$K =$	Shaft diameter, mm
K_a	$D \leq 300$ (12 in.)
$\frac{1}{2}(K_a + K_o)$	$300 < D \leq 600$
$\frac{1}{3}(K_a + K_o + K_p)$	$D > 600$ (or any D for slump > 70 mm)

In cemented sands you should try to ascertain the cohesion intercept and use a perimeter × cohesion × L term. If this is not practical you might consider using about 0.8 to $0.9K_p$.

The data base for this table includes tension tests on cast-in-place concrete piles ranging from 150 to 1066 mm (6 to 42 in.) in diameter. The rationale for these K values is that, with the smaller-diameter piles, arching in the wet concrete does not develop much lateral pressure against the shaft soil, whereas the larger-diameter shafts (greater than 600 mm) allow full lateral pressure from the wet concrete to develop so that a relatively high interface pressure is obtained.

16-15 LATERALLY LOADED PILES

Piles in groups are often subject to both axial and lateral loads. Designers into the mid-1960s usually assumed piles could carry only axial loads; lateral loads were carried by batter piles, where the lateral load was a component of the axial load in those piles. Graphical methods were used to find the individual pile loads in a group, and the resulting force polygon could close only if there were batter piles for the lateral loads.

Sign posts, power poles, and many marine pilings represented a large class of partially embedded piles subject to lateral loads that tended to be designed as "laterally loaded poles." Current practice (or at least in this textbook) considers the full range of slender vertical (or battered) laterally loaded structural members, fully or partially embedded in the ground, as *laterally loaded piles*.

A large number of load tests have fully validated that vertical piles can carry lateral loads via shear, bending, and lateral soil resistance rather than as *axially* loaded members. It is also common to use superposition to compute pile stresses when both axial and lateral loads are present. Bowles (1974*a*) produced a computer program to analyze pile stresses when both lateral and axial loads were present [including the $P - \Delta$ effect (see Fig. 16-21)] and for the general case of a pile fully or partially embedded and battered. This analysis is beyond the scope of this text, partly because it requires load-transfer curves of the type shown in Fig. 16-18*b*, which are almost never available. Therefore, the conventional analysis for a laterally loaded pile, fully or partly embedded, with no axial load is the type considered in the following paragraphs.

Early attempts to analyze a laterally loaded pile used the finite-difference method (FDM), as described by Howe (1955), Matlock and Reese (1960), and Bowles in the first edition of this text (1968).

Matlock and Reese (ca. 1956) used the FDM to obtain a series of nondimensional curves so that a user could enter the appropriate curve with the given lateral load and estimate the ground-line deflection and maximum bending moment in the pile shaft. Later Matlock and Reese (1960) extended the earlier curves to include selected variations of soil modulus with depth.

Although the nondimensional curves of Matlock and Reese were widely used, the author has never recommended their use. A pile foundation is costly, and computers have been available—together with computer programs—for this type of analysis since at least 1960. That is, better tools are now available for these analyses.

THE *p–y* METHOD. The initial work on the FDM lateral pile solution [see McClelland and Focht (1958)] involved using node springs p and lateral node displacements y, so that users of this method began calling it the "p–y method." Work continued on this FDM computer program to allow use of different soil node springs along the pile shaft—each node having its own p–y curve [see Reese (1977)]. Since p–y curves were stated by their author to represent a line loading q (in units of kip/ft, which is also the unit of a soil spring), user confusion and uncertainty of what they represent has developed. This uncertainty has not been helped by the practice of actually using the p part of the p–y curve as a node spring but with a 1-ft node spacing so that it is difficult to identify exactly how p is to be interpreted. The product of node spring and node displacement y gives $p \cdot y$ = a node force similar to spring forces computed in the more recognizable form of force $= K \cdot X$.

The data to produce a p–y curve are usually obtained from empirical equations developed from lateral load tests in the southwestern United States along the Gulf Coast. In theory, one obtains a p–y curve for each node along the pile shaft. In practice, where a lateral load test is back-computed to obtain these curves, a single curve is about all that one can develop that has any real validity since the only known deflections are at or above the ground line unless a hollow-pipe pile is used with telltale devices installed. If the node deflection is not known, a p–y curve can be developed with a computer, but it will only be an approximation.

The FDM is not easy to program since the end and interior difference equations are not the same; however, by using 1-ft elements, interior equations can be used for the ends with little error. The equations for the pile head will also depend on whether it is free or either translation and/or rotation is restrained. Other difficulties are encountered if the pile section is not constant, and soil stratification or other considerations suggest use of variable length segments. Of course, one can account for all these factors. When using 1-ft segments, just shift the critical point: The maximum shift (or error) would only be 0.5 ft.

The FDM matrix is of size $N \times N$, where N = number of nodes. This matrix size and a large node spacing were advantages on early computers (of the late 1950s) with limited memory; however, it was quickly found that closer node spacings (and increases in N) produced better pile design data. For example, it is often useful to have a close node spacing in about the upper one-third of a pile.

The FDM would require all nodes to have equal spacing. For a 0.3-m spacing on a 36-m pile, 121 nodes would be required for a matrix of size $N \times N$ = 14 641 words or 58.6 kbytes (4 bytes/word in single precision). This size would probably require double precision, so the matrix would then use 117 kbytes.

THE FEM LATERAL PILE/PIER ANALYSIS. The author initially used the FDM for lateral piles (see first edition of this text for a program); however, it soon became apparent that the FEM offered a significant improvement. Using the beam element requires 2 degrees of freedom per node, but the matrix is always symmetrical and can be banded into an array of size

$$2 \times \text{number of nodes} \times \text{Bandwidth}$$

This array is always 2 × NNODES × 4, thus, a pile with 100 nodes would have a stiffness matrix of 2 × 100 × 4 = 800 words. This is 3200 bytes or 3.2k of memory and in double precision only requires 6.4k bytes.

One advantage of the FEM over the FDM is the FEM has both node translation and rotation, whereas the FDM only has translation. The elastic curve is somewhat better defined using both translation and rotation.

Another advantage is that the element lengths, widths, and moments of inertia can vary with only slightly extra input effort. One can even use composite piles. The pile modulus of elasticity is usually input as a constant since most piles are of a single material, but it is trivial to modify the moment of inertia for a composite section so that the program computes the EI/L value correctly. This value is determined by computing a modified moment of inertia I_m as in Eq. (13-4).

When using variable element lengths it is suggested that one should try to keep the ratio of adjacent element lengths (longest/shortest) < 3 or 4.

A major advantage of the FEM is the way in which one can specify boundary cases (nodes with either zero rotation or translation) and lateral loads. The FDM usually requires the load and boundary points be pre-identified; the FEM allows any node to be used as a load point or to have known translation or rotation—the known value is usually 0.0 but can be nonzero as well.

A final advantage is that the FEM for a lateral pile program can be used for a lateral pier (piles with a larger cross section) or beam-on-elastic-foundation design. It is only necessary to input several additional control parameters so the program knows what type of problem is to be solved. Thus, one only has to learn to use one fairly simple program in order to solve several classes of problems. Your sheet-pile program FADSPABW (B-9) is a special case of this method. It was separately written, although several subroutines are the same, because there are special features involved in sheet-pile design. These additional considerations would introduce unnecessary complexity into a program for lateral piles so that it would be a little more difficult to use. Many consider it difficult in any case to use a program written by someone else, so the author's philosophy has been to limit what a program does so that it is easier to use.

Refer to Sec. 9-8 for the derivation of the stiffness matrix and other matrices for the beam-on-elastic foundation and also used for the lateral pile. The only difference is that the beam-on-elastic foundation is rotated 90° clockwise for the lateral pile P–X coding and the end springs are not doubled (see Fig. 16-19). You must know how the finite-element model is coded and how the element force orientations (direction of arrowheads on force, moment, and rotation vectors) are specified either to order the input loads or to interpret the output element moments and node displacements.

USING THE FEM COMPUTER PROGRAM. The general approach to setting up an FEM model for using your diskette program FADBEMLP (B-5) to analyze lateral piles is this:

1. Divide the pile into a convenient number of elements (or segments) as in Fig. 16-19. From experience it has been found that the top third of the embedment depth is usually critical for moments and displacements, so use shorter element lengths in this region. Avoid very short elements adjacent to long elements; place nodes at pile cross-sectional changes, at soil strata changes, and where forces or boundary conditions are being applied. Generally 10 to 15 elements are adequate, with 4 to 8 in the upper third of the embedded shaft length.

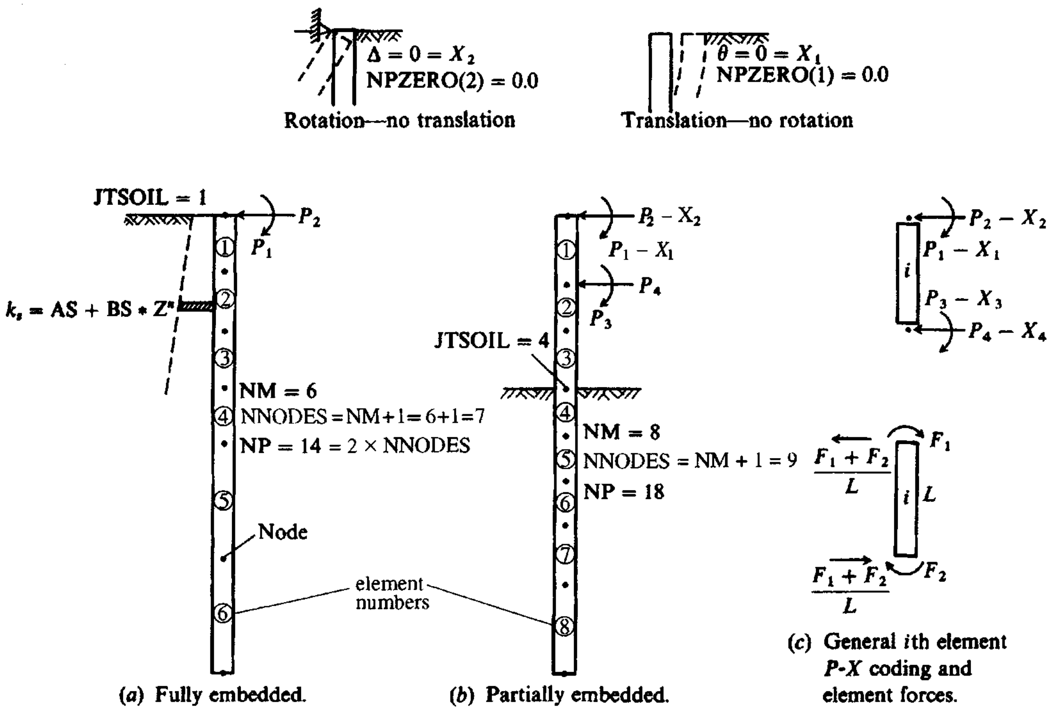

Figure 16-19 Laterally loaded pile using finite elements. Typical loadings shown in (*a*) and (*b*). Note that elements do not have to be same size or length. Generally use short elements near ground surface and longer elements near pile point where moments are less critical.

2. Partially embedded piles are readily analyzed by using JTSOIL equal to the node where soil starts (same as for sheet-pile wall). Use JTSOIL = 1 if ground line is at first pile node.
3. Identify any nodes with zero translation and/or rotation. NZX = number of *X*s of zero displacement. Use element coding to identify those *X* values that are input using NXZERO(I).
4. Make some estimate of the modulus of subgrade reaction and its depth variation (AS, BS, EXPO). Note that either AS or BS can be zero; EXPO = 0.5, 0.75, 1.0, or 1.5 may be appropriate; EXPO is the exponent of Z^n. You can also estimate a k_s-value [and XMAX(I)] for each node to input similar to the sheet-pile program.
5. Back-compute lateral load test data, if they are available, for the best estimate of k_s. One should not try to back-compute an exact fit since site variability and changes in pile type (pipe versus **HP**) preclude the existence of a unique value of k_s. The large number of pile tests reported by Alizadeh and Davisson (1970) clearly shows that great refinement in back computations is not required. One should, however, use in a load test the lateral load that is closest to the working load for best results.

WHAT TO USE FOR THE MODULUS OF SUBGRADE REACTION k_S.[5] The modulus of subgrade reaction is seldom measured in a lateral loaded pile test. Instead, loads and deflec-

[5]It should be understood that even though the term k_s is used in the same way as for the beam-on-elastic foundation, it is a vertical value here. The type (vertical or horizontal) is identified to the user by the context of usage.

tions are usually obtained as well as, sometimes, bending moments in the top 1 to 3 m of the embedded pile. From these one might work back using one's favorite equation for lateral modulus (or whatever) and obtain values to substantiate the design for that site.

Node values (or an equation for node values) of k_s are required in the FEM solution for lateral piles. Equation (9-10), given in Chap. 9 and used in Chap. 13, can also be used here. For convenience the equation is repeated here:

$$k_s = A_s + B_s Z^n \tag{9-10}$$

If there is concern that the k_s profile does not increase without bound use $B_s = 0$ or use B_s in one of the following forms:

$$B_s\left(\frac{Z}{D}\right)^n = \frac{B_s}{D^n}Z^n = B_s'Z^n \qquad \text{(now input } B_s' \text{ for } B_s\text{)}$$

or use $B_s(Z)^n$ where $n < 1$ (but not < 0)

where Z = current depth from ground surface to any node
D = total pile length below ground

The form of Eq. (9-10) for k_s just presented is preprogrammed into program FADBEMLP (B-5) on your diskette together with the means to reduce the ground line node and next lower node k_s (FAC1, FAC2 as for your sheet-pile program). You can also input values for the individual nodes since the soil is often stratified and the only means of estimating k_s is from SPT or CPT data. In this latter case you would adjust the ground line k_s before input, then input FAC1 = FAC2 = 1.0.

The program then computes node springs based on the area A_c contributing to the node, as in the following example:

Example 16-9. Compute the first four node springs for the pile shown in Fig. E16-9. The soil modulus is $k_s = 100 + 50Z^{0.5}$. From the k_s profile and using the average end area formula:

$$K_i = \frac{BL}{6}(2k_{s,i} + k_{s,i-1}) \qquad \text{or} \qquad \frac{BL}{6}(2k_{s,i} + k_{s,i+1})$$

$$K_1 = H(1) \times B(1)(2k_{s,1} + k_{s,2})/6 = 1.0 \times 0.45(2 \times 100 + 150)/6 = 26.3$$

$$K_2 = H(1) \times B(1)(2k_{s,2} + k_{s,1})/6 = 1.0 \times 0.45(2 \times 150 + 100)/6 = 30.0$$

$$K_2' = H(2) \times B(2)(2k_{s,2} + k_{s,3})/6 = 1.0 \times 0.45(2 \times 150 + 174.2)/6 = 42.7$$

$$K_3 = H(3) \times B(3)(2k_{s,3} + k_{s,2})/6 = 1.0 \times 0.45(2 \times 174.2 + 150)/6 = 44.9$$

$$K_3' = 1.0 \times 0.30(2 \times 174.2 + 189.4)/6 = 26.9$$

$$K_4 = 1.0 \times 0.30(2 \times 189.4 + 174.2)/6 = 27.7$$

Summary.

$$K_1 = \mathbf{26.3}\ \text{kN/m}$$

$$K_2 = K_2 + K_2' = 30.0 + 42.7 = \mathbf{72.7}\ \text{kN/m}$$

$$K_3 = K_3 + K_3' = 44.9 + 26.9 = \mathbf{71.8}\ \text{kN/m}$$

$$K_4 = 27.7 + 29.1 = \mathbf{56.8}\ \text{kN/m}, \ldots, \text{etc.}$$

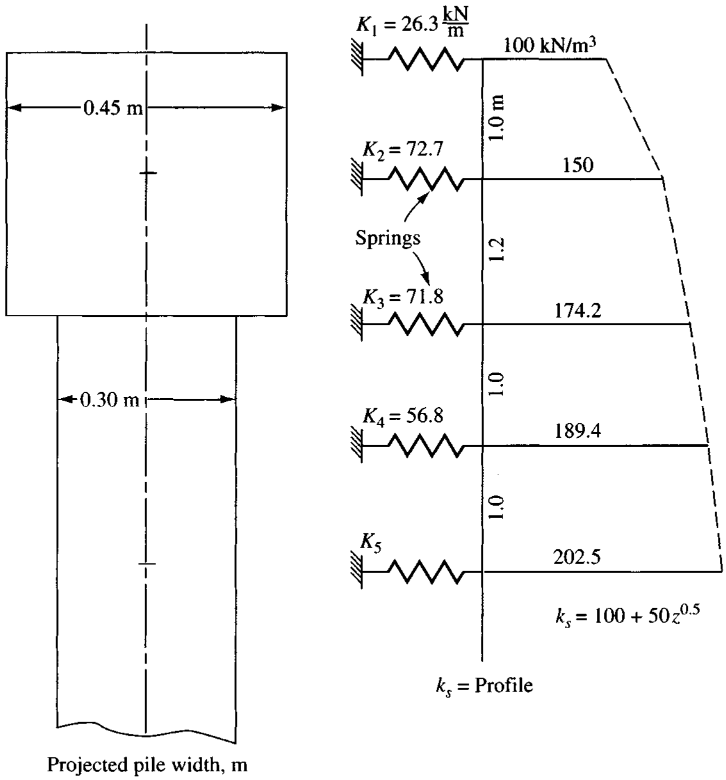

Figure E16–9

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Example 16-9 illustrates a basic difference between this and the sheet-pile program. The sheet-pile section is of constant width whereas a pile can (and the pier or beam-on-elastic foundation often does) have elements of different width.

This program does not allow as many forms of Eq. (9-10) as in FADSPABW; however, clever adjustment of the BS term and being able to input node values are deemed sufficient for any cases that are likely to be encountered.

In addition to the program computing soil springs, you can input $k_s = 0$ so all the springs are computed as $K_i = 0$ and then input a select few to model structures other than lateral piles. Offshore drilling platforms and the like are often mounted on long piles embedded in the soil below the water surface. The drilling platform attaches to the pile top and often at several other points down the pile and above the water line. These attachments may be modeled as springs of the AE/L type. Treating these as springs gives a partially embedded pile model—with possibly a fixed top and with intermediate nonsoil springs and/or node loads—with the base laterally supported by an elastic foundation (the soil).

Since the pile flexural stiffness EI is several orders of magnitude larger than that of the soil, the specific value(s) of k_s are not nearly so important as their being in the range of 50 to about 200 percent of correct. You find this comparison by making trial executions using a k_s, then doubling it and halving it, and observing that the output moments (and shears) do not vary much. The most troublesome piece of data you discover is that the ground line displacement is heavily dependent on what is used for k_s. What is necessary is to use a pile stiff enough and keep the lateral load small enough that any computed (or actual) lateral displacement is tolerable.

A number of persons do not like to use the modulus of subgrade reaction for anything—beams, mats or lateral piles. Generally they have some mathematical model that purportedly works for them and that they would like for others to adopt. In spite of this the k_s concept has remained popular—partly because of its simplicity; partly because (if properly used) it gives answers at least as good as some of the more esoteric methods; and, most importantly, because k_s is about as easy to estimate as it is to estimate the stress-strain modulus E_s and Poisson's ratio μ.

WHAT PILE SECTION TO USE. It is usual to use the moment of inertia I of the actual pile section for both **HP** and other piles such as timber and concrete. For reinforced concrete piles, there is the possibility of the section cracking. The moment of inertia I of a cracked section is less than that of the uncracked section, so the first step in cracked section analysis is to recompute *I based on a solid transformed section,* as this may be adequate.

It is suggested that it is seldom necessary to allow for section cracking. First, one should not design a pile for a lateral load so large that the tension stresses from the moment produce cracking—instead, increase the pile cross section or the number of piles. Alternatively, use steel or prestressed concrete piles.

The possibility of concrete pile cracking under lateral load is most likely to occur when partially embedded piles are used. The unsupported length above the ground line may undergo lateral displacements sufficiently large that the section cracks from the resulting moment-induced tension stresses. The unsupported pile length must be treated similarly to an unsupported column for the structural design, so a larger cross section may be required—at least in the upper portion of the pile.

16-15.1 Empirical Equations for Estimating k_s

Where pile-load tests are not available, some value of k_s that is not totally unrealistic must be estimated, one hopes in the range between ± 50 and ± 200 percent[6] of the correct value. The following equations can be used to make reasonable estimates for the lateral modulus of subgrade reaction.

An *approximation* proposed by the author is to *double* Eq. (9-9) since the soil surrounds the pile, producing a considerable side shear resistance. For input you obtain A_s, B_s values and multiply by two. Using the bearing-capacity components of Eq. (13-1) to give the needed parts of Eq. (9-9), we have

$$A_s = \text{AS} = C(cN_c + 0.5\gamma B_p N_\gamma)$$
$$B_s Z^n = \text{BS} * (\text{Z\^{}N}) = C(\gamma N_q Z^1)$$

where $C = 40$ for SI, 12 for Fps. It was also suggested that the following values could be used, depending on the actual lateral displacement:

[6]Two hundred percent is double the true value, and 50 percent is one-half the true value.

For ΔH,		C		
SI (m)	Fps (in.)	SI	Fps	2C
0.0254	1	40	(12)	80
0.006	$\frac{1}{4}$	170	(48)	340
0.012	$\frac{1}{2}$	80	(24)	160
0.020	$\frac{3}{4}$	50	(36)	100

16-15.2 Size and Shape Factors

The idea of *doubling* the lateral modulus was to account for side shear developed as the pile shaft moves laterally under load, both bearing against the soil in front and shearing the soil on parts of the sides as qualitatively illustrated in Fig. 16-20. Clearly, for piles with a small projected D or B, the side shear would probably be close to the face bearing (consisting of 1.0 for face $+2 \times 0.5$ for two sides $= 2.0$). This statement would not be true for larger D or B values. The side shear has some limiting value after which the front provides the load resistance. Without substantiating data, let us assume this ratio, two side shears to one face, of 1:1 reaches its limit at $B = D = 0.457$ m (18 in.). If this is the case then the *size factor* multiplier (or ratio) C_m should *for single piles* be about as follows (the 1.0 is the face contribution):

For	Ratio, C_m
Lateral loads of both P_x and P_y (face + 1 side)	1.0 + 0.5
$B = D \leq 0.457$ m	$1.0 + 2 \times 0.5$
$B = D > 0.457$	$1.0 + \left(\frac{457}{D, \text{mm}}\right)^{0.75} \geq 1.5$
	use 1.0 + 0.25 for $D > 1200$mm

You should keep the foregoing contributing factors in mind, for they will be used later where the *face* and *side* contributions may not be 1.0 and 0.5, respectively.

Now with C_m, rewrite Eq. (13-1) as used in Sec. 16-15.1 to read

$$\left.\begin{aligned} A_s &= \text{AS} = C_m C(cN_c + 0.5\gamma B_p N_\gamma) \\ B_s Z^n &= \text{BS} * \text{Z\^{}N} = C_m C(\gamma N_q Z^n) \end{aligned}\right\} \tag{16-26}$$

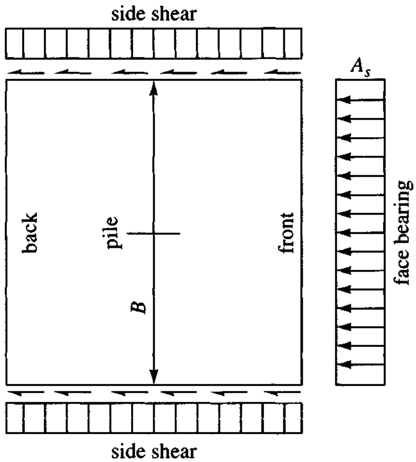
It is also suggested that the BS term should use an exponent n that is on the order of 0.4 to 0.6 so that k_s does not increase without bound with depth.

Research by the author by back-computing k_s from piles in cohesionless soils at the same site indicates that Eq. (9-10) should be further rewritten to read

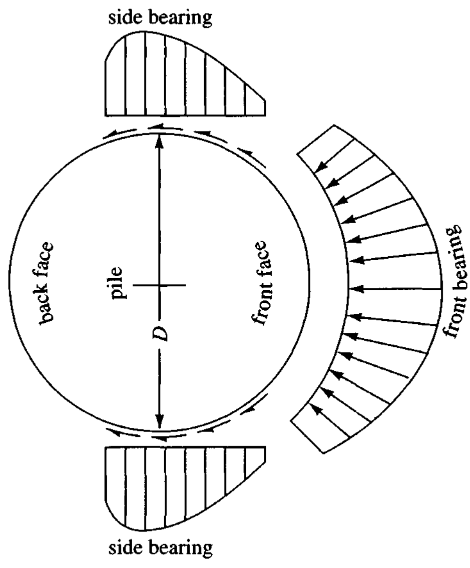
$$\left.\begin{aligned} A_s &= \text{AS} = F_{w1} C_m C(cN_c + 0.5\gamma B_p N_\gamma) \\ B_s Z^n &= \text{BS} * \text{Z\^{}N} = F_{w2} C_m C(\gamma N_q Z^n) \end{aligned}\right\} \tag{16-26a}$$

where F_{w1}, F_{w2} = **1.0** for square and **HP** piles (reference modulus)

F_{w1} = 1.3 to 1.7; F_{w2} = 2.0 to 4.4 for round piles



(*a*) Square of rectangular



(*b*) Circular pile

Figure 16-20 Qualitative front and side resistances for a lateral pile.

One probably should apply the F_i factors only to the *face* term (not side shear) for round piles. Whether these shape factors actually result from a different soil response for round piles or are due to erroneous reported data from neglecting the distortion of the hollow pipe (laterally into an oblate shape) under lateral load is not known at this time. Gleser (1983) and others have observed that the response of a round pile is different from that of a square or **HP** pile, in general agreement with the foregoing except in a case where a comparison of a 100-mm **HP** pile to a 180-mm diameter pipe pile was claimed not to produce any noticeable difference.

Size and projection widths would make it very difficult to note any differences in this case, particularly if the pipe wall thickness was such that the diameter did not tend to oblate (flatten).

USING THE GIVEN BEARING CAPACITY. If we have only the allowable bearing pressure q_a, we can use Eq. (16-26) as follows (but may neglect the N_q term):

$$k_s = F_{w,1} \times \text{SF} \times C_m C \times q_a + F_{w,2} \times C_m C \gamma Z^n N_q \tag{16-26b}$$

where SF = safety factor used to obtain q_a (usually 3 for clay; 2 for cohesionless soil)
N_q = value from Table 4-4 or from Eq. (16-7) or (16-7*d*)
n = exponent as previously defined; 1 is probably too large so use about 0.4 to 0.6 so k_s does not increase too much with depth

If you use either Eq. (16-26) or (16-26a) you should plot k_s for the pile depth using several values of exponent n to make a best selection.

It has been found that the use of Eq. (16-26) produces values within the middle to upper range of values obtained by other methods.

If we take $q_a = q_u$ (unconfined compression test) and omit the N_q term in Eq. (16-26*a*), the value of k_s in *Fps* units for a pile of unknown B is

$$k_s = C_m \times 12 \times \text{SF} \times q_u = 2 \times 3 \times 12 \times q_u = 72q_u$$

Davisson and Robinson (1965) suggested a value of $k_s \approx 67s_u$, which was about half of $72q_u$. Later Robinson (1978) found that $67s_u$ was about half the value of k_s indicated by a series of lateral load tests [that is, $72q_u$ (or $240q_u$, kPa) was about the correct value].

The API (1984) suggests that the lateral bearing capacity for soft clay ($c \le 50$ kPa) be limited to $9c$ and for stiff clay from $8c$ to $12c$ [see Gazioglu and O'Neill (1985) for detailed discussion]. In soft clay this bearing capacity would give, according to Eq. (16-26*a*), the value

$$k_s = C_m(40)(9c) = 360C_m c \qquad (\text{kN/m}^3)$$

which does not appear unreasonable.

You may indirectly obtain k_s from the following type of in situ tests:

a. Borehole pressuremeter tests where E_{pm} = pressuremeter modulus

$$k_s = \frac{3.3E_{\text{pm}}}{B_p} \tag{16-27}$$

For cohesionless soils [see Chen (1978)]:

$$k_s = \frac{3E_{\text{pm}}}{B} \tag{16-27a}$$

And for cohesive soils:

$$k_s = \frac{1.6E_{\text{pm}}}{B} \tag{16-27b}$$

b. Flat dilatometer tests:

$$k_s = \frac{E_d F_p}{3.7B} \tag{16-28}$$

where E_d = dilatometer modulus, kPa or ksf
F_p = pile shape factor: 1.5 to 4.0 for round piles; 1.0 for **HP** or square piles

For these values of k_s you would compute values as close to your pile nodes as possible and input the several node values, not just a single value for the full depth.

The stress-strain modulus E_s can be used in Eq. (16-31) following [or Vesić's Eq. (9-6), given earlier] to compute k_s. Estimate E_s from your equation or method or one of the following:

a. Triaxial tests and using the secant modulus E_s between 0 and 0.25 to 0.5 of the peak deviator stress. The initial tangent modulus may also be used. Do not use a plane strain E_s.

b. The standard penetration test [see Yoshida and Yoshinaka (1972)] to obtain

$$E_s = 650N \qquad \text{kPa} \tag{16-29}$$

This equation has a maximum error of about 100 percent with an average error of close to ± 20 percent. Assume that N in Eq. (16-29) is N_{70} (see under donut hammer of Table 3-3).

For CPT data convert to equivalent SPT N and use Eq. (16-29).

c. Use consolidation test data to obtain m_v to compute the stress-strain modulus by combining Eqs. (2-43) and Eq. (*f*) of Sec. 2-14 and noting

$$\frac{\Delta H}{\Delta p H} = \frac{1}{E_b}$$

to obtain

$$E_s = \frac{3(1 - 2\mu)}{m_v} \tag{16-30}$$

Any of these three values of E_s can be used to compute k_s in clay using any of the following three equations cited by Pyke and Beikae (1983):

$$k_s = \frac{0.48 \text{ to } 0.90E_s}{B} \tag{a}$$

where 0.48 is for **HP** piles; 0.9 for round piles (i.e., a shape factor $F_{w1} \approx 2$);

$$k_s = \frac{1.8E_s}{B} \tag{b}$$

and for sands

$$k_s = \frac{E_s}{B} \tag{c}$$

where in Eq. (*c*) E_s = triaxial test value at about $\epsilon \approx 0.01$. You may also use these stress-strain moduli values in the following equation [Glick (1948)] to obtain a modified k_s' that is then used in Eq. (16-32):

$$k_s' = \frac{22.4E_s(1-\mu)}{(1+\mu)(3-4\mu)[2\ln(2L_p/B) - 0.433]} \qquad \text{(units of } E_s\text{)} \tag{16-31}$$

where L_p = pile length, m or ft
B = pile width, m or ft

After computing k_s', convert it to the usual k_s using the following:

$$k_s = \frac{k_s}{B} \tag{16-32}$$

Since this value of k_s' has the same meaning as the Vesić value given by Eq. (9-6), we can use that equation with the following suggested modification:

$$k_s = \frac{k_{s,v}z^n}{B} \tag{16-32a}$$

The z^n term is suggested to allow some controlled increase in k_s with depth.

The *NAFAC* Design Manual DM7.2 (1982) suggests the following:

$$k_s = \frac{fz}{D} \tag{16-33}$$

where f = factor from following table, kN/m^3 or k/ft^3
D = pile diameter or width, m or ft
z = depth; m or ft gives $k_s = 0$ at ground surface and a large value for long piles at the tips. A better result might be had using $(z/D)^n$ where n ranges from about 0.4 to 0.7.

Values for f (use linear interpolation)

	q_u	D_r	f
Fine-grained:	20	0	200
	40		350
	60		550
	80	15	800
Coarse-grained:		30	800
	110	40	1400
	150	50	2000
	190	60	2800
	230	70	3400
	270	80	4200
	310	90	4900
	370		

TABLE 16-4
Representative range of values of lateral modulus of subgrade reaction (value of A_s in the equation $k_s = A_s + Bz^n$

Soil*	k_s, kcf	k_s, MN/m^3
Dense sandy gravel	1400–2500	220–400
Medium dense coarse sand	1000–2000	157–300
Medium sand	700–1800	110–280
Fine or silty, fine sand	500–1200	80–200
Stiff clay (wet)	350–1400	60–220
Stiff clay (saturated)	175–700	30–110
Medium clay (wet)	250–900	39–140
Medium clay (saturated)	75–500	10–80
Soft clay	10–250	2–40

*Either wet or dry unless otherwise indicated.

Table 16-4 gives ranges of k_s for several soils, which are intended as a guide for probable values using more precise methods—or at least using the site soil for guidance. They should be taken as reasonably representative of the $A_s + B_s$ terms at a depth from about 3 to 6 m and for pile diameters or widths under 500 mm.

16-15.3 Nonlinear Effects

It is well known that doubling the load on a lateral pile usually more than doubles the lateral displacement and increases the bending moment. The moment increase results from both the increase in δ_h and the greater depth in which lateral displacements occur. Both of these effects result from nonlinear soil behavior idealized by the curve shown in Fig. 2-43*c*, particularly at higher stress levels σ that result from larger lateral loads. Usually the lateral displacements in the load range of interest are in that part of the σ–δ curve that is approximately linear.

In the curve of Fig. 2-43*c* the modulus of subgrade reaction is taken as a "secant" line from the origin through some convenient stress value σ. Ideally one should have a curve such as this for each node point (see Fig. E13-1*e*) for a lateral pile. Then, as a displacement is computed one would enter the curve, obtain a revised secant modulus k_s, and recompute the displacements until the δ_h value used $=$ δ_h value obtained.

This approach is seldom practical since these curves are difficult to obtain—usually a pipe pile must be used for the test so that lateral measurements can be taken at nodes below the ground line. A pipe pile, however, has a shape factor, so the results are not directly usable for other pile shapes.

Most lateral piles are designed on the basis of using penetration testing of some kind, supplemented with unconfined compression data if the soil is cohesive. For these cases the two-branch nonlinear model proposed by the author (see Fig. 9-9*c*) will generally be adequate.

The program FADBEMLP on your diskette allows you to model the two-branch nonlinear node displacement curve for the soil as you did in program FADSPABW. That is, you can input the maximum linear displacement at each node as XMAX(I) and activate a nonlinear check using the control parameter NONLIN $>$ 0. Here a negative displacement is not a soil separation, but rather the pile has deflected forward such that the elastic line has produced a displacement at a lower node against the soil behind the pile. An extensive discussion of XMAX(I) was given in Chap. 13 that will not be repeated here except to note that the nonlinear check is $|X(I)| \leq XMAX(I)$.

CYCLIC LOADING. The k_s for cyclic loading should be reduced from 10 to 50 percent of that for static loading. The amount of reduction depends heavily on the displacements during the first and subsequent cycles.

Quasi-dynamic analysis of offshore piles subject to wave forces can be obtained by applying the instant wave force on the nodes in the water zone for several closely spaced discrete time intervals.

DISPLACEMENTS FROM SOIL CREEP. Lateral displacement from long-term loading, producing secondary consolidation or creep, has not been much addressed for lateral piles. Kuppusamy and Buslov (1987) gave some suggestions; however, the parameters needed for the necessary equations are difficult to obtain. Although one could consult that reference, their equations are little better than simply suggesting that, if the lateral load is kept under 50 percent of the ultimate, the creep displacement for sand after several years is not likely to exceed 10 percent of the initial lateral displacement.

For clay, the creep will depend on whether it is organic or inorganic. The creep displacement may be as much as the initial displacement for an organic clay but only about 15 to 20 percent for an inorganic one. One might compute a lateral influence depth of approximately $5 \times$ projected width of pile/pier $= H_f$ and use Eq. (2-49) for a numerical estimate if you have a secondary compression coefficient C_α.

Laterally loaded piles in permafrost also undergo creep. Here the creep depends on the temperature, quantity and type of ice, and the lateral pressure, generally expressed as a "creep" parameter. Neukirchner (1987) claims to have a reasonable solution, but the creep parameter is so elusive that there is substantial uncertainty in any permafrost creep estimate.

When lateral piles undergo creep, the effect is to increase the lateral displacement and bending moment. The goal is an estimate of the final lateral displacement and bending moment. The bending moment might be obtained in any situation where creep is involved by simply measuring the displacements and, using the current lateral displacement as the specified displacement in program FADBEMLP, computing the moment produced by that displacement.

Alternatively since creep decreases approximately logarithmically it might be obtained by plotting the displacement at several time intervals (long enough to be meaningful) and numerically integrating the curve to find the anticipated total lateral displacement for input so as to compute the lateral pile bending moments.

16-15.4 Including the P–Δ Effect

The P–Δ effect can be included for lateral piles (refer to Fig. 16-21) in a straightforward manner as follows:

1. Draw the partially embedded pile to rough scale, code the nodes, and locate the node JTSOIL. We will use JTSOIL as the reference node.
2. Make an execution of the data with the horizontal force P_h located at the correct node above JTSOIL. This will generally be at the top of the pile where the vertical load P_v also

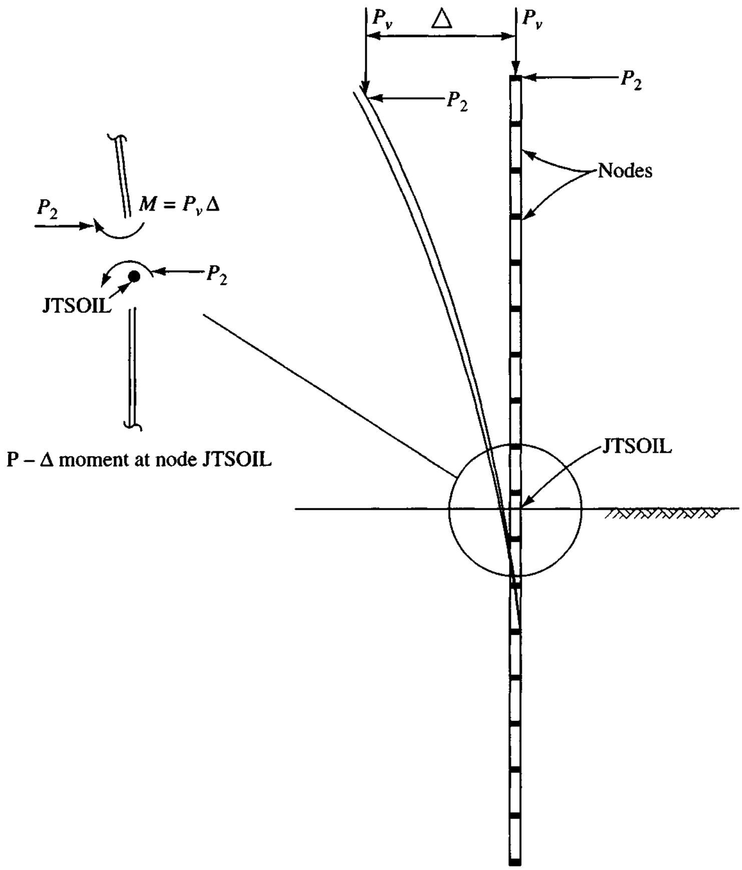
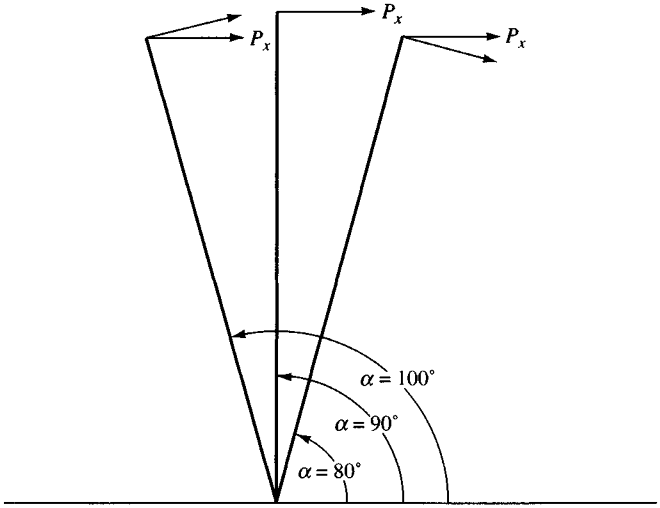


Figure 16-21 The geometric P–Δ effect for laterally loaded piles.

acts. Until you become familiar with program FADBEMLP you should use the pile and load geometry which corresponds to Fig. 16-21.

3. Inspect the output, and at the top node where P_v acts there will be a lateral displacement (let us use, say, $\Delta = 0.40$ m and a vertical force $P_v = 60$ kN). From the lateral displacement, which is with respect to the original position of node JTSOIL, a P–Δ moment can be computed (see inset of Fig. 16-21) of $60 \times 0.40 = 24$ kN·m.
4. Make a copy of the original data and change NNZP from 1 (for the horizontal force only) to 2 to include both the original horizontal force and the P–Δ moment just computed of 24 kN· m. If we assume JTSOIL = 11, the moment NP location is $2 \times 11 - 1 = 21$.
5. In the data file you can see the horizontal load and its NP number. Just below, enter 21 and the moment value of 24. Note from the inset, however, that the moment has a negative sign. The two load matrix entries would now read

Node	Load	
2	P_h	(this is the problem value)
21	−24.0	



Given: $\phi = 32°$; $\beta = 0°$; $\delta = 20°$

unfactored $k_s = 200 + 40Z^n$; Coulomb $K_p = 6.89$ ($\alpha = 90°$)

$\alpha = 100° \rightarrow K_{pb} = 11.35$ (use prog. FFACTOR)	$\alpha = 80° \rightarrow K_{pb} = 4.89$
$C_m = 11.35/6.89 + 2(0.5) = 2.65$	$C_m = 4.89/6.89 + 2(0.5) = 1.71$
$k_s = 2.65(200 + 400Z^n)$	$k_s = 1.71(200 + 400Z^n)$
$= 530 + 1060Z^n$	$= 342 + 684Z^n$

(a) Definition of batter angle α for adjustment of C_m for k_s.

Figure 16-22 Adjusting k_s factor C_m for pile batter and spacing and/or location in group.

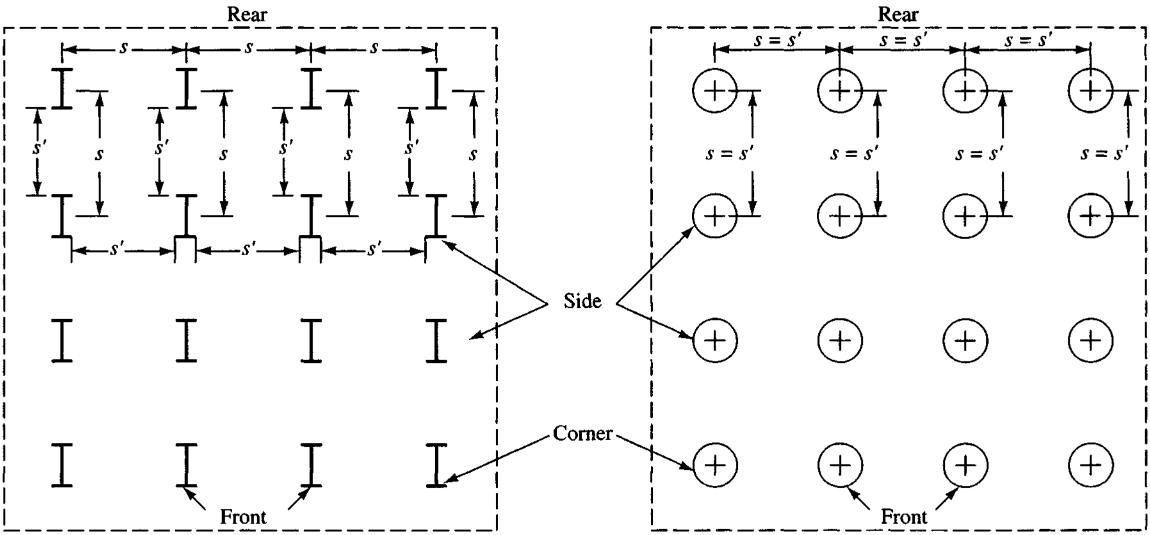
6. Now execute this data set (if the sign is correct the top node displacement Δ will slightly increase). Obtain the displacements, and if the previous $\Delta_p - \Delta_{\text{current}} \leq$ some convergence (not in program but decided by the user), say, 0.005 m or less, stop. Otherwise continue to compute a new P–Δ moment and recycle.

Note that the second data set has two changes initially: (1) to increase NNZP by 1 and (2) to input the P–Δ moment. After this, the only change to that second data set is to reinput the new P–Δ moment until the problem converges.

The node JTSOIL will probably move laterally also, and the most critical P–Δ moment is not the difference between the top node and node JTSOIL but between the top node and some node farther down that does not move laterally. You could, of course, put the P–Δ moment at this location, but the foregoing suggested solution is generally adequate. You can also use the difference between the top translation and the computed translation at node JTSOIL, but this is less conservative.

16-15.5 Lateral Piles on Slopes

Laterally loaded piles are frequently sited on slopes, for example, power poles and bridge foundations. It is suggested that the same procedure be used to reduce the lateral k_s values as was used for the sheet-pile wall case. That is, use program WEDGE or FFACTOR to



(*b*) Pile spacing s' and location for c_m adjustments for k_s.

compute the passive force (or coefficient K_p) case for the horizontal ground line and for the actual ground slope and use the ratio RF as in Eq. (13-3). Because the side shear part from factor C_m is not required to be reduced, you should apply the slope ratio RF *only to the face* (or bearing) part of k_s. For example, compute $k_s = 2000$ based on using $C_m = 2$; RF $= 0.6$. This calculation gives $k_{s1} = 2000/C_m = 1000 = k_{s2}$. The revised $k_s = k_{s2} + \text{RF} \times k_{s1} = 1000 + 0.6 \times 1000 =$ **1600**.

16-15.6 Battered Piles

The k_s for battered piles has not been addressed much in the literature. In the absence of substantiating data the author suggests (see Fig. 16-22*a*) the following:

1. Compute the Coulomb passive earth pressure coefficient K_p for a vertical wall ($\alpha = 90°$), including any slope angle β. A lateral pile is a "passive" earth-pressure case but requires including side shear effects since the Coulomb case is one of plane strain.
2. Next draw the battered pile and place a perpendicular load on the pile with the (+) direction as shown on Fig. 16-22*a*. The perpendicular load direction should correspond to that used to establish the batter direction [will be either (+) or (−)]. Draw a horizontal component line as, say, P_x as shown.
3. Now measure (or compute) the batter angle α. It is *counterclockwise* from a horizontal line at the pile tip for the (+) load perpendicular; it is *clockwise* for a (−) load perpendicular. For the (+) perpendicular shown on Fig. 16-22*a* we have $\alpha > 90°$ if the horizontal component is below and $\alpha < 90°$ if the horizontal component is above the perpendicular.
4. Compute a Coulomb passive pressure coefficient K_{pb} for the applicable batter angle α. Use program FFACTOR. You probably should include a pile-to-soil friction angle δ.

5. Compute a revised k_s as

$$k_s = \left(1.0 \times \frac{K_{pb}}{K_p}\right) + (2 \times 0.5)$$

This calculation should give the expected result of a larger k_s for $\alpha > 90^\circ$ and a smaller k_s for $\alpha < 90^\circ$ for the (+) case shown on Fig. 16-22*a*.

Note: We only adjust the face or bearing part of k_s because the side shear should be about the same for either a vertical or a battered pile.

16-15.7 Adjusting k_s for Spacing

It is generally accepted that there is a reduction in the lateral subgrade modulus k_s when piles are closely spaced. Poulos (1979) suggested using factors from curves developed using an elastic analysis of pile-soil interaction (i.e., E_s, μ), which are then combined to give a *group* factor. This method does not seem to be used much at present.

The following method (refer to Fig. 16-22*b*) is suggested as an easy-to-visualize alternative to obtain the lateral modulus for individual piles in a group:

1. Referring to the Boussinesq pressure bulb (Fig. 5-4) beneath a rectangular footing, we see that at a $D/B > 6$ the pressure increase on the soil is negligible. So, using a *clear* pile spacing s' for depth D and pile projected width for B, we can say that if $s'/B > 6$ no adjustment in k_s is necessary.
2. For spacings of $s'/B < 6$ use Fig. 5-4 ("Continuous") and multiply the face bearing term by (1.0 − interpolated pressure intensity factor). For example at $s'/B = 2$, we obtain 0.29, and the face term is $1.0 \times (1.0 - 0.29) = \mathbf{0.71}$ (here 0.29, or 29 percent, of the pressure is carried by the front pile). This is the *face factor* contribution to C_m (= 2 sides + face $= (2 \times 0.5) + 0.71 = \mathbf{1.71}$).
3. For the *side shear factor* contribution to C_m we have two considerations:
   a. Location (corner, front, side, interior, or rear)
   b. What reduction factor (if any) to use

Clearly for side and corner piles one side is not affected by any adjacent pile so for those we have some interior side interaction factor Ψ + an exterior factor of 0.5. For front, interior, and rear piles we have a side interaction factor of 2Ψ.

One option is to consider that any pile insertion increases the lateral pressure so that the use of $\Psi = 0.5$ is adequate. Another option is to consider that enough remolding takes place that the soil is in a residual stress state and to reduce the 0.5 side factor to

$$\Psi = 0.5 \frac{\text{Residual strength}}{\text{Undisturbed strength}}$$

16-15.8 Estimating Required Length of a Laterally Loaded Pile

The required length of a laterally loaded pile has not been directly addressed in the literature. Obviously, it should be long enough to provide lateral stability, and if there is an axial load, the pile must be long enough to develop the required axial capacity.

We can obtain the required pile embedment length for lateral stability (it was previously noted that usually the upper one-third of the pile actively resists the lateral loads) as follows:

1. Compute the embedment length required for any axial load. If there is no axial load initially, try some reasonable length, say, L'.
2. Use computer program B-5 with your lateral load P_h and obtain a set of output.
3. Inspect the horizontal displacement δ_{hp} at the pile base (or point). If the absolute value of $\delta_{hp} \approx 0.0$, the pile length is adequate. If $|\delta_{hp}| > 0.0$, you have to decide whether the length is adequate, since this amount of displacement may be indicative of a toe kickout (lateral soil failure). Also check that the active (zone of significant bending moment) depth is approximately $L'/3$. Now do two other checks:
   a. Depending on how you initialized L', you may want to increase it by 20 to 30 percent to allow for a modest stability number (SF).
   b. Make two additional program executions using 1/2 and 2 times the initial value of lateral subgrade modulus k_s. If both these executions give $\delta_{hp} \approx 0.0$, you have an adequate pile embedment depth L'. If $\delta_{hp} > 0.0$ (particularly for the $k_s/2$ case), you probably should increase L'.

   If you increase L' based on either (*a*) or (*b*), you should recycle to step 2. When you find an L' value that satisfies the toe-movement criteria, you have a suitable pile embedment depth. The total pile length is then $L_p = L' +$ pile length above soil line.

16-15.9 Pile Constants for Pile Group Analyses

The lateral pile program B-5 can be used to obtain the pile constants needed for the group analysis of Chap. 18. Figure 16-19 illustrates how the node displacements are specified in order to obtain the required computer output. Figure E16-13*c* illustrates how the output is plotted to obtain curve slopes that are the desired constants. The units of these constants produce either shear springs (translation for P/δ) or rotational springs (M/θ). The specific procedure for a given pile is outlined in Example 16-13 following. The general procedure is (*for either partially or fully embedded piles*) to select one of the two axes and do the following:

1. Fix the pile head against translation [NZX = 1 and NXZERO(1) = 2 since NP = 2 is the translation NP at node 1]. Apply a series of moments for NP = 1 (or only one moment if a linear model is assumed). The computer output gives the corresponding rotations at node 1, which are plotted versus M. Also plot the unbalanced force (required to restrain translation) versus M as in Fig. E16-13*c* curve A. The slopes of these two curves are two of the required pile constants.
2. Fix the pile head against rotation [NZX = 1, NXZERO(1) = 1]. Apply a series of lateral loads for NP = 2 (or a single load if a linear model is assumed). The computer outputs translations at node 1, which are plotted versus input load P. Also plot the "near" end moment in element 1 (the rotation-fixed node) versus P. These two plots are shown in Fig. E16-13*c* curve B. The slopes of these two curves are also two of the required pile constants.
3. If the pile is round, the preceding two items complete the necessary computer usage since either axis gives the same output. If the pile is rectangular or an **HP** pile, one set of data

(for four constants) uses the moment of inertia about the x axis and a second set (the other four constants) uses the moment of inertia about the y axis.

4. Strictly, there will be a set of constants for each of the corner, side, front, interior, and rear piles (including batter effects) in a pile group, although some of the constants may be the same for several piles depending on the group geometry. The reason is that the lateral soil modulus k_s will be different for the several piles (although many analyses have been done using a single k_s and set of pile constants for the group). A single k_s is used for Example 16-13 and for the group examples in Chap. 18 to save text space and make the examples easier to follow.

16-16 LATERALLY LOADED PILE EXAMPLES

The following several examples will illustrate computing k_s for a laterally loaded pile and using your program FADBEMLP to analyze lateral piles.

Example 16-10.

Given. A soft silty clay with average $q_u = 47.5$ kPa and, from a consolidation test, $m_v = 5.32 \times 10^{-5}$ m²/kN. An **HP** 310 × 174 pile ($d = 324$; $b = 327$ mm; and $I_x = 394 \times 10^{-6}$ m⁴) is to be used.

Required. What is the lateral k_s by Vesić's Eq. (9-6) and Bowles' method?

Solution.

a. Use Vesić's Eq. (9-6) and take $\mu = 0.45$. We find

$$E_s = \frac{3(1-2\mu)}{m_v} = \frac{3(1-2\times 0.45)}{0.000\,0532} = \mathbf{5639}\text{ kPa} \qquad \text{[from Eq. (16-30)]}$$

$$E_s \approx 200 s_u = 100 q_u = 100 \times 50 = 5000 \text{ kPa}$$

Use $E_s = \mathbf{5300}$ kPa

Using Eq. (9-6) with $E_s = 5300$; $E_{\text{pile}} = 200\,000$ MPa; $B = 327$ mm (0.307 m), we obtain

$$k_s B = 2 \times 0.65 \sqrt[12]{\frac{E_s B^4}{E_p I_p}} \times \frac{E_s}{1-\mu^2} = 1.3 \sqrt[12]{\frac{5300.0 \times 0.327^4}{200 \times 394}} \times \frac{5300.0}{1-0.45^2}$$

$$= 1.3 \times 0.550 \times 5300/0.798 = 4749 \text{ kPa}$$

$$k_s = 4749/0.327 = \mathbf{14\,520} Z^n \text{ kN/m}^3 \qquad \text{(slight rounding)}$$

b. Using Bowles' method and $q_a = q_u$ with an SF = 3, a square pile gives $F_{w,i} = 1.0$, and doubling for side shear, $C_m = 2.0$. Then

$$k_s = F_{w,1} \times 2 \times C \times \text{SF} \times q_u = 1 \times 2 \times 40 \times 3 \times 50 = \mathbf{12\,000} Z^n \text{ kN/m}^3$$

Note that C has units of 1/m.

Check the API method where $q_{\text{ult}} = 9c = 4.5 q_u$.

$$k_s = F_{w,1} \times 2 \times C \times q_{\text{ult}} = 1 \times 2 \times 40 \times 4.5 \times 50 = \mathbf{18\,000} Z^n \text{ kN/m}^3$$

If q_u is the average for the range of the embedment depth of the pile, one would use the exponent $n = 0$.

What would you recommend for k_s for this pile(s)? The author would be reluctant to use much over **10 000Z^n** kN/m based on the range of the three computed values shown.

////

Example 16-11. Given the soil profile of Fig. E16-6 containing average blow counts for each 2.4 m (8 ft) of depth as follows: 10, 15, 20, and 25. Compute a reasonable equation in the form of

$$k_s = \text{AS} + \text{BS} * Z^n$$

Solution. Using Eq. (16-29) and converting the N values given to N_{70}, we obtain k_s at these points:

−1.2	$650 \times N = 650 \times 10(55/70) = 5100$ (rounding)
−3.6	$650 \times 15 \times 0.786 = 7600$
−6.0	$650 \times 20 \times 0.786 = 10\,200$
−8.4	$650 \times 25 \times 0.786 = 12\,700$

These values are used to plot a curve of Z versus k_s, which is approximately linear. If we extend it to $Z = 0$, the intercept is AS = 4000. With this value and at $Z = 8.4$ we solve

$$\text{AS} + \text{BS} \times Z^1 = 12\,700 = 4000 + \text{BS} \times 8.4$$
$$\text{BS} = 1036 \qquad \text{(rounded)}$$

The resulting equation is

$$k_s = 4000 + 1036Z$$

In using this equation we would want to use FAC1 and FAC2 on the first two nodes since sand would have little lateral capacity at $Z = 0$.

////

Example 16-12. This and Example 16-13 require that you use program FADBEMLP on your diskette. The data set for this example is EX1612.DTA. Its use illustrates using several load cases in a single execution—four in this example.

Given. The pile-soil geometry shown in Fig. E16-12*a*, which is from a series of lateral pile tests for a lock and dam on the Arkansas River in the mid-1960s. The approximate data can be found in Alizadeh and Davisson (1970) in Fps units, but the author had access to one of the original reports provided to the U.S. Army Corps of Engineers (who built the lock and dam). The 406-mm (16-in.) diameter pile test was selected for this example. The test used four loads as given in the table on Fig. E16-12*a*.

Solution.

Step 1. Divide the pile into a number of segments. The pile was loaded 0.03 m (0.1 ft) above the ground surface, but this will be neglected. We will take the top two elements as 0.335 m and 0.3 m and increase the lengths to 0.6 for four elements, etc. as shown on the output sheet Fig. E16-12*b*. The pile moment of inertia was given in the report as 0.3489×10^{-3} m^4 (838.2 in.^{4}). The pipe being steel, $E_{\text{pile}} = 200\,000$ MPa. The length was given as 16.12 m (52.8 ft). The width is the pipe diameter, or 0.406 m.

Step 2. Estimate k_s. Use Eq. (16-26*a*) with $C_m = 2.0$; and the shape factors $F_{w,1} = 1.5$ and $F_{w,2} = 3.2$. Obtain from Table 4-4 $N_q = 23.2$ and $N_\gamma = 20.8$; use no depth or shape factors.

$$k_s = 2 \times 40(F_{w,1}\tfrac{1}{2}\gamma' BN_\gamma + F_{w,2}\gamma' N_q Z^1)$$

Loading LC	P_h, kN	Node	M_{max}, kN•m Computed	Measured
1	93.4	5	81	81 *
2	140.1	5	128	115 *
3	191.3	5	166	163
4	249.1	5	216	206

Node 5 = −1.835 m.
Computed moments rounded.
*Only 2 cases for Fig. E16-12b.

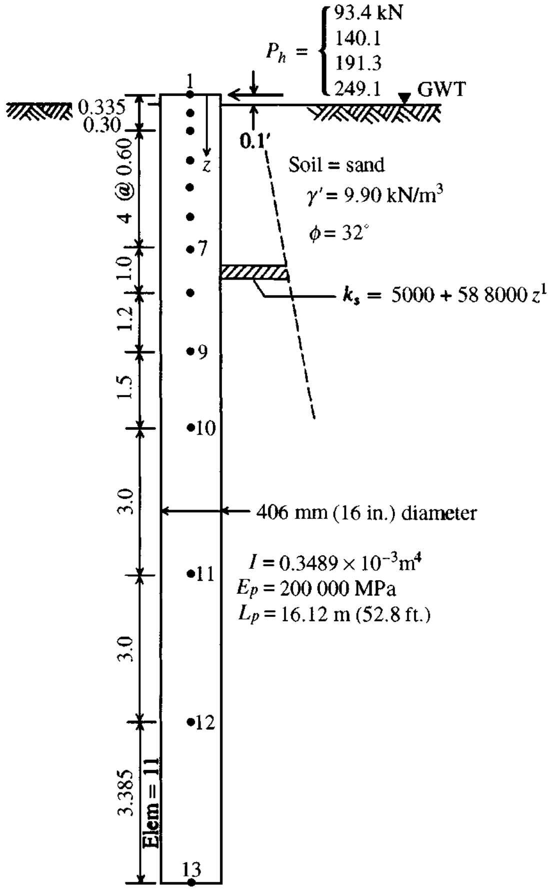


Figure E16-12a

Making substitutions ($\gamma' = 9.8$ kN/m^3), we obtain

$$k_s = 80 \times 1.5 \times 0.5 \times 9.9 \times 0.406 \times 20.8 + 80 \times 3.2 \times 9.9 \times 23.2Z^1$$

$$k_s = 5000 + 58\,800Z^1 \qquad \text{(using minor rounding)}$$

These values are input to the program (and shown on Fig. E16-12*b*). The modulus reduction factors FAC1, FAC2 = 1.0. For node 1 the lateral displacement $\delta_h = 0.00817$ m = 8.17 mm versus about 6.6 mm measured for the 140.1 kN load.

This output compares quite well both in displacements and maximum moment (and its location), and this aside from the fact the lateral modulus was computed only one time using the foregoing input. The results might be somewhat improved using an exponent of 0.4 or 0.6 instead of 0.5, but this supposition is left as a reader exercise. Certainly the output is well within the scatter one would expect in testing several piles at a site.

The file EX1612.DTA was edited to use only two load cases for text output; all four load cases are in the file for reader use.

You have a plot file option in this program by which you can save data to a disk file for later plotting using a CAD plotting program. The file contents are output to paper (but only if the plot

ARKANSAS LOCK AND DAM TEST PILE NO. 2--406 MM (16-IN) PIPE

++++++++++++++++ THIS OUTPUT FOR DATA FILE: EX1612.DTA

SOLUTION FOR LATERALLY LOADED PILE--ITYPE = 1 ++++++++++

NO OF NP = 26 NO OF ELEMENTS, NM = 12 NO OF NON-ZERO P, NNZP = 1
NO OF LOAD CASES, NLC = 2 NO OF CYCLES NCYC = 1
NODE SOIL STARTS JTSOIL = 1
NONLINEAR (IF > 0) = 0 NO OF BOUNDARY CONDIT NZX = 0
MODULUS KCODE = 2 LIST BAND IF > 0 = 0
IMET (SI > 0) = 1

MEMNO	NP1	NP2	NP3	NP4	LENGTH	WIDTH	INERTIA, M**4
1	1	2	3	4	.335	.406	.34890E-03
2	3	4	5	6	.300	.406	.34890E-03
3	5	6	7	8	.600	.406	.34890E-03
4	7	8	9	10	.600	.406	.34890E-03
5	9	10	11	12	.600	.406	.34890E-03
6	11	12	13	14	.600	.406	.34890E-03
7	13	14	15	16	1.000	.406	.34890E-03
8	15	16	17	18	1.200	.406	.34890E-03
9	17	18	19	20	1.500	.406	.34890E-03
10	19	20	21	22	3.000	.406	.34890E-03
11	21	22	23	24	3.000	.406	.34890E-03
12	23	24	25	26	3.385	.406	.34890E-03

THE INITIAL INPUT P-MATRIX ENTRIES

NP	LC	P(NP,LC)
2	1	93.400
2	2	140.100

MOD OF ELASTICITY E = 200000. MPA
GROUND NODE REDUCTION FACTORS FOR PILES, FAC1,FAC2 = 1.00 1.00
EQUATION FOR KS = 5000.0 + 58800.0*Z**1.00

THE NODE SOIL MODULUS, SPRINGS AND MAX DEFL:

NODE	SOIL MODULUS	SPRING,KN/M	MAX DEFL, M
1	5000.0	786.5	.0250
2	24698.0	3095.3	.0250
3	42338.0	8809.4	.0250
4	77618.0	18907.7	.0250
5	112898.0	27502.0	.0250
6	148178.0	36096.2	.0250
7	183458.0	62133.6	.0250
8	242258.0	109943.1	.0250
9	312818.0	174678.4	.0250
10	401018.0	393186.8	.0250
11	577418.0	703295.1	.0250
12	753818.0	986845.7	.0250
13	952855.9	609169.8	.0000

Figure E16-12*b*

BASE SUM OF NODE SPRINGS = 3134450.0 KN/M NO ADJUSTMENTS
* = NODE SPRINGS HAND COMPUTED AND INPUT

MEMBER MOMENTS, NODE REACTIONS, DEFLECTIONS, SOIL PRESSURE, AND LAST USED P-MATRIX FOR LC = 1

MEMNO	MOMENTS--NEAR END 1ST, KN-M		NODE	SPG FORCE, KN	ROT, RADS	DEFL, M	SOIL Q, KPA	P-, KN-M	P-, KN
1	-.001	29.855	1	4.28	-.00299	.00544	27.22	.00	93.40
2	-29.857	52.463	2	13.78	-.00292	.00445	109.93	.00	.00
3	-52.462	78.643	3	31.72	-.00274	.00360	152.44	.00	.00
4	-78.643	80.827	4	40.00	-.00217	.00212	164.19	.00	.00
5	-80.827	66.255	5	27.92	-.00149	.00102	114.63	.00	.00
6	-66.255	44.799	6	11.47	-.00086	.00032	47.10	.00	.00
7	-44.799	11.754	7	-2.71	-.00038	-.00004	8.02	.00	.00
8	-11.754	-4.037	8	-19.89	.00003	-.00018	43.82	.00	.00
9	4.037	-2.094	9	-14.45	.00009	-.00008	25.89	.00	.00
10	2.094	.523	10	.42	.00003	.00000	.43	.00	.00
11	-.523	-.101	11	1.08	-.00001	.00000	.89	.00	.00
12	.101	.000	12	-.24	.00000	.00000	.18	.00	.00
			13	.03	.00000	.00000	.05	.00	.00

SUM SPRING FORCES = 93.41 VS SUM APPLIED FORCES = 93.40 KN

(*) = SOIL DISPLACEMENT > XMAX SO SPRING FORCE AND Q = XMAX*VALUE ++++++++++++
NOTE THAT P-MATRIX ABOVE INCLUDES ANY EFFECTS FROM X > XMAX ON LAST CYCLE ++++++++++

MEMBER MOMENTS, NODE REACTIONS, DEFLECTIONS, SOIL PRESSURE, AND LAST USED P-MATRIX FOR LC = 2

MEMNO	MOMENTS--NEAR END 1ST, KN-M		NODE	SPG FORCE, KN	ROT, RADS	DEFL, M	SOIL Q, KPA	P-, KN-M	P-, KN
1	.001	44.782	1	6.42	-.00448	.00817	40.83	.00	140.10
2	-44.783	78.696	2	20.67	-.00437	.00668	164.90	.00	.00
3	-78.694	117.965	3	47.58	-.00411	.00540	228.66	.00	.00
4	-117.965	121.240	4	60.00	-.00326	.00317	246.29	.00	.00
5	-121.240	99.383	5	41.89	-.00223	.00152	171.95	.00	.00
6	-99.383	67.199	6	17.21	-.00129	.00048	70.65	.00	.00
7	-67.199	17.631	7	-4.07	-.00057	-.00007	12.02	.00	.00
8	-17.631	-6.056	8	-29.83	.00004	-.00027	65.73	.00	.00
9	6.056	-3.141	9	-21.68	.00014	-.00012	38.83	.00	.00
10	3.141	.784	10	.63	.00004	.00000	.65	.00	.00
11	-.784	-.151	11	1.62	-.00001	.00000	1.33	.00	.00
12	.151	.000	12	-.36	.00000	.00000	.27	.00	.00
			13	.04	.00000	.00000	.07	.00	.00

SUM SPRING FORCES = 140.12 VS SUM APPLIED FORCES = 140.10 KN

(*) = SOIL DISPLACEMENT > XMAX SO SPRING FORCE AND Q = XMAX*VALUE ++++++++++++
NOTE THAT P-MATRIX ABOVE INCLUDES ANY EFFECTS FROM X > XMAX ON LAST CYCLE ++++++++++

Figure E16-12*b* (*continued*)

file is created) with headings so you can identify the contents of the plot file. *You can use the paper output to plot shear and moment diagrams by hand if you do not have a plotting program.*

////

Example 16-13. This example illustrates how to obtain pile constants as required for the pile cap analysis using computer program FAD3DPG (B-10) or program B-28. For this analysis an **HP360** $\times$ 174 is used with the required data of $d = 361$ mm; $b = 378$ mm; $I_x = 0.5080 \times 10^{-3}$ m^4; $I_y = 0.1840 \times 10^{-3}$ m^4. These and selected other data are shown in Fig. E16-13*a*, including the element lengths and number of nodes. The soil modulus is somewhat arbitrarily taken as

$$k_s = 200 + 50Z^{0.5}$$

partly to illustrate using an exponent less than 1.0. A spring taken as 0.9 $\times$ computed value is input for the cases of translation but no rotation (the first node spring can be anything since it is not used for the case of no translation but node rotation). The input of a spring here is to illustrate how it is done.

To obtain four sets of pile constants we must make two executions with respect to each principal axis of the pile. In one execution node 1 is fixed to allow rotation but no translation (data set EX1613A.DTA); in the second execution the node is fixed to allow translation but no rotation (EX1613B.DTA). You have this sample output set as Fig. E16-13*b*. Data sets EX1613C.DTA and EX1613D.DTA are similar but with respect to the y axis.

From execution of all the data sets one can plot the Curves A and B of Fig. E16-13*c*. The loads were somewhat arbitrarily chosen after making several trial runs using different values of k_s so that displacements and rotations would be large enough to produce easily identifiable data for the textbook user.

Pile input data: HP360 $\times$ 174   Obtain I_x, b_f; I_y, d from Table A-1

$E = 200\,000$ MPa

10 elements: 3 @ 1, 2 @ 1.5, 2 @ 2, and 3 @ 3 m

$K_x = 200 + 50Z^{0.5}$

REDFAC = 0.9

Comments. (see figures on pages following)

1. $P_h = 50.78$ kN is plotted versus $\delta = 0.06206$ m for one curve with respect to the x axis.
2. The fixed-end moment (from no rotation) of 208.483 kN · m is plotted versus $\delta = 0.06206$ m for a second curve, also with respect to the x axis.
3. The other two curves with respect to the x axis are obtained from executing data set EX1613A.DTA.

////

16-17 BUCKLING OF FULLY AND PARTIALLY EMBEDDED PILES AND POLES

The author, using a method presented by Wang (1967) for buckling of columns of variable cross section, developed a procedure that can be used to obtain the buckling load for piles either fully or partially embedded. The method is easier to use and considerably more versatile, if a computer program such as B-26 is available, than either the methods of Davisson and Robinson (1965) or those of Reddy and Valsangkar (1970). This method can be used to

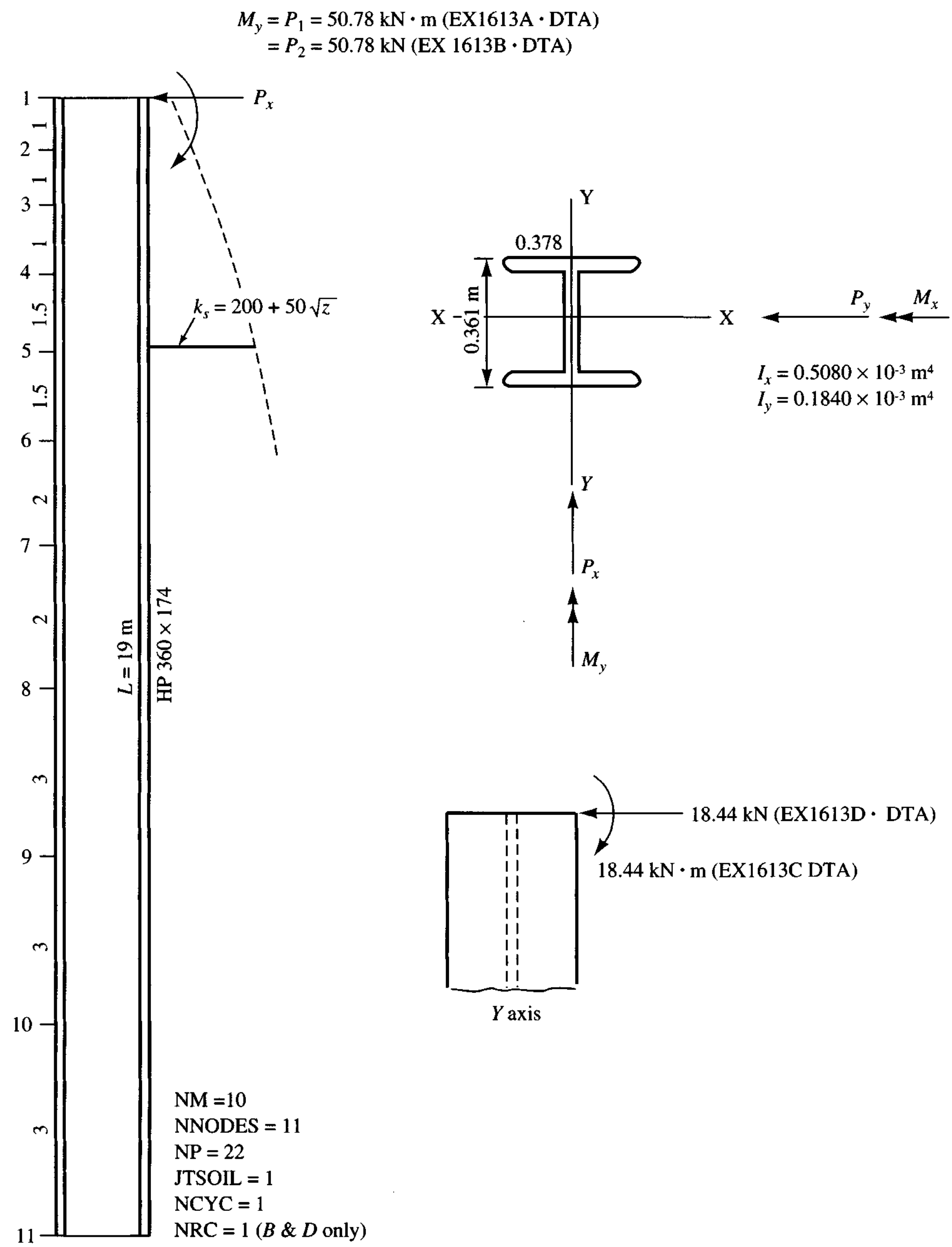


Figure E16-13*a*

USING H360 X 174 TO OBTAIN PILE CONST FOR EXAM 18-7--TRANSL--NO ROTAT

++++++++++++++++ THIS OUTPUT FOR DATA FILE: EX1613B.DTA

SOLUTION FOR LATERALLY LOADED PILE--ITYPE = 1 ++++++++++

```
NO OF NP =  22   NO OF ELEMENTS, NM = 10   NO OF NON-ZERO P, NNZP =  1
  NO OF LOAD CASES, NLC =  1                      NO OF CYCLES NCYC =  1
NODE SOIL STARTS JTSOIL =  1
     NONLINEAR (IF > 0) =  0            NO OF BOUNDARY CONDIT NZX =  1
          MODULUS KCODE =  2                       LIST BAND IF > 0 =  0
                                                       IMET (SI > 0) =  1
```

MEMNO	NP1	NP2	NP3	NP4	LENGTH	WIDTH	INERTIA, M**4
1	1	2	3	4	1.000	.378	.50800E-03
2	3	4	5	6	1.000	.378	.50800E-03
3	5	6	7	8	1.000	.378	.50800E-03
4	7	8	9	10	1.500	.378	.50800E-03
5	9	10	11	12	1.500	.378	.50800E-03
6	11	12	13	14	2.000	.378	.50800E-03
7	13	14	15	16	2.000	.378	.50800E-03
8	15	16	17	18	3.000	.378	.50800E-03
9	17	18	19	20	3.000	.378	.50800E-03
10	19	20	21	22	3.000	.378	.50800E-03

NX BOUNDARY CONDITIONS = 1

BOUNDARY VALUES XSPEC = .0000

THE INITIAL INPUT P-MATRIX ENTRIES

NP	LC	P(NP,LC)
2	1	50.780

MOD OF ELASTICITY E = 200000. MPA

GROUND NODE REDUCTION FACTORS FOR PILES, FAC1,FAC2 = 1.00 1.00

EQUATION FOR KS = 200.0 + 50.0*Z** .50

+++++NUMBER OF NODE SPRINGS INPUT = 1

Figure E16-13*b*

THE NODE SOIL MODULUS, SPRINGS AND MAX DEFL:

NODE	SOIL MODULUS	SPRING, KN/M	MAX DEFL, M
1	200.0	36.8*	.0250
2	250.0	92.7	.0250
3	270.7	102.0	.0250
4	286.6	136.3	.0250
5	306.1	173.3	.0250
6	322.5	214.2	.0250
7	341.4	257.8	.0250
8	358.1	340.5	.0250
9	380.3	430.8	.0250
10	400.0	453.3	.0250
11	417.9	233.6	.0250

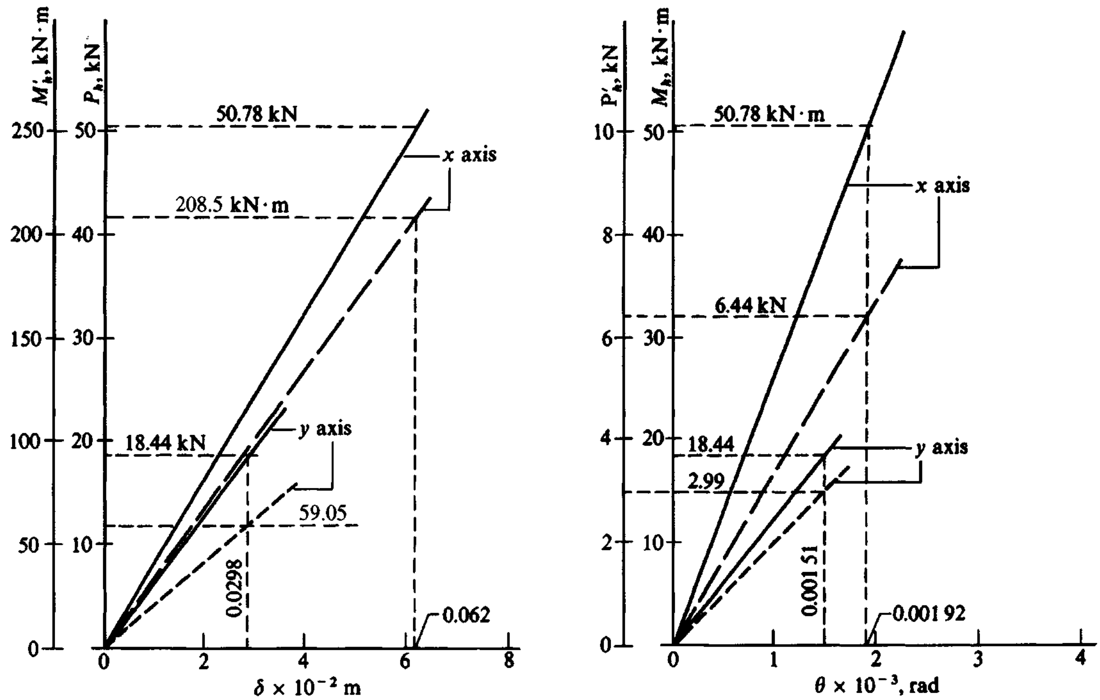
BASE SUM OF NODE SPRINGS = 2475.2 KN/M NO ADJUSTMENTS
* = NODE SPRINGS HAND COMPUTED AND INPUT

MEMBER MOMENTS, NODE REACTIONS, DEFLECTIONS, SOIL PRESSURE, AND LAST USED P-MATRIX FOR LC = 1

MEMNO	MOMENTS--NEAR END 1ST, KN-M		NODE	SPG FORCE, KN	ROT, RADS	DEFL, M	SOIL Q, KPA	P-, KN-M	P-, KN
1	208.483	-159.990	1	2.29	.00000	.06206	12.41	.00	50.78
2	159.988	-117.168	2	5.66	-.00181	.06112	15.28	.00	.00
3	117.161	-80.323	3	5.98	-.00318	.05859	15.86	.00	.00
4	80.320	-36.280	4	7.48	-.00415	.05489	15.73	.00	.00
5	36.281	-4.697	5	8.31	-.00501	.04794	14.67	.00	.00
6	4.698	20.219	6	8.60	-.00531	.04014	12.94	.00	.00
7	-20.218	29.876	7	7.63	-.00516	.02959	10.10	.00	.00
8	-29.876	24.209	8	6.72	-.00467	.01973	7.07	.00	.00
9	-24.209	9.535	9	3.00	-.00387	.00697	2.65	.00	.00
10	-9.535	.000	10	-1.71	-.00337	-.00378	1.51	.00	.00
			11	-3.18	-.00323	-.01361	5.69	.00	.00

SUM SPRING FORCES = 50.77 VS SUM APPLIED FORCES = 50.78 KN

(*) = SOIL DISPLACEMENT > XMAX SO SPRING FORCE AND Q = XMAX*VALUE ++++++++++++
NOTE THAT P-MATRIX ABOVE INCLUDES ANY EFFECTS FROM X > XMAX ON LAST CYCLE ++++++++++



Curve A: translation but no rotation.

Curve B: rotation but no translation.

Figure E16-13c

analyze the buckling load of other pole structures such as steel power-transmission poles [see ASCE (1974) and Dewey and Kempner (1975)] or even columns of varying end conditions.

The method used in program B-26 consists in the following steps:

1. Build the $\mathbf{ASA}^{\mathrm{T}}$ matrix and obtain the $\mathbf{ASA}^{\mathrm{T}}$ inverse of the pile system for whatever the embedment geometry. It is necessary in this inverse, however, to develop the matrix such as shown in Fig. 16-23*a*. All the rotation *P–X* are coded first, then the translation *P–X* values. The resultant matrix can be partitioned as

$$\frac{P_m}{P_s} = \left|\frac{A_1}{A_2}\right|\frac{A_2}{A_3}\left|\frac{X_R}{X_s}\right.$$

2. From the lower right corner of the $\mathbf{ASA}^{\mathrm{T}}$ inverse (Fig. 16-23*b*) take a new matrix called the *D* matrix (of size $NX_s \times NX_s$), identifying the translation or sidesway *X*'s as

$$X_s = \mathbf{D}P_s \qquad (a)$$

3. Develop a "second-order string matrix" considering one node deflection at a time as Fig. 16-24*b*:

$$P'_s = \mathbf{G}X_sP_{\mathrm{cr}} \qquad (b)$$

4. Since P'_s must be equal to P_s, substitute (*b*) into (*a*), noting that P_{cr} is a critical load column matrix for which the placing order is not critical, to obtain

$$X_s = P_{\mathrm{cr}}\{\mathbf{DG}\}X_s \qquad (16\text{-}34)$$

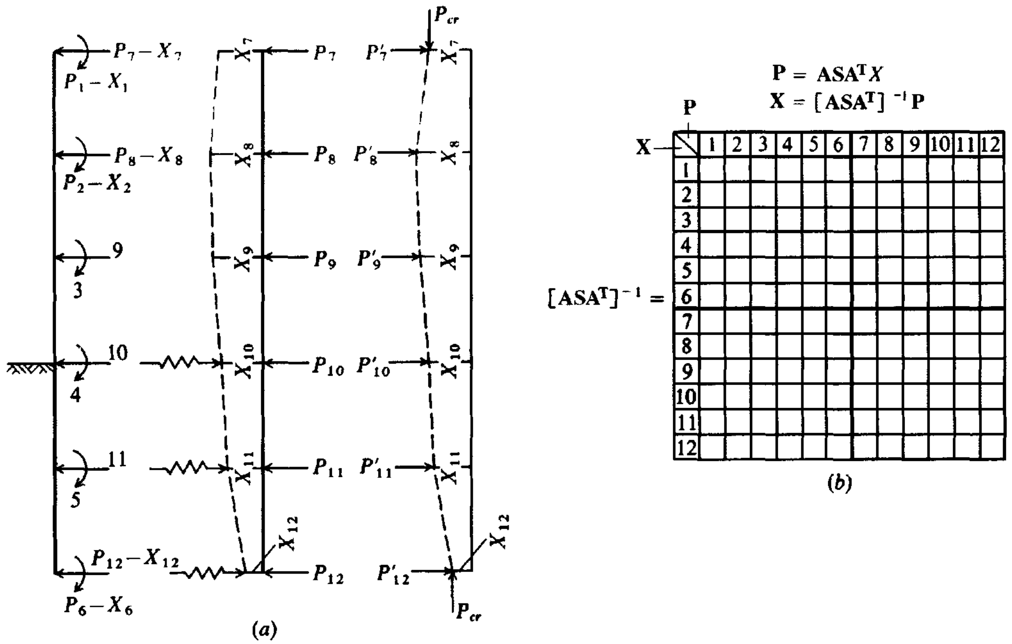


Figure 16-23 (*a*) General coding and notation used in the pile-buckling problem. The ground line can be specified at any node. Develop the $\mathbf{ASA}^T$, invert it, and obtain the **D** matrix from the location shown in (*b*).

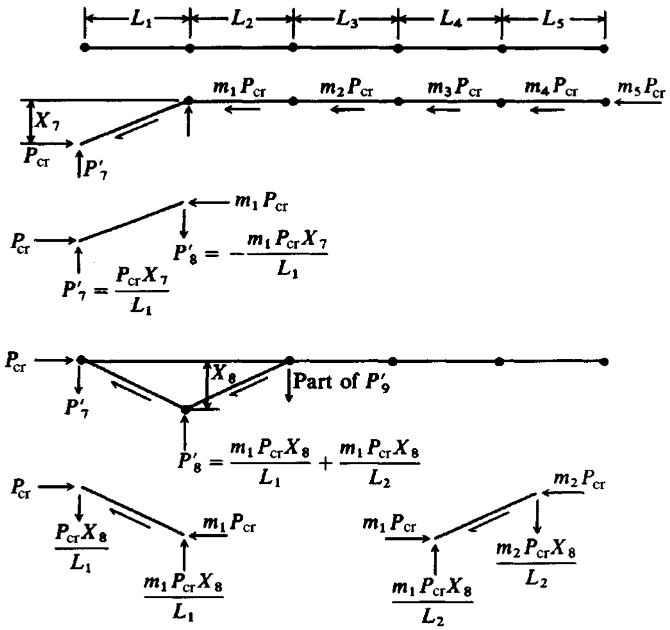
This is an eigenvalue problem, which can be solved to some predetermined degree of exactness (say, $\Delta X = 0.000\,000\,1$) by an iteration process proposed by Wang as follows:

1. Calculate the matrix product of **DG** (size $NX_s \times NX_s$) and hold.
2. As a first approximation set the column matrix $X_s(i) = 1.00$.
3. Calculate a matrix $X_s' = \mathbf{DG}X_s$ using the value 1.00.
4. Normalize the X_s' matrix just computed by dividing all the values by the largest value.
5. Compare the differences of $X_s - X_s' \leq \Delta X$ and repeat steps 2 through 5 until the difference criterion is satisfied. On the second and later cycles the current matrix values of X_s are computed from the values of X_s from one cycle back.
6. When the convergence criterion has been satisfied, compute the buckling load using the largest current values in the X_s' and X_s matrix as

$$P_{\text{cr}} = \frac{X'_{s,\max}}{X_{s,\max}}$$

This step is simply solving Eq. (16-34) for P_{cr} with the left side being the current computation of X_s using the preceding cycle X_s' on the right side.

If higher buckling modes are desired, and one should always compute at least the first two since this method does not always give the lowest buckling load on the first mode (especially



(*a*) Use one node deflection at a time to develop the G matrix.

[G] =

P'_s \ X'_s	7	8	9	10	11	12
7	$\frac{1}{L_1}$	$-\frac{m_1}{L_1}$				
8	$-\frac{1}{L_1}$	$\frac{m_1}{L_1}+\frac{m_2}{L_2}$	$-\frac{m_2}{L_2}$			
9		$-\frac{m_1}{L_2}$	$\frac{m_2}{L_2}+\frac{m_2}{L_3}$	$-\frac{m_3}{L_3}$		
10			$-\frac{m_2}{L_3}$	$\frac{m_3}{L_3}+\frac{m_3}{L_4}$	$-\frac{m_4}{L_4}$	
11				$-\frac{m_3}{L_4}$	$\frac{m_4}{L_4}+\frac{m_4}{L_5}$	$-\frac{m_5}{L_5}$
12					$-\frac{m_4}{L_5}$	$\frac{m_5}{L_5}$

(*b*) The G matrix for the number of elements given in (*a*).

Figure 16-24 The **G** matrix. For partially embedded piles *m* will be 1 until the soil line is encountered.

if the values are close together), one may continue steps 1 through 6 using a revised **DG** matrix for step 1 obtained from the following matrix operation:

$$\{\mathbf{DG}\}_{i+1} = \{\mathbf{DG}\}_i - \frac{1}{(P_{cr}X_s^T\mathbf{G}X_s)_i}(X_s\{\mathbf{G}X_s\}^T)_i \qquad (16\text{-}35)$$

where i identifies the current mode and $i+1$ is the next higher mode. For proof of the validity

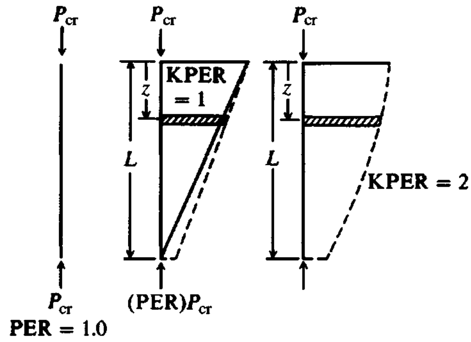


Figure 16-25 Variation of P_{cr} with depth of embedment of the pile or pole. PER = computer program variable used by the author relating the assumed amount of P_{cr} at the point. KPER = computer variable to specify type of skin resistance reduction as shown.

of Eq. (16-35) see Wang (1967). The values of P_{cr} and X are obtained as the values of the ith buckling mode.

Any variation of skin resistance to reduce P_{cr}, as illustrated in Fig. 16-25 to develop the string matrix, can be used. Note that no skin resistance is used in developing the $\mathbf{ASA}^T$ and corresponding **D** matrix since the assumption of small values of rotation and translation for vertical piles does not produce any skin-resistance effect. Note also that the lateral soil resistance effect is included only in the $\mathbf{ASA}^T$ matrix and not in the **G** matrix.

This solution can be readily compared with the theoretical solutions by applying one large soil spring at the top and bottom of the pile and no intermediate values (i.e., the pile becomes a beam column). It is possible to use a method (similar to that in your included computer program B-5) of zeroing boundary conditions, except that this will not work for the case of a fully embedded pile with top and bottom both specified zero. Satisfactory results can usually be obtained with 8 to 15 finite elements.

Example 16-14. To illustrate pile buckling and the effect of soil on buckling of piles, the following example will be presented. Its solution requires use of program FADPILB (B-26), but you can see how buckling loads are affected by the soil from careful study of the example.

Given. A 254-mm diam × 6.35-mm wall (10 × 0.25 in.) pipe pile that is 12 m in length. It is embedded 5 m in an extremely soft soil (average q_u for full depth is only 10 kPa) with the point on rock as shown in Fig. E16-14*a*. We would like to estimate the buckling load. Assume the point carries 50 percent of the buckling load (side friction carries a significant amount of the load of any pile in any soil—even though this is a point-bearing design). Assume further that the side friction distribution is parabolic (KPER = 2) as shown in Fig. 16-23. The first soil spring is reduced 25 percent for driving damage.

Solution. First draw a sketch and locate the pile nodes. Note the *P–X* coding here is automatically done as in Fig. 16-23. That is, the rotation *P–X* values are numbered first, then the translation *P–X* values. The program will also compute the moment of inertia of round solid, round pipe, tapered, and square piles so all you have to input (in this case) is the diameter and wall thickness.

We will have to input k_s, and we will use Eq. (16-26*b*) and not use the N_q-term, giving

$$k_s = F_{w,1} \times \text{SFC}_m C \times q_a = 1.3 \times 3 \times 2 \times 40 \times 10 = \mathbf{3100}\ \text{kN/m}^3 \qquad \text{(rounded)}$$

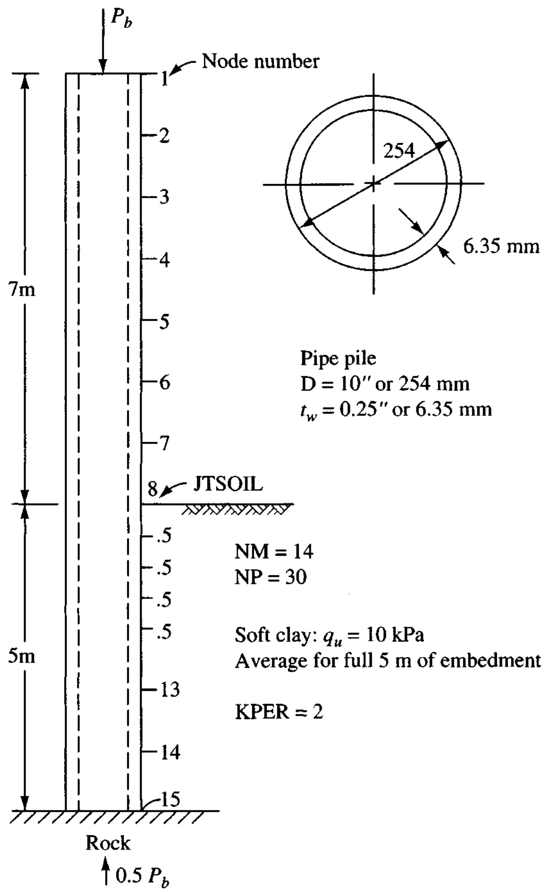


Figure E16-14*a*

The resulting computer output is shown on Fig. E16-14*b*. The *Euler load* shown is for a column fixed at the ground line (making the effective column length 14 m). The Euler equation used is

$$P_{cr} = \frac{\pi^2 EI}{kL^2}$$

where k = 1 for members pinned on each end; = 2 for members fixed on one end; = 0.5 for members fixed on both ends

L = length of column or member

Other terms have been previously defined.

The program uses JTSOIL; when it is 1 (fully embedded pile) the Euler critical load is computed for a column pinned at each end. Of course, if $k_s = 0$ the program inputs lateral node springs $K_i = 0$ so it is actually a column pinned at each end.

The program allows the user to specify boundary cases of fixing one or more nodes, however, in the case of columns one of the nodes should be fixed by inputting a very large spring.

An alternate Euler load for this example would be for a column that is fixed on one end but 12 m in length (effective length = 24 m). Inspection of the Euler load of 381.7 kN versus the computed buckling (or critical) load of 198.0 kN (first mode) seems reasonable. We would expect a partially

254 MM X 6.35 MM TW 12 M L X 5 M EMBEDDED IN SOFT CLAY

+++++++++ NAME OF DATA FILE USED FOR THIS EXECUTION: EX1614.DTA

DIAMETER OF ROUND SECTION = .25400 M WALL THICK = .006350 M

NO OF PILE ELEMENTS = 14
NODE SOIL STARTS = 8 NO OF BUCKLING MODES REQD = 2
PERCENT POINT LOAD = 50.00 % NO OF NODES W/SPRINGS INPUT = 0
GROUND LINE REDUCT FAC = .750
MODULUS OF ELASTICITY = 200000. MPA TOTAL PILE LENGTH = 12.00 M

PARABOLIC SKIN RESISTANCE REDUCTION--KPER = 2

PILE EMBEDMENT DEPTH, DEMB = 5.00 M
EMBED DEPTH SOIL MOD, KS = 3120.000 + .000Z**1.000 KN/M**3

+++++++EULER BUCKLING LOAD = 381.7 KN
BASED ON AVERAGE I = .000038 M**4
LENGTH (OR L ABOVE GROUND) USED = 7.00 M

MEMNO	NP1	NP2	NP3	NP4	ELEM L	WIDTH	I, M**4	NODE	SOIL MOD	SOIL SPRNG	ELEM FRIC
1	1	2	16	17	1.000	.000	.37900E-04	1	.0	.0	1.000
2	2	3	17	18	1.000	.000	.37900E-04	2	.0	.0	1.000
3	3	4	18	19	1.000	.000	.37900E-04	3	.0	.0	1.000
4	4	5	19	20	1.000	.000	.37900E-04	4	.0	.0	1.000
5	5	6	20	21	1.000	.000	.37900E-04	5	.0	.0	1.000
6	6	7	21	22	1.000	.000	.37900E-04	6	.0	.0	1.000
7	7	8	22	23	1.000	.000	.37900E-04	7	.0	.0	1.000
8	8	9	23	24	.500	.254	.37900E-04	8	3120.0	148.6$	1.000
9	9	10	24	25	.500	.254	.37900E-04	9	3120.0	396.2	.995
10	10	11	25	26	.500	.254	.37900E-04	10	3120.0	396.2	.980
11	11	12	26	27	.500	.254	.37900E-04	11	3120.0	396.2	.955
12	12	13	27	28	1.000	.254	.37900E-04	12	3120.0	594.4	.920
13	13	14	28	29	1.000	.254	.37900E-04	13	3120.0	792.5	.820
14	14	15	29	30	1.000	.254	.37900E-04	14	3120.0	792.5	.680
								15	3120.0	396.2	.500

$ = NODE SPRING REDUCED BY FAC = .750

THE BUCKLING MODE SHOWN ON OUTPUT IS USED AS A COUNTER--INSPECTION
OF THE UNIT DEFLECTIONS WILL GIVE THE CURRENT BUCKLING MODE

THE BUCKLING LOAD IS 198.0 KN FOR MODE 1 AFTER 8 ITERATIONS

THE BUCKLING LOAD IS 1712.1 KN FOR MODE 2 AFTER 19 ITERATIONS

NODE DISPLACEMENTS--MAXIMUM OF 3 OUTPUT

MODE NO =	1		2	
NODE	ACTUAL	NORMALIZED	ACTUAL	NORMALIZED
1	.00505	1.00000	.00043	.51830
2	.00424	.83892	.00054	.74109
3	.00344	.68203	.00062	.91452
4	.00269	.53341	.00065	1.00000
5	.00200	.39693	.00062	.97812
6	.00139	.27613	.00053	.85291
7	.00088	.17416	.00040	.65099
8	.00047	.09366	.00025	.41577
9	.00031	.06211	.00018	.30146
10	.00018	.03639	.00012	.19675
11	.00008	.01605	.00006	.10472
12	.00000	.00038	.00002	.02638
13	-.00010	-.02046	-.00006	-.09345
14	-.00017	-.03315	-.00011	-.17971
15	-.00022	-.04303	-.00015	-.25211

Figure E16-14*b*

embedded pile in a very soft soil not to have a buckling load as large as the Euler load of the free-standing part fixed on one end. The computed buckling load of 198 kN should be larger than that of a 12-m column fixed on only one end. This idea is left for the reader to check.

The critical buckling load of 1712.1 kN for the second mode is larger than the first mode. This increase is generally the case, but if the second mode is smaller than the first, then the second buckling mode governs. You should always obtain two buckling modes using a program such as this.

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PROBLEMS

Few answers are provided since a major part of pile design is selection of parameters. When parameters are provided all one does is solve a given equation.

16-1. A 460-mm diameter pipe pile is driven closed-end 15 m into a cohesionless soil with an estimated ϕ angle of 34°. The soil has a $\gamma_{wet} = 16.50$ kN/m^3 and $\gamma' = 8.60$ kN/m^3. The GWT is 6 m below the ground surface. Estimate the ultimate pile capacity P_u using the β method and friction angle $\delta = 22°$.

Answer: $P_u \approx 510$ kN (using $K = 1.5K_o$)

16-2. A **HP**360 × 152 pile is driven into a cohesionless soil with a ϕ angle = 34°. The soil has $\gamma_{wet} = 17.3$ kN/m^3; $\gamma' = 10.1$ kN/m^3 and the GWT is 3 m below the ground surface. Estimate the pile capacity P_u using a pile length of 16 m, the β method, and $\delta = 22°$ soil-to-steel and 26° soil-to-soil (in web zone). Use $K = 1.0$.

16-3. A pile is driven through a soft cohesive deposit overlying a stiff clay. The GWT is 5 m below the ground surface and the stiff clay is at the 8-m depth. Other data:

	Soft clay	Stiff clay
γ_{wet}	17.5	19.3 kN/m^3
γ'	9.5	10.6 kN/m^3
s_u	50	165 kPa

Estimate the length of a 550-mm diam pile to carry an allowable load $P_a = 420$ kN using an SF = 4 and the λ method.

Answer: $L \approx 13$ m

16-4. Redo Problem 16-3 using an **HP**360 × 109 pile.

Answer: $L \approx 16$ to 16.5 m

16-5. A **J** taper Union Monotube pile with a top diam of 457 mm and a taper of 1 : 48 and a length of 12.2 m is driven into a medium stiff clay deposit with an average $s_u = 67$ kPa. The pile will later be filled with concrete. Estimate the ultimate capacity P_u using the α method and the API value.

16-6. A Union Monotube **F** taper shell is driven into a cohesionless deposit with an average $\phi = 34°$. The $\gamma_{wet} = 17.8$ and $\gamma' = 9.8$ kN/m^3, and the GWT is 5 m below the ground surface. The pile top diam = 460 mm and the taper is 1 : 48. For a length of 20 m what is the ultimate pile capacity using Eq. (16-19)?

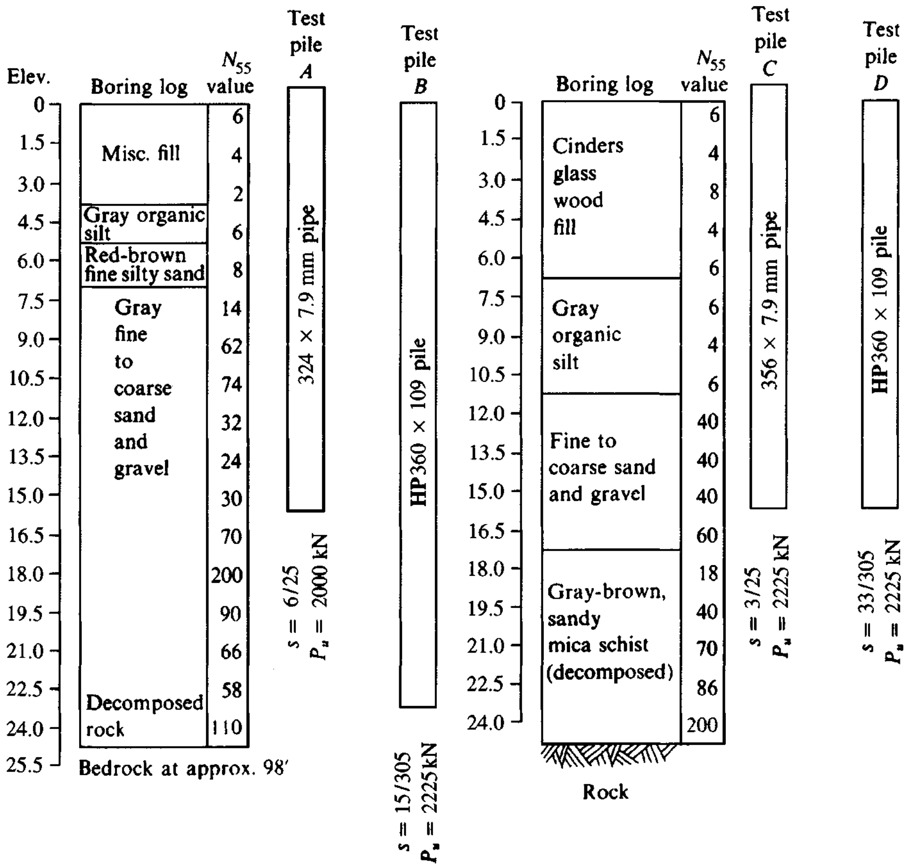

Figure P16-7

16-7. For the assigned boring log and pile (A, B, C, or D) of Fig. P16-7 estimate the pile capacity using Meyerhof's or Vesić's equations for skin resistance and point capacity. These are actual boring logs that have been converted to SI.

16-8. What is the approximate ultimate pullout resistance T_u for a tension pile in a medium dense sand with $\phi \approx 36°$, $\gamma = 18.2$ kN/m^3, and using an 800-mm diameter concrete pile with a length of 5 m (and no bell)?

16-9. For the same data of Prob. 16-8 what is T_u if the diameter is only 300 mm, both without *and* with a 1-m diameter bell?

16-10. Verify the skin resistance of the sand layers given on Fig. E16-7*b*.

16-11. Verify the skin resistance of the clay layers given on Fig. E16-7*b*. Recompute the α values. Also, what is the effect if you use a single 27-m layer with $\alpha = 1$ instead of the three layers of the example?

16-12. See if you can reproduce the settlement computed and shown on the output sheets of Fig. E16-7*b*.

16-13. Redo Example 16-8 for $P_a = 170$ kN and with $c_s = 0.22$ g·cal.

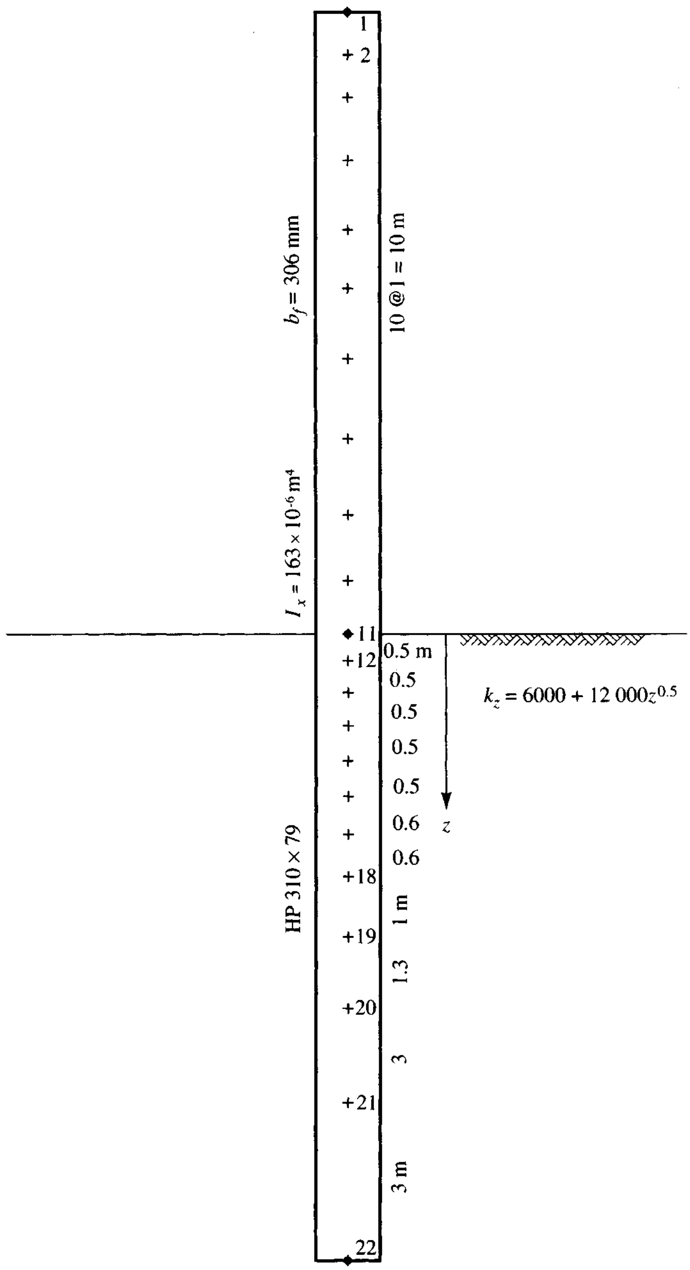

Figure P16-18

16-14. Check the side resistance of Example 16-8 and estimate if creep will be a problem. If creep is a problem, how can you reduce its effect?

16-15. What is P_u for Fig. 16-18 if the pile perimeter = 1.3 m; AE = 2600 MN; L_i = 2 m (for all three elements); and Δy_p = 3 mm? Assume the point load P_p = 40 kN.
Answer: $P_u \approx 657$ kN

16-16. Do Example 16-12 for the other two load cases and, together with those given on Fig. E16-12*b*, make a plot of P_h versus displacement δ. Also plot the shear and moment diagrams for the assigned load case. If the P_h versus δ plot is linear, what can be done to make it somewhat nonlinear since real plots of this type are seldom linear except near the origin?

16-17. Make a copy of data set EX1612.DTA as EX1612A.DTA and apply a lateral load of P_h = 40 kN at node 1. Then make a second copy and fix node 4 against translation; make a third copy and input a zero spring at node 4. Compare the results and answer the following:

a. What external cause could produce a fixed node 4?
b. What would reduce the spring at node 4 to 0?

16-18. Referring to Fig. P16-18 (see previous page), code and make an estimate of the P–Δ effect [i.e., solve with the horizontal load, then resolve where you input a moment (need a 2nd NZX) produced by the vertical load $P_v \times \Delta_{top}$ with respect to the dredge line, continue doing this until δ_{top} converges within about 0.01 m]. The two initial data sets are included as HP1619.DTA and HP1619A.DTA on your program diskette.

16-19. Redo Example 16-13 using loads as follows:

x-axis	*y*-axis
$P_h = M_y = 40$	$P_h = M_x = 20$ kN or kN · m

Plot the results and see if there is any difference in the computed curve slopes. Explain why there is or is not a difference.

16-20. Compute the Euler load for the pile of Example 16-14, assuming it is 14 m long and fixed at the end bearing on rock, and compare your result with the buckling load shown on Fig. E16-14*b*.

16-21. Verify that the moment of inertia for the concrete base of Problem 16-22 would be input as 1.744 ft^4 so that E_{st} = 30 000 ksi applies to all the pile elements. The E_c = 4000 ksi.

16-22. If you have the pile buckling program FADPILB (B-26) compute the buckling load for the tapered power transmission pole shown in Fig. P16-22. All element lengths are equal.

$$L = 10 \text{ ft (element lengths—use average diameter for } I)$$
$$E_{steel} = 30\,000 \text{ ksi} \qquad E_c = 4\,000 \text{ ksi}$$

Element I in order from top down:

0.07, 0.095, 0.125, 0.155, 0.190, 0.240
0.295, 0.350, 0.410, 0.475, 0.550, 0.640
0.735, 0.825, 1.744, 1.744

$$\text{Use } k_s = 100 + 100Z^1$$

Answer: P_{cr} = 216.5 kips (requires program B-26)

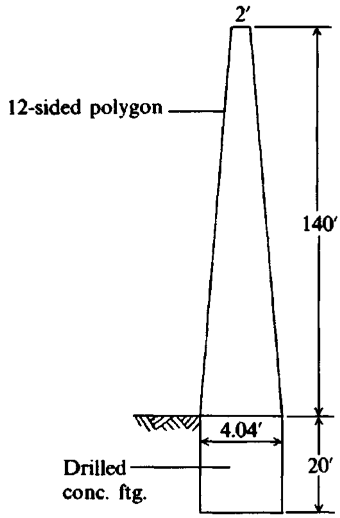

Figure P16-22