs at DT = 32 when the force F = 964.3 kN. This data locates a curve point at $\sigma = F/A = 964.3/0.0139 = 69.4$ MPa versus 1/0.0407 = 24.6 → 25 (blows/cm). At DT = 43 we have $\sigma = 833/0.0139 = 59.9$ MPa versus 1/0.724 cm = 1.38 (blows/cm). We can plot the nonintegers, but the curve user can carry out only integer blow counts. The reader should obtain several additional points and draw a curve similar to Fig. 17-4*b*.

General Comments on the Wave Equation

There have been a number of modifications to the original wave equation to include what the programmer asserts to be better modeling of the soil effects on the shaft sides [$R(M)$], of the interface elements (ram, anvil, capblock, etc.) to the pile, and in the case of the diesel hammer, to model the fuel-mixture explosion. In all these cases the result is little better than the original Smith proposal (if proper allowances are made) for a number of reasons. The point and shaft resistances and quake are at best factors that make the program give a solution. The hammer impacts and resulting pile vibration will reduce the soil immediately adjacent to the pile shaft and point to a viscous fluid. The "viscosity" probably does increase with depth but this problem can be accounted for by inputting an $R(M)$ different for each pile segment. Since a wide range of quake gives solutions with not much difference, it is evident that this is a "make it work factor," although certain factors do work better than others. Those recommended by Smith work as well as any. A similar statement can be made for the side and point damping factors.

Modeling the pile-hammer interface is at best an exercise in computational tenacity. The different hammers have different anvil configurations (and dimensions), the driving cap varies widely, and the capblock "spring" varies widely (even during driving the same pile) depending on how much it has been used. Pile input energy is heavily dependent on the mechanical state of the hammer. Considering all these variables, it is suggested that the simplest form of the wave equation is adequate. Any comparison between computer output and predicted pile capacity within a 30 percent deviation is likely to be a happy coincidence of input data [see also the comprehensive study by Tavenas and Audibert (1977)] rather than computer program sophistication. It is relatively easy with any of the wave equation programs to back-compute excellent correlation with a load test. It is less easy to predict the load test results in advance, however.

Since the wave equation is really concerned with the energy that the pile segments "receive," it should be evident that the energy input to the program is only an estimate unless it is directly measured via strain gauges or velocity- or acceleration-measuring devices attached to one or more of the upper pile segments. This approach is essentially that of Rausche and Goble (1979) where the force/acceleration measurements are then directly input into a wave equation type of program.

A number of programs purport to model the input energy of the diesel hammer using the "blast energy." Since the fuel-explosion energy is somewhat indeterminate and as previously stated the energy output depends on the mechanical condition of the hammer, it is evident that the earlier programs, which are much simpler, can as easily be used. It is only necessary to input the correct energy (i.e., adjust either ram weight or height of fall h) so that the energy output is the same as assumed for the blast force. The capblock "spring" can be adjusted to account for the interfacing of the diesel hammer elements, which might be different from a steam hammer. Again, the problem is solved if the first pile segment is instrumented to obtain the energy input.

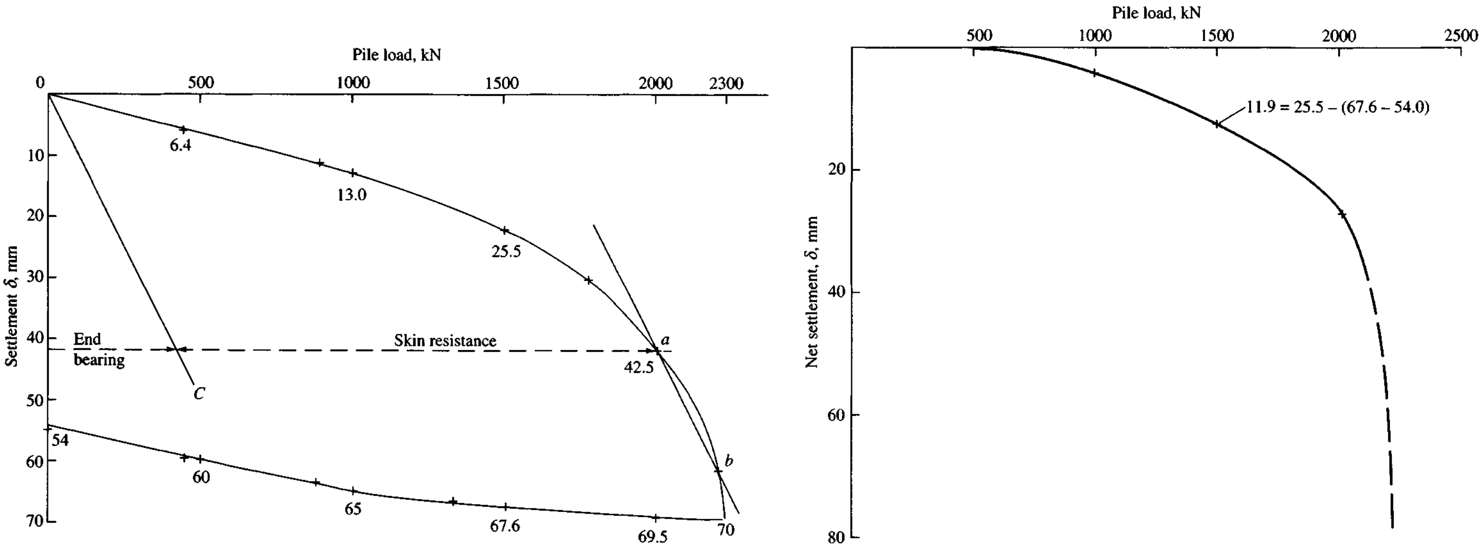
A number of the early wave equation programs had an interface modeling error [in Smith's original paper; found by the author when developing a wave equation for the vibratory pile driver (unpublished)]. This error could affect the output by as much as 5 percent. This kind of error is difficult to find since minor variations in input and order of magnitude of the output forces are such that small errors are usually insignificant.

17-7 PILE-LOAD TESTS

The most reliable method to determine the load capacity of a pile is to load-test it. This consists in driving the pile to the design depth and applying a series of loads by some means. The usual procedure is to drive several of the piles in a group and use two or more of the adjacent piles for reactions to apply the load. A rigid beam spans across the test pile and is securely attached to the reaction piles. A large-capacity jack is placed between the reaction beam and the top of the test pile to produce the test load increments. The general setup (Fig. 17-7*c*) is similar to the plate load test shown in Fig. 4-8 with the plate being replaced by the pile. The test has been standardized as ASTM D 1143; however, local building codes may stipulate the load increments and time sequence. Somewhat similar means are used to test laterally loaded piles. Here the lateral load may be applied by jacking two adjacent piles apart or suitably connecting several piles for the lateral reaction.

Figure 17-7 illustrates typical data from a pile-load test. Figure 17-7*a* is the usual plot for a load test.

The ultimate pile load is commonly taken as the load where the load-settlement curve approaches a vertical asymptote as for the 2200 kN load shown in Fig. 17-7*a*, or as the load corresponding to some amount of butt settlement, say, 25 mm, based on the general shape of the load-settlement curve, design load of the pile, and local building code (if any). The



(*a*) Usual method of presenting data.

(*b*) Plot of load vs. net settlement computed as shown on the figure using data from (*a*).

Figure 17-7 Pile load-test data. This is the pile shown in Fig. P16-7 (356 diam × 7.9 mm wall × 15.24 m long). The method of estimating end bearing and side resistance shown in (*a*) was suggested by Van Weele (1957).

load-settlement curve must be drawn to a suitably large settlement scale so that the shape (and slope) is well defined. Referring to Fig. 17-7*a*, we see that reducing the vertical scale by a factor of one-half would make it very difficult to determine that the curve is becoming nearly vertical between the 2000 and 2200 kN load.

An alternative method of interpreting Fig. 17-7*a* is based on the concept that the load is carried mostly by skin resistance until the shaft slip is sufficient to mobilize the limiting value. When the limiting skin resistance is mobilized, the point load increases nearly linearly until the ultimate point capacity is reached. At this point further applied load results in direct settlement (load curve becomes vertical). Referring to Fig. 17-7*a*, these statements translate as follows:

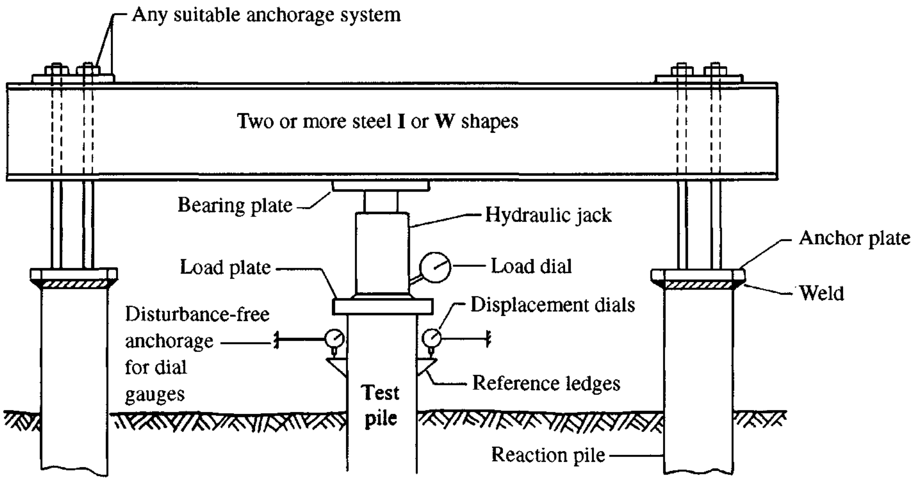
1. From 0 to point *a* the capacity is based on the skin resistance plus any small point contribution. The skin resistance capacity is the principal load-carrying mechanism in this region. Point *a* usually requires some visual interpretation since there is seldom a sharp break in the curve.
2. From point *a* to *b* the load capacity is the sum of the limiting skin resistance (now a constant) plus the point capacity.
3. From point *b* the curve becomes vertical as the ultimate point capacity is reached. Often the vertical asymptote is anticipated (or the load to some value is adequate) and the test terminated before a "vertical" curve branch is established.

This concept was introduced by Van Weele (1957) and has since been used by others [e.g., Brierley et al. (1979), Leonards and Lovell (1979), among others]. According to Van Weele, if we draw the dashed line 0 to *c* through the origin and parallel to the point capacity region from *a* to *b*, the load-carrying components of the pile are as shown on Fig. 17-7*a*. In this figure we have at settlement $\delta = 25$ mm the load carried as follows:

$$
\begin{aligned}
&\text{Point} &&= \ \ 250 \text{ kN} \\
&\text{Skin resistance} &&= \underline{1350 \text{ kN}} = 1600 - 250 \text{ kN} \\
&\text{Total} &&= 1600 \text{ kN} \quad \text{shown on figure}
\end{aligned}
$$

Local building codes usually stipulate how the load test is to be run and interpreted and pile design loads above which a load test is required (usually $P_d > 200$ kN). For example, the Chicago building code stipulates the test as follows:

1. Apply load increments of 25 percent of the proposed working load.
2. Carry the loading to two times the proposed working load. This requires seven or eight load increments.
3. Apply the loads after a specified time lapse or after the settlement rate is some small value.
4. The allowable pile load is taken as one-half that load that causes a net settlement of not more than 1 mm/35 kN. For example, in referring to Fig. 17-7*b*, the allowable pile load is about 1100 kN (so $2 \times 1100/35 = 63$ mm versus about 70 mm measured).
5. The building codes limit the minimum value of hammer energy E_h.
6. The codes require a minimum number of test piles per project.



(*c*) **Typical pile load test setup using adjacent piles in group for reaction.**

Figure 17-7(*c*) Typical pile load test setup using adjacent piles in group for reaction.

Piles in *granular* soils are often tested 24 to 48 hr after driving when load test arrangements have been made. This time lapse is usually sufficient for excess pore pressures to dissipate; however, Samson and Authier (1986) show that up to a 70 percent capacity gain may occur if load tests are made two to three weeks after driving.

Piles in *cohesive* soils should be tested after sufficient lapse for excess pore pressures to dissipate. This time lapse is commonly on the order of 30 to 90 days giving also some additional strength gain from thixotropic effects.

In any soil sufficient time should elapse before testing to allow partial dissipation of residual compression stresses in the lower shaft and point load from negative skin resistance on the upper shaft caused by shaft expansion upward as the hammer energy is released. Residual stresses and/or forces have been observed in a number of reports and summarized by Vesić (1977). It appears that pile load testing of the load-unload-reload type is more likely to produce residual stresses than driving.

ASTM D1143 gives the "standard" pile load test procedure and outlines in considerable detail the data to be collected in addition to load versus butt displacement. It would, of course, be most worthwhile for the various organizations that publish technical papers (such as ASCE and CGJ) to establish a similar checklist of information that would be the minimum to be included for the paper to be accepted for publication. This would give readers sufficient information to verify or provide alternative conclusions as well as to create a useful data base for future correlations that are more reliable. This is particularly important for piles since, as noted in Chap. 16, such a large amount of conflicting test data have been published.

17-8 PILE-DRIVING STRESSES

A pile must be adequately sized to satisfy both the static and dynamic (or driving) stresses. The driving stresses are difficult to determine except as approximations. Stresses are computed as P_d/A, and the limitations inherent in the dynamic equations exist for computing the driving force P_d so that a stress can be computed.

The wave equation seems to provide the best means to estimate the driving force P_d, both for compression in all piles and tension in concrete piles, and to find compressive and tension loads in the pile elements.

Figure 17-6, which is a printout of a wave equation trial, shows that the maximum force $P_d = 1341.6$ kN; this occurred in element 4 at DT = 13. Because this was a metal pile we do not need tension forces, but the pile had some (with the proper option activated, the program would also collect the largest negative forces in the elements as well). The option should always be activated for concrete piles.

The pile element forces depend on two factors in a wave equation analysis:

1. The estimated ultimate load $P_u = R_u$ (used as RU in program)
2. The amount of load estimated to be carried by the point

For the pile of Fig. 17-5 we have 100 percent point load and 0.0 point load—the two extremes. In this case the maximum loads are these:

Point load	Pile element	P_d, kN	At DT
100%	10 (bottom)	1808.2	34 (of 59)
0% (Fig. 17-6)	4 (near top)	1341.6	13 (of 62)

Also DT = 1/4000 = 0.000 25 sec. These data are for an estimated $P_u = 900$ kN, so it appears that the driving stresses can be from 50 to 100 percent larger than the estimated ultimate load.

The dynamic equations (such as the ENR and Hiley types) can also be used to estimate driving stresses and set. The use of the Hiley equation is illustrated in Example 17-4 following.

Since the pile driving supervisor can only obtain blow counts in the field, it is useful to present the data as illustrated in Fig. 17-4 or in E17-4. It should be evident, however, that the curves in these two figures represent particular pile-hammer combinations. A change in either invalidates the curves.

It should also be evident that a measurement of "set" is not straightforward, rather it must be done indirectly. The reason is that there is both "set" and axial compression (PL/AE) during driving. This makes it necessary to attach some type of scribing device to the pile head (for measurements when the approximate design depth is reached) so that the scribe moves down at impact and back up but not to the original starting point. The difference between the starting point and the final point (below the initial) is assumed as the "set" for that blow.

There is a question of what the limit should be on driving stresses. Since they are temporary and always higher than the design load stresses, some leeway must be allowed. Driving on the order of $0.85f'_c$ has resulted in fracture of concrete piles, so it would appear that their stresses should be limited to about 0.5 to $0.6f'_c$.

Driving stresses for wood piles should also be limited to about 0.5 to $0.6f_{ult}$ because of knots and other interior flaws.

Steel piles can probably be limited to stresses on the order of 0.8 to $0.9f_y$. If steel piles are stressed into the yield zone the principal result is increased possibility of corrosion from flaking off of mill scale as Luder (or slip) lines form. There is also opinion that driving stresses for steel piles can be from f_y to as much as $1.15f_y$ because of strain-hardening. The author suggests not over $0.9f_y$ as a reasonable compromise, knowing that we are being optimistic if the driving stresses are not over ±20 percent of the estimate.

Example 17-4. Make a set versus driving resistance curve using the Hiley equation [Eq. (17-2)] with the following data:

DE-30 hammer (get data from Table A-2, in Appendix)
$W_r = 12.45$ kN $\quad E_h = W_r h = 22\,700$ to $30\,400 \rightarrow 27\,000$ kN·m
Efficiency $e_h = 0.85$ (not 1.0); $n = 0.40$ (Table 17-3)
Pile and other data: 406-mm (16-in.) OD with $t_w = 4.8$ mm
$A_p = 0.006\,02$ m^2 $\quad E_p = 200\,000$ MPa; Pile length = 18.3 m
Driven open-end but later cleaned and filled with concrete
Design load = 900 kN $\quad \gamma_s' = 9.0$ kN/m^3 $\quad \gamma_{st} = 77$ kN/m^3
Take SF = 1 for driving stresses $\quad f_y = 250$ MPa

Solution. The Hiley equation [Eq. (17-2)] is as follows:

$$P_u = \left[\frac{e_h E_h}{s + \frac{1}{2}(k_1 + k_2 + k_3)}\right]\left[\frac{W_r + n^2 W_p}{W_r + W_p}\right] \qquad k_2 = \frac{P_u L}{AE}$$

$$AE = 0.00602 \times 200\,000 = \mathbf{1204}\ \text{MN}$$

(Where 10^3 values cancel they will not be shown.) Obtain $k_1 = 2.5$ mm (given). Estimate $k_3 = \mathbf{2.0}$ mm (in range of 0 to 5 mm given earlier). Compute pile weight including plug as

$$\text{ID area} = 0.7854(0.406 - 2 \times 0.0048)^2 = \mathbf{0.123}\ \text{m}^2$$

$$W_p = \text{Weight of steel + cap + soil plug}$$

$$W_p = 0.00602 \times 77 \times 18.3 + 2.67 + 0.123 \times 9.0 \times 18.3$$

$$= 8.5 + 2.7 + 20.3 = \mathbf{31.5}\ \text{kN}$$

Making substitutions into the Hiley equation, we obtain

$$P_u = \left[\frac{0.85 \times 27\,000}{s + 0.5(2.5 + 2.0 + P_u L/AE)}\right]\left[\frac{12.45 + 0.16 \times 31.5}{12.45 + 31.5}\right]$$

Collecting terms, we obtain

$$P_u = \left[\frac{22\,950}{s + 2.25 + P_u(18.3/2408)}\right]\left[\frac{17.49}{44.0}\right] = \frac{9123}{s + 2.25 + P_u(18.3/2408)}$$

In this form the equation was programmed (since P_u is on both sides of the equation) for selected values of "set" in millimeters with the following output (Table E17-4) for plotting curves of set versus P_u and number of blows N/cm versus f_s as in Fig. E17-4. Note again that it is necessary to use the set in centimeters (cm) to obtain meaningful values—that is, divide by mm but multiply by 10. Since this step is equivalent to using centimeters we should call it that.

Notes.

1. We must initialize P_u to start computations. I used $P_u = 900$ kN.
2. We must use the pile area as the area of steel (0.00602 m^2), since the pipe must be filled with concrete after it is driven.
3. Adequate convergence is taken as 10 kN. That is, the difference between computed and used P_u is not over 10 kN.
4. You can use program FFACTOR (Hiley option 12) for these computations.

TABLE E17-4

$k = \frac{1}{2}(2.5 + 2.0) = 2.25$ mm

Set s_s, mm (cm)	C, mm $k + k_3$	Current P_u	Previous P_u, kN	Blows/cm N	Driving stress $f_s = P_u/A_p$, MPa
.0	9.522	958.1	(956.9)	.0	159.1
1.0	9.120	901.4	(904.0)	10.0	149.7
2.0	8.735	849.8	(853.4)	5.0	141.2
4.0	8.032	758.2	(760.8)	2.5	125.9
6.0	7.436	679.0	(682.4)	1.7	112.8
8.0	6.925	611.3	(615.1)	1.3	101.5
10.0 (1.0)	6.486	553.4	(557.4)	1.0	91.9
25.0 (2.5)	4.632	307.9	(313.5)	.4	51.1
50.0 (5.0)	3.607	170.2	(178.5)	.2	28.3
60.0 (6.0)	3.403	143.9	To plot s, cm vs. P_u		
100.0 (10.0)	2.973	88.6			

at s = 1.0 mm: $C = 2.25 + 904(18.3)/2408 = 9.120$ mm $(904 - 901.4 = 2.6 < 10)$

$f_s = 901.4/(0.00602 \times 1000) = 149.7$ MPa

Blows/cm $= 1/s \times 10 = 1/1.0 \times 10 = 10.0\ldots$ etc.

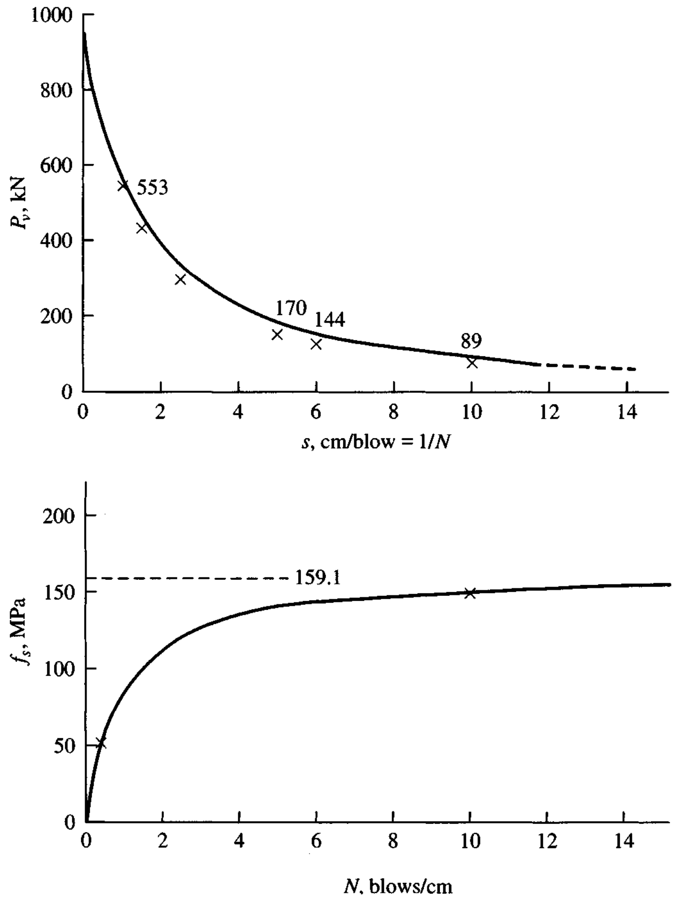


Figure E17-4

Question.

Would a better estimate of k_1 be 4 mm instead of the 2.5 used?

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17-9 GENERAL COMMENTS ON PILE DRIVING

Alignment of piles can be difficult to get exactly correct, and often the driven piles are not exactly located in plan. A tolerance of 50 to 100 mm is usually considered allowable. Larger deviations may require additional substructure design to account for eccentricities, or more piles may have to be driven. Alignment of pipe piles may be checked by lowering a light into the tube. If the light source disappears, the alignment is not true. Pile groups should be driven from the interior outward because the lateral displacement of soil may cause excessively hard driving and heaving of already driven piles.

Damage to piles may be avoided or reduced by squaring the driving head with the energy source. Appropriate pile-driving caps and/or cushions should be used. When the required driving resistance is encountered, driving should be stopped. These driving resistances may be arbitrarily taken as

Timber piles	4–5 blows/25 mm
Concrete piles	6–8 blows/25 mm
Steel piles	12-15 blows/25 mm

Driving may require corrective action if the head of a timber pile becomes damaged; e.g., use a cap or metal band or cut the head of the pile more carefully. If during driving any pile changes direction, or the penetration becomes irregular or suddenly increases, the pile may already be broken or bent. Damaged piles will have to be pulled; pulling a broken timber pile is not a trivial task—particularly the lower broken part.

Pile driving may induce heave in saturated, fine-grained, non-quick-draining soils, where the displaced soil increases the pore water pressure so that the void ratio cannot rapidly change. As the pore pressure dissipates, the amount of heave may be reduced. Piles already driven in this material may be uplifted, the problem being especially aggravated if the piles are closely spaced [Klohn (1961)]. The problem may or may not be serious, depending on how the heave takes place [Nordlund (1962)], and may be more serious for point-bearing piles if they are driven to refusal and then heave takes place, since excessive settlements may result after the structure is built as the piles reseat themselves under load. If heave is anticipated, survey benchmarks should be established, and elevations taken on the piles after they are driven and as other piles are driven in the vicinity.

Since heave is caused by volume displacement, it can be somewhat controlled by using small-volume displacement piles (**HP** or open-end pipes). Heave can be controlled by predrilling an undersized hole for timber and closed-end pipe piles to reduce the volume displacement.

In granular soils a rearrangement of the soil structure from the driving vibrations may result in a subsidence of the adjacent area. Already driven piles may be preloaded to some extent by this phenomenon. A pile driven in a zone within about three pile diameters of an already driven pile will be more difficult to drive because the soil in this zone will be densified.

Continuity of cast-in-place piles is verified by computing the volume of concrete used to fill the pile cavity and comparing this with the theoretical cavity volume.

PROBLEMS

Pile hammer data are obtainable in Table A-2 of the Appendix.

17-1. A pile-load test provides the following data:

Pile = 406-mm diameter pipe $L_p = 16.8$ m
$A = 0.01539\ \text{m}^2$ $E_{st} = 200\,000$ MPa wt = 1.2 kN/m
Weight includes attachments for instrumentation.
Hammer = Vulcan 140C $e_h = 0.75$
Set = 8 mm/blow for last 300 mm
Pile cap = 7.61 kN (driven open-end)

Find P_u and P_a by Hiley, ENR, and Gates equations.
Answer: $P_u = 1735$ kN (load test); by ENR $P_u = 3485$; Gates $P_a = 340$ kN

17-2. A pile-load test provides the following data:

Pile = 406 mm square concrete $L_p = 13.7$ m
$A = 0.1648\ \text{m}^2$ $E_c = 43\,430$ MPa
Weight/m = 3.89 kN/m
Hammer = Vulcan 140C $e_h = 0.78$
Set = 13.8 mm/blow
Pile cap (uses cushion) = 7.604 kN

Find P_u and P_a by Hiley, ENR, and Janbu equations.
Answer: $P_u = 1512$ kN (load test); by Janbu $P_u \cong 1400$ kN

17-3. A pile-load test provides the following data:

Pile = 400 mm square concrete $L_p = 16.0$ m
$E_c = 27\,800$ MPa ($f_c' = 35$)
Hammer = Vulcan 140C $e_h = 0.85$
Set = 6 mm/blow for last 300 mm
Weight of pile cap = 7.61 kN

Required: Compute ultimate and allowable pile capacity using the ENR equation [Eq. (17-5)].
Answer: $P_u = 2130$ kN (load test), ENR $P_u = 3950$ kN, $P_a = 660$ kN

17-4. A pile-load test provides the following data:

Pile = timber 0.116 m² butt, 0.058 m² tip $L_p = 12.2$ m
$E_w = 11\,000$ MPa wood = 20.6 kN/m³
Hammer = Vulcan 65C $e_h = 0.76$
Set = 13.3 mm/blow
Weight of pile cap = 4.23 kN

Required: Compute the ultimate and allowable pile capacity using the Gates and CNBC equations from Table 17-1.

Answer: P_u = 712 kN (load test); by Gates P_u = 627, by CNBC P_u = 477 kN

17-5. Plot a curve of P_u versus $1/s$ and stress versus $1/s$ for the pile of Prob. 17-3 using the equation from Table 17-1 as assigned by the instructor.

17-6. Plot a curve of P_u versus $1/s$ and stress versus $1/s$ for the pile of Prob. 17-4 using the Hiley equation.

17-7. What is the allowable load on the pile of Prob. 17-3 using the PCUBC equation?

17-8. What is the allowable load on the pile of Prob. 17-4 using the PCUBC equation?

17-9. Plot the assigned load-test data from the following two actual load tests, and select the allowable design load based on pile and load-test data.

	Test No. 1 HP 360 × 109, L = 15.2 m		Test No. 2 324 × 8 mm pipe*, L = 16.8 m		
P, kN	**Load, mm**	**Unload, mm**	**P, kN**	**Load, mm**	**Unload, mm**
0		0.6			25.4
445	5.0	20.3	445	03.0	29.2
890	9.0	25	890	05.6	31.8
1335	12.5	29	1330	10.2	34.3
1780	20.3	32	1780	16.5	37.8
2220	30.5		2000	31.8	
	33.0 (24 hr)			38.1 (24 hr)	

*Filled with concrete of f'_c = 28 MPa.

Use the building code in your area or the Chicago code method given in Sec. 17-7.

17-10. Compute P_u for the piles shown in Fig. P16-7 using a dynamic equation assigned by the instructor, and compare the solution to the load-test values of P_u shown. The driving hammer in all cases was a Vulcan No. 0 single-acting hammer.

17-11. Refer to Fig. 17-6 (wave equation output). Why is there no set at DT = 31 and how is the value of 0.40762 obtained for the "set" at DT = 32? What is the difference between total point displacement D and "set" at DT = 42? Can you draw any conclusions about the point displacement and set?

17-12. From the DT data of Fig. 17-6, make a plot of DT versus set and point displacement from DT = 10 to DT = 62.

17-13. What is the maximum stress (in MPa) in element 8 of the pile model of Fig. 17-6?

17-14. Verify that the first pile "spring" = 666 666.6 kN/m as shown on the output sheet (Fig. 17-6).

17-15. If the first pile element (element 4) were assumed also to carry an equal part of the 900 kN load, what would the side resistances be (they are 112.5 kN excluding the first pile element of Fig. 17-6)?

17-16. If you have access to a wave equation program, verify the output given in Fig. 17-6. Also verify that using 100 percent point load gives approximately the maximum load given in the textbook. Note that different programs may give slightly different answers. Also vary the point percent using 0.0, 0.25, 0.50, and 0.75 of P_u = 900 kN. The base data is on files FIG175.DTA and FIG175A.DTA on your diskette for using the Bowles program B-27.