CHAPTER 12

MECHANICALLY STABILIZED EARTH AND CONCRETE RETAINING WALLS

12-1 INTRODUCTION

Retaining walls are used to prevent retained material from assuming its natural slope. Wall structures are commonly used to support earth, coal, ore piles, and water. Most retaining structures are vertical or nearly so; however, if the α angle in the Coulomb earth-pressure coefficient of Eq. (11-3) is larger than 90°, there is a reduction in lateral pressure that can be of substantial importance where the wall is high and a wall tilt into the backfill is acceptable.

Retaining walls may be classified according to how they produce stability:

- 1. Mechanically reinforced earth-also sometimes called a "gravity" wall
- 2. Gravity-either reinforced earth, masonry, or concrete
- 3. Cantilever-concrete or sheet-pile
- 4. Anchored-sheet-pile and certain configurations of reinforced earth

At present, the mechanically stabilized earth and gravity walls are probably the most used—particularly for roadwork where deep cuts or hillside road locations require retaining walls to hold the earth in place. These walls eliminate the need for using natural slopes and result in savings in both right-of-way costs and fill requirements.

Cantilever walls of reinforced concrete are still fairly common in urban areas because they are less susceptible to vandalism and often do not require select backfill. Typically they compete well in costs where the wall is short (20 to 50 m in length) and not very high (say, under 4 m). They are also widely used for basement walls and the like in buildings.

This chapter will investigate the basic principles of the reinforced earth, gravity, and concrete cantilever wall; the sheet-pile cantilever and anchored walls will be considered separately in the next two chapters.

12-2 MECHANICALLY REINFORCED EARTH WALLS

The mechanically reinforced earth wall of Fig. 12-1 uses the principle of placing reinforcing into the backfill using devices such as metal strips and rods, geotextile strips and sheets and grids, or wire grids. There is little conceptual difference in reinforcing soil or concrete masses—reinforcement carries the tension stresses developed by the applied loads for either material. Bond stresses resist rebar pullout in concrete; soil relies on friction stresses developed based on the angle of friction δ between soil and reinforcement or a combination of friction and passive resistance with geo- and wire grids.

The principle of reinforced earth is not new. Straw, bamboo rods, and similar alternative materials have long been used in technologically unsophisticated cultures to reinforce mud bricks and mud walls. Nevertheless, in spite of this long usage French architect H. Vidal was able to obtain a patent (ca. mid-1960s) on the general configuration of Fig. 12-1, which he termed "reinforced earth." We see three basic components in this figure:

- 1. The earth fill—usually select granular material with less than 15 percent passing the No. 200 sieve.
- 2. Reinforcement—strips or rods of metal, strips or sheets of geotextiles, wire grids, or chain link fencing or geogrids (grids made from plastic) fastened to the facing unit and extending into the backfill some distance. Vidal used only metal strips.
- **3.** Facing unit—not necessary but usually used to maintain appearance and to avoid soil erosion between the reinforcements.



Figure 12-1 The reinforced earth concept. [After Vidal (1969).]

These three components are combined to form a wall whose side view is shown in Fig. 12-2a. The facing units may be curved or flat metal plates or precast concrete strips or plates (see Fig. 12-2b). Where geotextiles are used the sheet may lap, as in Fig. 12-3, to produce the facing unit.

When wire mesh or or other reinforcement with discontinuities (grid voids) is used, a portion may be bent, similar to the sheet of Fig. 12-3, to form a facing unit. Grid-type







(b) Front face of a reinforced earth wall under construction for a bridge approach fill using patented precast concrete wall face units (c) Backside of wall in (b), which shows the reinforcing strips attached to the wall face units. Note the drain pipe to carry runoff from the future road surface. Recent rain has eroded soil beneath reinforcement strips at wall, which will have to be carefully replaced. Also shown are interlocking dowels and lifting devices (D rings), which weigh around 2 kips each.





(d) A low reinforced earth wall showing a different concrete facing unit pattern (also patented). Note top cap includes a drainage depression that empties into a drop inlet barely seen at forward end.

reinforcements strengthen the soil through a combination of friction and passive pressure pullout resistance. The bent-up portion used as a facing piece provides some erosion control until the wall is completed.

The exposed reinforcements are usually sprayed with concrete mortar or gunite (material similar to mortar) in lifts to produce a thickness on the order of 150 to 200 mm. This is both to improve the appearance and to control erosion. For metals this covering also helps



Figure 12-3 Using geotextile sheets for reinforcement with the facing unit formed by lapping the sheet as shown. Critical dimensions are L_e , L'_o , and L_o . Distances L_e and L_o are variable but for this wall produce a constant length $L_{\rm con} = L_o + L_e$. The Rankine $\rho \neq 45^\circ + \phi/2$ for backfill β as shown. Use your program SMTWEDGE (B-7) to find ρ , and make a scaled plot to check computed lengths.

control rust, and for geotextiles it provides protection from the ultraviolet rays¹ in sunlight and discourages vandalism.

The basic principle² of reinforced earth is shown in Fig. 12-4 where we see a wall acted on by either the Rankine or Coulomb active earth wedge. Full-scale tests have verified that the earth force developed from the active earth wedge at any depth z is carried by reinforcing strip tension.

Strip tension is developed in the zone outside the active earth wedge from the friction angle δ between strip and soil and the vertical earth pressure γz on the strip. With no lateral earth pressure left to be carried by the wall facings they can be quite thin and flexible with the principal functions of erosion control and appearance.

The following several factors enter into the design of a reinforced earth wall:

1. Backfill soil is usually specified to be granular; however, recent research indicates that we can use cohesive soil if a *porous* geotextile is used for reinforcement to allow backfill drainage. This allows one to use the drained friction angle ϕ' to calculate friction between the soil and reinforcing.

For cohesive materials, either use a narrow vertical back face zone of granular material or, alternatively, use strips of a permeable geotextile for vertical drainage.

¹Most geotextiles have a rating of strength loss versus amount of ultraviolet exposure. ASTM D 4355 gives a standard in which geotextile strength loss is reported for 150 hours of exposure.

²An extensive literature survey along with a number of applications, primarily in Europe, is given by Ingold (1982).



Figure 12-4 The general concept of reinforced earth is that $\sum T_i = P_a \cos \delta$, so the earth force against the wall (or facing units) = 0.

2. Backfill soil should be compacted, taking care not to get equipment too close to the facing unit, so that it is not pulled from the reinforcement.

It is also necessary to exercise care with geotextile fabrics not to tear the fabric in the direction parallel to the wall. A partial tear of this type would reduce the amount of tension the fabric can carry.

- 3. Tests with experimental walls indicate that the Rankine wedge (of angle $\rho = 45^{\circ} + \phi/2$) adequately defines the "soil wedge." This angle should be routinely checked using the trial wedge method (or computer program) for large backfill β angles.
- 4. The wall should be sufficiently flexible that the active earth pressure wedge forms and any settlement/subsidence does not tear the facing unit from the reinforcement.
- 5. It is usual to assume all the tension stresses are in the reinforcement outside the assumed soil wedge zone—typically the distance L_e of Fig. 12-5.
- 6. The wall failure will occur in one of three ways:
 - a. Tension in the reinforcements
 - b. Bearing-capacity failure of the base soil supporting the wall, as along the baseline AB of Figs. 12-3 and 12-6.
 - c. Sliding of the full-wall block (ACDB of Fig. 12-6) along base AB.
- 7. Surcharges (as in Fig. 12-6) are allowed on the backfill. These require analysis to ascertain whether they are permanent (such as a roadway) or temporary and where located. For example:
 - *a*. Temporary surcharges within the reinforcement zone will increase the lateral pressure, which in turn increases the tension in the reinforcements but does not contribute to reinforcement stability.



Figure 12-5 Length of reinforcements $L_o = L_R + L_e$ as required but must extend beyond Rankine/Coulomb earth-pressure wedge.

- b. Permanent surcharges within the reinforcement zone will increase the lateral pressure and tension in the reinforcements and will contribute additional vertical pressure for the reinforcement friction.
- c. Temporary or permanent surcharges outside the reinforcement zone contribute a lateral pressure, which tends to overturn the wall.

In most cases the lateral pressure from a backfill surcharge can be estimated using the Theory of Elasticity equation [Eq. (11-20)]. One can also use the Boussinesq equation for vertical pressure, but it may be sufficiently accurate to use the 2 : 1 (2 on 1) method [Eq. (5-2)] adjusted for plane strain to give

$$q_v = \frac{Q}{B+z}$$

- where $Q = Bq_o$ for the strip width (side view) and average contact pressure produced by the surcharge; for point loads use either a unit width (0.3 m or 1 ft) or Eq. (5-3). Since these two methods give greatly differing vertical pressures (the 2 : 1 is high and Eq. (5-3) is very low) you may have to use some judgment in what to use—perhaps an average of the two methods.
 - B = strip width; you are implicitly using L = 1 unit of width.



Figure 12-6 General wall case with surcharge on backfill as from a road or other construction. Linearizing the surcharge pressure profile as shown is sufficiently accurate.

Laba and Kennedy (1986) used the 2:1 vertical pressure method [Eq. (5-2)] as shown in Fig. 12-5 with reasonably good results. In this figure Eq. (5-2) is being used to get a pressure increase in the zone L_1 so that the friction resistance F_R for the effective lengths $(L_e = L_1 + L_2)$ is

$$F_r = \tan \delta[(\gamma z + \Delta q)L_1 + \gamma z L_2]$$

where terms are identified in Fig. 12-5.

- 8. Corrosion may be a factor where metal reinforcements are used. It is common to increase the theoretical strip thickness somewhat to allow for possible corrosion within the design period, which may be on the order of 50 to 100 years.
- 9. Where aesthetics is critical, a number of concrete facing unit configurations are available in a wide range of architecturally pleasing facades, which can either outline the wall or blend it into the landscape (Figs. 12-2b, d).
- 10. There will be two safety factors SF involved. One SF is used to reduce the ultimate strength of the reinforcements to a "design" value. The other SF is used to increase the computed length L_e required to allow for any uncertainty in the backfill properties and soil-to-reinforcement friction angle δ .

12-3 DESIGN OF REINFORCED EARTH WALLS

The design of a reinforced earth wall proceeds basically as follows:

- 1. Estimate the vertical and horizontal spacing of the reinforcement strips as in Fig. 12-7. Horizontal spacing s is meaningless for both wire grids and geotextile sheets but one must find a suitable vertical spacing h for those materials. The vertical spacing may range from about 0.2 to 1.5 m (8 to 60 in.) and can vary with depth; the horizontal strip spacing may be on the order of 0.8 to 1.5 m (30 to 60 in.). The lateral-earth-pressure diagram is based on a unit width of the wall but is directly proportional to horizontal spacing s.
- 2. Compute the tensile loads of the several reinforcements as the area of the pressure diagram contributing to the strip. This calculation can usually be done with sufficient accuracy by computing the total lateral pressure at the strip (see Fig. 12-6) level,

$$q_{h,i} = q_h + \Delta q_h \tag{12-1}$$

- where q_h = Rankine or Coulomb lateral earth pressure, taking into account backfill slope and any uniform surcharge
 - Δq_h = lateral pressure from any concentrated backfill surcharge; obtain using your computer program SMBLP1

With the average pressure obtained from Eq. (12-1), the strip tensile force can be computed as

$$T_i = A_c q_{h,i} \tag{12-1a}$$

where $A_c = \text{contributory area, computed (including the horizontal spacing s) as}$

$$A_c = \frac{h_i + h_{i+1}}{2}s$$

One should routinely make a computational check:

$$\sum T_i = s \times (P_{ah} + \text{area of } \Delta q \text{ diagram})$$
(12-1*b*)





That is, the sum of the several tensile reinforcement forces should equal the lateral-earthpressure diagram ratioed from a unit width to the actual reinforcement spacing *s*.

Although Fig. 12-6 does not show the correct pressure profile for a surcharge q_o and $\beta > 0$ (for that case refer to Fig. 11-9c and use K_a , which includes the effect of β), it is a common case. The other common case is a sloping backfill (Fig. 11-9b) but no concentrated surcharge.

3. Compute the strip lengths L_e of Fig. 12-5 that are required to develop a friction resistance $F_r = T_i \times SF$ (or $L_{e,design} = L_{e,computed} \times SF$). From these lengths and the Rankine wedge zone we can then determine the overall strip length L_o to use. It is common to use a single length for the full wall height so that the assembly crew does not have to be concerned with using an incorrect length at different elevations; however, this choice is a designer's prerogative. The friction length is based on soil-to-strip friction of $f = \tan \delta$, where $\delta =$ some fraction of ϕ such as 1.0, 0.8, 0.6 ϕ . What to use depends on the roughness of the strip (or geotextile sheet). For rough materials use $\delta = \phi$; for smooth metal strips use $\delta \approx 20$ to 25°.

For strips of $b \times L_e$ or geotextile sheets of base width $\times L_e$, both sides resist in friction. For round bars the perimeter resists friction. In both cases friction is the product of $f \times$ normal pressure on the reinforcement, computed as $p_o = \gamma z_i$ where $z_i = average$ depth from ground surface to reinforcement. Using consistent units, this approach gives the following reinforcements:

- Strip: $F_r = 2(\gamma z_i)(b \times L_e) \tan \delta \ge T_i \times SF$ (12-2*a*)
- Rod: $F_r = \pi D(\gamma z_i) L_e \tan \delta \ge T_i \times SF$ (12-2b)

Sheet:
$$F_r = 2(\gamma z_i)(1 \times L_e) \tan \delta \ge T_i \times SF$$
 (12-2c)

where b = strip width, D = rod diameter, and 1 = unit sheet width. Manufacturers provide geotextiles in rolls of various lengths and widths.³ For the year 1993 and earlier, the *Specifier's Guide* of fabric specifications listed roll dimensions of geotextiles the given manufacturer could supply. For 1994 and later, the roll dimensions are no longer supplied. The supplier should be contacted prior to design to see what fabric dimensions can be provided.

4. Next compute the reinforcement area for strips $b \times t$ and for rods with bar diameter D. For wire and geotextile grids, obtain the tension force per some unit of width. For geotextile sheets look in the manufacturer's catalog to find a fabric with a suitable strength.

For these materials a suitable SF must be used to reduce the ultimate tensile strength of metal strips and bars to a design value or the geotextile strength (which is, by the way, orientation-sensitive) to a design value. For metals it is common to use some SF such as 1.5 to 1.67; however, for both metals and geotextiles we can compute an SF based on partial safety factors as follows:

$$T_{\text{allow}} = T_{\text{ult}} \left(\frac{1}{\text{SF}_{\text{id}} \times \text{SF}_{\text{cr}} \times \text{SF}_{\text{cd}} \times \text{SF}_{\text{bd}} \times \text{SF}_{\text{if}} \times \text{SF}_{\phi}} \right)$$
(12-3)

³The Industrial Fabrics Association International, 345 Cedar St., Suite 800, St. Paul, MN, 55101, Tel. 612-222-2508, publishes a quarterly magazine *Geotextile Fabrics Report* and an annual *Specifier's Guide*, which tabulates available geotextile fabrics and select engineering properties such as tensile strength and permeability.

where T_{allow} = allowable tensile stress

- $T_{\rm ult}$ = ultimate tensile stress
- SF_{id} = installation damage factor, 1.1 to 1.5 for geotextiles; 1 for metal
- SF_{cr} = creep factor (1.0 to 3.0 for geotextiles; 1 for metal)
- SF_{cd} = factor for chemical damage or corrosion (about 1.0 to 1.5 for geotextiles; 1.0 to 1.2 for metal)
- SF_{bd} = factor for biological degradation (about 1.0 to 1.3 for geotextiles; 1.0 to 1.2 for metal)
- SF_{if} = importance factor (1.0 to 1.5)
- SF_{ϕ} = general factor; (about 1.0 for geotextiles; about 1.3 to 1.4 for metal)

Koerner (1990 in Table 2-12, p. 115) gives some ranges for the partial factors of safety. The preceding values (not all are in his table) can be used, since you have to estimate them anyway.

Let us compute an allowable tensile stress f_a for a steel strip based on 350 MPa steel (factors not shown are 1.0) as

$$f_a = 350 \frac{1}{1.1 \times 1.2 \times 1.3} = \frac{350}{1.716} = 204 \rightarrow 200 \text{ MPa}$$

Let us now consider a geotextile example. From the 1995 *Specifier's Guide* we find an Amoco 2044 woven (W) geotextile with a wide-width tensile strength, using the ASTM D 4595 method, of 70.05 kN/m in both the MD (along the roll) and XD (across the roll) directions. The allowable tensile strength is computed using Eq. (12-3). Substituting some estimated values, we obtain

$$T_{\text{allow}} = 70.05 \frac{1}{1.5 \times 2.0 \times 1.2 \times 1.1 \times 1.1 \times 1.0} = \frac{70.05}{4.356}$$

= 16.08 \rightarrow 16.0 kN/m

12-3.1 General Comments

For geotextiles we have a problem in that the fabric strength varies

- 1. Between manufacturers.
- 2. With fabric type and grade. For example, woven fabric is usually stronger than film fabric and additionally has a larger coefficient of friction.
- 3. With direction. The MD direction (machine direction, also warp; that is, with the roll) is stronger than (or as strong as) the XD direction (cross-machine, or fill; that is, across the roll—transverse to the roll length). Sometimes the strength difference is on the order of $XD \approx 0.5MD$. This means that attention to the strength direction during placing may be critical.

We must test (or have tested by the mill, or use an independent testing laboratory) the fabric to obtain the strength, usually in kN/m (or lb/in.) of width. From the several choices we choose a strip so that

Strip width b \times design strength/unit width $\ge T_i$

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Strip design may require several iterations to set the horizontal and vertical reinforcement spacing. Since fabric cost is relatively small compared with other costs (engineering time, backfill, etc.) and since there is some uncertainty in this type of analysis, a modest amount of overdesign is acceptable.

Metal reinforcement strips currently available are on the order of b = 75 to 100 mm and t on the order of 3 to 5 mm, with 1 mm on each face excluded for corrosion. Concrete reinforcing rods are often used for their roughness, but with one end prepared for attachment to the face piece—by welding or threads. Rod diameters should be at least three times larger than the average (D_{50}) particle diameters of the granular backfill so adequate friction contact is developed. Particle diameter is less critical with wire grids since the grid bars perpendicular to the tension rods provide considerable additional pullout resistance.

The pullout forces and resistance are assumed to be developed as shown in Fig. 12-4 where a tension from the wall face to the Rankine/Coulomb rupture zone defined by the angle ρ develops to a maximum at the wedge line. Even with a sloping backfill and/or surcharges the Rankine wedge shown is generally used. This tension is resisted by the friction developing outside the zone along length L_e of Fig. 12-5, so we can write, from the differential equation shown on Fig. 12-4,

$$T = \int_0^{L_e} 2b(p_o \tan \delta) \, dL$$

This expression may be somewhat of a simplification, and 2b must be replaced with the perimeter (πD) for round bars, but it seems to allow an adequate wall design.

Most of the construction technology currently used for reinforced earth walls is under patent protection; however, it is important to understand the principles involved and methods of analysis both in order to make a reasonable decision on the best system for a site and because the patents on some of the walls will expire shortly and the method(s) will transfer to the public domain.

12-3.2 Soil Nailing

Using "nails" to reinforce the earth is a relatively recent (about 20 years old) method for soil reinforcement. Basically this consists in either driving small-diameter rods (on the order of 25 to 30 mm) into the earth or drilling holes on the order of 150 to 200 mm, inserting the required diameter (again 25- to 30-mm) rod, and filling the remainder of the hole with grout (usually a cement-sand mixture with a low enough viscosity that it can be pumped).

The essential difference between soil nailing and tieback walls (of Chap. 14) is that there is little prestress applied to the soil nails, whereas the tieback wall requires prestressing the rods.

Soil nailing has the advantage of being suitable both for walls and for excavation support. For walls one starts the wall upward and at specified levels inserts "nails" into the backfill. The wall then proceeds and the nail is attached to the wall (often through a prepared hole with a face plate and a nut for fastening). In excavations some depth is excavated, the nails are inserted, and wall is added and attached as for the retaining wall. The next level is excavated, nails are inserted, wall facing is added and attached, etc.

The rods are usually inserted or drilled at a slope from the horizontal of about 15° , but near the upper part of the wall the slope may be larger (20 to 25°) to avoid underground utilities.

The latest soil nail insertion technique consists in using a compressed-air driver that fires (or launches) the nail at high velocity into the soil. The tip is the launch point, so the nail rod is pulled rather than driven into position. Pulling avoids rod buckling, since the nail diameters for current air launchers are on the order of 25 to 40 mm for depths of 3 to 6 m—larger diameters may be used but smaller penetration depths result. This type of device can fire a nail at any orientation and at a rate of up to 15 per hour. The nail head is normally prefitted with a threaded portion or prefastened to an arresting collar so that it is not fired too far into the ground for accessibility.

Rod spacing varies between 1 and 4 m^2 of wall surface area depending on factors such as type of retained soil, wall height, available space behind the wall for rod penetration, rod diameter, and designer caution.

Although the analysis is somewhat similar to other reinforced earth walls there are some differences. Usually the analysis consists in a global stability analysis using a slope stability program. The slope stability program must be specifically modified to allow locating the rods (if they protrude through the trial circle arc).

It may also require modification to use a portion of a logarithmic spiral as the failure surface rather than part of a circle. A rod stability analysis for both tension (or pullout) and bending (on the potential slip plane) is also required—but often just for pullout.

One can make a reasonable wall design with reference to Fig. 12-8 as follows:

1. Estimate the rod tension T_i using the appropriate pressure diagram of Fig. 12-8b (see similarity with Fig. 14-5), the position of the rod (upper $\frac{1}{4}$, middle $\frac{1}{2}$, or lower $\frac{1}{4}$), and the spacing. Use the equation shown on the figure for T_i . You should compute a table with the several values of T_i . Since all the rods should be the same diameter D, select the largest T_i .





(b) Approximate soil pressure diagrams with pressure intensities and soil types



2. Compute the required rod diameter D for this tension using a suitable SF so that $f_a = f_y/SF$ of rod steel (or other rod material). With T_i and f_a , compute

$$D = \sqrt{\frac{T_i}{0.7854f_a}}$$

3. Estimate the nail friction resistance (outside the modified Rankine wedge zone of Fig. 12-8*a*) using Eq. (12-2*b*). Use the actual rod diameter if the rod is driven, but use the grouted diameter if the rod is put in a drilled hole and grouted. Use $\tan \delta =$ estimated value for soil-metal interface based on metal roughness. Use $\delta = \phi$ for grouted rods. For sloping rods use an average depth z_i in the length outside the wedge zone. One must use a trial process for finding the computed distance $L_{e,comp}$ —that is, assume a length and compute the resistance $F_r \geq T_i$. Several values may be tried, depending on whether all rods are to be the same length, or variable lengths (depending on wall location) are to be used. In any case increase the computed length as

$$L_{e,des} \geq SF \cdot L_{e,comp}$$

Compute the total rod (nail) length L_{tot} at any location as the length just computed for pullout resistance $L_{e,des}$ + length L_R to penetrate through the Rankine wedge zone, giving the following:

$$L_{\rm tot} = L_{e,\rm des} + L_R$$

It will be useful to make a table of nail lengths L_{tot} versus depth z to obtain the final design length(s). One has the option of either using a single nail length or of locating elevations where the nail length changes occur if different nail lengths are used.

4. Make a scaled plot of the wall height, modified Rankine wedge, rod locations, and their slopes and lengths. Use this plot to make your slope stability analysis. Clearly one possibility is to use a regular slope stability computer program and ignore the "nails."

There is already an enormous amount of literature as well as at least three separate design procedures for nailed walls. The reader is referred to Jewell and Pedley (1992), Juran et al. (1990) and ASCE Geotechnical SP No. 12 (1987) for design information sources or to build confidence in the procedure outlined above.

12-3.3 Examples

We will examine the reinforced earth methodology further in the following three examples.

Example 12-1. Analyze the wall of Fig. E12-1 using strip reinforcement. The strips will be tentatively spaced at s = 1 m and h = 1 m and centered on the concrete wall facing units. We will use interlocking reinforced concrete facing units, shaped as indicated, that are 200 mm thick (with a mass of about 1000 kg or 9.807 kN each). A wall footing will be poured to provide alignment and to spread the facing unit load somewhat, since their total mass is more than an equivalent volume of soil. A 150-mm thick reinforced cap will be placed on top of the wall to maintain top alignment and appearance.

Required. Analyze a typical interior vertical section and select tension strips based on $f_y = 250$ MPa and $f_a = 250/1.786 = 140$ MPa. Other data: $\phi = 34^\circ$; $\gamma = 17.30$ kN/m³; and assume $\delta = 0.7 \times 34 = 24^\circ$.



Figure E12-1

Solution. From Table 11-3 obtain $K_a = 0.283$

 $f = \tan \delta \rightarrow \tan 24^\circ = 0.445$

Set up the following table from wall data (L_e is computed after T_i and strip width b are computed):

Strip no.	z _i , m	$T_i = \gamma z_i (1 \times 1) K_a,$ kN	$L_e = \frac{T_i \times SF}{2b \tan \delta(\gamma z_i)},$
1	0.5	2.45	4.77
2	1.5	7.34	1
3	2.5	12.24	
4	3.5	17.14	
5	4.5	22.03	Ì
6	5.5	26.93	
7	6.5	31.82	
8	7.5	36.72	
9	8.5	41.62	Ļ
10	9.5	46.51	4.77
		$\sum T_i = 2\overline{44.80}$ kN	

Check:

$$P_a = \frac{1}{2}\gamma H^2 K_a$$
 [Eq. (11-9) and $s = 1 \text{ m} = \text{unit width}]
$$P_a = \frac{1}{2}(17.30)(10^2)(0.283) = 244.80 \text{ kN/m}$$$

Next we find the cross section of the reinforcement strips. Tentatively try b = 100 mm since the wall is 10 m high.

 $b \times t \times f_a = T_i$ (a SF is already on f_a)

The largest T_i is strip 10, so for T_{10} we have

0.100(t)(140) = 46.51 kN (using meters)

Solving (and inserting 1000 to convert MPa to KPa), we obtain

$$t = \frac{46.51}{0.10(140)1000} = 0.00332 \text{ m} \rightarrow 3.32 \text{ mm}$$
, so use $t = 5.0 \text{ mm}$

This value allows a little less than 1-mm loss on each side for corrosion. Next find the strip length for T_i and total strip length L_o . We equate $\tan \delta \times \text{vertical pressure } p_o$ on both sides of strip of width $b \times L_e$ to the strip tension $T_i \times \text{SF}$. Get T_i from the preceding table and use an SF = 1.5:

$$2b(\tan\delta)(\gamma z_i)L_e = T_i(SF)$$

Rearranging into solution form for L_e , we have

$$L_e = \frac{(\text{SF})T_i}{2b(\tan\delta)(\gamma z_i)} = \frac{1.5T_i}{2(0.10)(0.445 \times 17.30z_i)}$$

This equation can be programmed. The first value (for $z_i = 0.50$ m) is

$$L_e = \frac{1.5(2.45)}{1.5397(0.50)} = 4.77 \text{ m}$$

Other values for $z_i = 1.5, 2.5, 3.5, \dots, 9.5$ are similarly computed and we find them constant as shown in the preceding table. We now find total strip lengths L_o as follows:

$$\rho = 45^{\circ} + \phi/2 = 45^{\circ} + 34^{\circ}/2 = 62^{\circ}$$

The Rankine zone at 9.5 m (strip 1) is

$$L_R = 9.5 \times \tan(90^\circ - 62^\circ) = 9.5 \times \tan 28^\circ = 5.05 \text{ m}$$

 $L_o = L_R + L_e = 5.05 + 4.77 = 9.82 \text{ m}$

We can use this length for all of the strips or, noting that the Rankine zone has a linear variation, we can use a linear variation in the strip lengths and apply careful construction inspection to ensure the correct strip lengths are used. This wall is high, so considerable savings can be had by using variable strip lengths. Do it this way:

At 0.5 m above base:	$L_o =$	$0.5 \times$	tan 28°	+ 4.77	=	5.04 m
At 4.5 m above base:	$L_o =$	4.5 ×	tan 28°	+ 4.77	=	7.16 m
At 9.5 m above base (top	strip):			L_o	=	9.82 m

As a check, plot the wall to scale, plot these three strip lengths, connect them with a line, and read off the other strip lengths.

Bearing capacity. We should check the bearing capacity for a unit width strip with a footing width B of either 9.82 or 5.04 m depending on strip configuration. Take all shape, depth, and inclination factors = 1.0. The poured footing for the concrete facing units will have a unit length but should have a B that is wide enough (greater than the 200-mm thickness of the wall units) that the bearing pressures for backfill and facing units are approximately equal to avoid settlement of the facing units and possibly tearing out the reinforcement strips.

Sliding resistance. The wall should resist sliding. Assuming a linear variation of reinforcement strips, we will have a block of soil that is one unit wide of weight $W = \gamma HB_{av}(1.0)$. Note that sliding is soil-to-soil, so take $\tan \delta = \tan \phi$. Inserting values, we have

$$W = 17.30(10) \frac{9.82 + 5.04}{2} \times 1 = 1286 \text{ kN}$$

$$F_R = W \tan \phi = 1286 \tan 34^\circ = 867 \text{ kN} \gg P_a = 244.8 \text{ kN}$$

Sliding stability = $\frac{867}{244.8} = 3.5$

The wall should be drawn to a reasonable scale with all critical dimensions shown to complete the design. Owing to limited text space this figure is not included here.

////

Example 12-2. Compute the reinforcement tension and friction resistance to obtain a tentative strip length L_o for the wall of Fig. E12-2 with a surcharge on the backfill. Check the strip at the 1.5-m depth (T_5) to illustrate the general procedure with a surcharge.

Soil data: $\gamma = 17.30 \text{ kN/m}^3$; $\phi = 32^\circ$ (backfill); take $f = \tan \delta = 0.4$ as the coefficient of friction between backfill soil and strip.

Strip data: h = 0.30 m; s = 0.60 m; width b = 75 mm; SF = 2.0 on steel of $f_y = 250$ MPa; SF = 2.0 on soil friction.

Solution. Obtain Rankine $K_a = 0.307$ from Table 11-3. Use your computer program SMBLP1 to obtain the lateral pressure profile for the surcharge. Assume plane strain, the *B* dimension of 1.5 m as shown, and a length of 1 m consistent with the Rankine wall pressures. A good approach is to use unit areas of 1.5/5 = 0.3 (NSQW = 5) and 1.0/4 = 0.25 (NSQL = 4) so that PSQR = $100(0.3 \times 0.25) = 7.5$ kN. When requested by the program, have the wall pressure profile output along with the total wall force so you can compare these to the values plotted on Fig. E12-2. You can use a "point" load at 2.25 m from wall with P = 150 kN and obtain almost "exact" pressures from 1.5 m down to the 6.0 m depth, but in the upper 1.5 m the pressures are somewhat in error.

At the 1.5-m depth the Rankine earth pressure is

$$q_R = \gamma z_i K_a = 17.30(1.5)(0.307) = 8.0$$
 kPa



Figure E12-2

At this depth (also 4.5 m above base) program SMBLP1 gives

$$\Delta q = 5.9 \,\mathrm{kPa}$$

The design pressure is the sum of the two pressures, giving

$$q_{\rm des} = q_R + \Delta q = 8.0 + 5.9 = 13.9 \,\rm kPa$$

The strip design force is

$$T_5 = F_{des} = q_{des}(h \times s)$$

= 13.9(0.30 × 0.60) = **2.50** kN/strip

The allowable strip tension $f_a = f_y/\text{SF} = 250/2 = 125$ MPa. The strip cross section of $b \times t$ with b = 75 mm is

$$b(t)f_a = T_5 \rightarrow t = \frac{F_{\text{des}}}{bf_a}$$

Inserting values, we obtain

$$t = \frac{2.5}{0.075 \times 125 \times 1000} = 0.000\,27 \text{ m} \rightarrow 0.27 \text{ mm}$$

Use $t = 3 \text{ mm}$ (to allow for corrosion)

The force $F_{des} = T_5$ must be resisted by friction developed on both sides of the strip of length L_e outside the Rankine wedge zone. This force will be assumed to be made of two parts, so L_e = $L_1 + L_2$.

From the sketch drawn to scale we can scale the length L_1 or directly compute it as follows:

Distance to right edge of surcharge = 1.5 + 1.5 = 3 m

.

Distance from wall =
$$L_R + L_1$$

= distance to right of surcharge + 1.5/2
 $L_R + L_1 = 3.0 + 1.5/2 = 3.75 \text{ m}$
 $L_1 = 3.75 - L_R$
 $L_1 = 3.75 - 4.5 \tan(90^\circ - \rho)$
= 3.75 - 2.49 = **1.26** m ($L_R = 2.49 \text{ m}$)

In this region the vertical pressure is

$$p_o = \gamma z_i + \frac{Q}{B + z_i}$$

 $p_o = 17.30(1.5) + \frac{1.5(100)}{1.5 + 2(0.75)} = 26 + 50 = 76 \text{ kPa}$

Now equating friction resistance to tension and using the given SF = 2 we have

$$2b[(p_o \tan \delta)L_1 + (\gamma z_i \tan \delta)L_2] = 2.50(\text{SF})$$

Inserting values (remember that $\tan \delta$ was given as 0.4), we obtain

$$2(0.075)[(76)(0.4)L_1 + 17.30(1.5)(0.4)L_2] = 2.50(2)$$

Thus, we have

$$4.56L_1 + 1.56L_2 = 5.0$$

It appears we do not need an L_2 contribution. If on solving for L_1 we obtain a value > 1.26, we will set $L_1 = 1.26$ and solve for the L_2 contribution,

$$L_1 = \frac{5.0}{4.56} = 1.09 \text{ m}$$
 (less than 1.26 m furnished, so result is O.K.)

////

The total length at this point is

 $L_o = L_R + L_1 \rightarrow 2.49 + 1.09 = 3.58 \text{ m}$

To complete the design, we must check other strip locations. Again one can use one length for all strips or use variable strip lengths, or use one strip length for the lower half of the wall and a different strip length for the upper half.

The remaining steps include the following:

1. Find the strip thickness based on the largest T_i . The Rankine earth pressure at $z_i = H = 5.85$ m is

$$q_R = 17.30 \times 5.85 \times 0.307 = 31.1$$
 kPa

and for the strip (including 0.5 for surcharge) is

$$T_{21} = (31.1 + 0.5)(0.3 \times 0.6) = 5.7 \text{ kN}$$

- 2. Check bearing capacity.
- 3. Check sliding stability.

Example 12-3. This example illustrates using geotextiles instead of strips for the wall design. The author's computer program GEOWALL will be used, since a substantial output is provided in a compact format and there is much busywork in this type of wall design. Refer to Fig. E12-3*a* and the following data:



Figure E12-3a

Backfill soil: $\gamma = 17.10 \text{ kN/m}^3$; $\phi = 36^\circ$; c = 0.0 kPa; backfill slope $\beta = 0^\circ$; Poisson's ratio $\mu = 0.0$ These are the input data here but the program also allows a concentrated backfill surcharge. Base soil: $\gamma = 18.10 \text{ kN/m}^3$; $\phi = 15^\circ$; c = 20 kPa; $\delta = 12^\circ$ (soil to fabric); cohesion reduction factor $\alpha = 0.8$ (so $c_\alpha = 0.8 \times 20 = 16 \text{ kPa}$)

Note all these data are shown on the output sheets (Fig. E12-3b).

The geotextile will be tentatively selected from the 1994 *Specifier's Guide* published annually by the Industrial Fabrics Association International in the "Geotextiles" section as a Carthage Mills 20 percent fabric with a wide-width tensile strength of 32.4 kN/m. It has a permeability of 0.55 $L/min/m^2$, which should be adequate for a sandy backfill.

A geotextile wall design consists in obtaining an optimum balance between fabric weight (a function of strength), spacing, and length. This can be done in a reasonable amount of time only by using a computer. What does the computer program do that otherwise one would do by hand?

1. Compute the Rankine wall pressures and any Boussinesq surcharge pressures (here there are no Boussinesq-type surcharges, but there is a uniform surcharge of 10 kPa). These are always output in the first listed table using equal spacings of 0.3 m (or 1 ft) down the wall (Fig. E12-3b). The Rankine and Boussinesq values are summed, as these would be used to compute fabric tensile force at these locations. Note that $10K_a = 10(0.2597) = 2.597$ kPa as top table entry.

Also found at this initial spacing are the total wall resultant (RFORC = 50.074 kN) for any surcharges + Rankine resultant and the location YBAR1 = \overline{y} = 1.552 m above the base.

- 2. Next the program checks sliding stability based on asking for an input value for N_s (usual range between 2 and 3—the author used 2). For this value of N_s a base fabric length of 3.0 m is required.
- 3. The program then outputs to the screen the first table shown and asks whether the user wants to change any of the vertical spacings. The author did, and elected to use 0.4-m (16-in.) layers for the upper 3.6 m of wall height and 0.3-m (12-in.) layers for the last 0.6 m (3.6 + 0.6 = 4.2). These values were chosen to give a reasonable balance between number of sheets and excessively thick soil layers. One could obtain a solution using 0.6 m for six layers and 0.3 m for two layers at the bottom for some savings; however, although 0.6 m (24-in.) might produce a more economical wall, the facing part may be at risk, and if one of the geotextile layers went bad, the internal spacing would be unacceptable in that region.
- 4. The program recomputes the earth pressures, the backfill, and any surcharges at the new spacing (the spacings can be changed any number of times—or repeated) and outputs this spacing (nine at 0.4 m and two at 0.3 m) to screen and asks whether this is O.K. or to change it. The author answered O.K., and this was used.
- 5. Next compute the fabric lengths for tension. This result is also output in a table as shown. The program has a preset SF = 1.4 here but also requires a preset minimum distance for fabric lengths L_e :
 - a. If the computed $L_e < 0.5$ m (18 in.) use 0.5 m.
 - b. If the computed L_e is $0.5 < L_e < 1$ m use 1.0 m.
 - c. If the computed $L_e > 1.0$ m use the computed value.

We need 3.00 m for the first layer—not for tension but for the sliding SF computed earlier. The top layer (layer 11) requires

$$L_{\text{tot}} = L_e + L_R = 0.500 + 1.936 = 2.436 \text{ m}$$

The program does not make "exact" computations here. It takes the distance from layer i - 1 to layer $i \times q_{h,i} \times SF = 1.4$ to compute sheet tension. Strictly, the tension force should use a zone centered (or nearly so) on the sheet, but the error from not doing this is negligible. In this example the preset minimum $L_e = 0.500$ m controls for the full wall height.

The required sheet length L_e is computed using the vertical distance from the backfill surface to the *i*th layer to compute the vertical sheet pressure. Both sides are used and with $\sigma_v \tan \delta$ and (if applicable) adhesion c_a .

On the basis of a screen display of this table the program asks what lengths the user wants to use. A single length or up to five different lengths can be used. From the table the author elected to use a single length for all layers of 3.00 m. This is less confusing to the construction crews, and besides in the upper several layers there are not much savings.

- 6. With the length selected the program next computes bearing capacity along AB of Fig. E12-3c using the length of layer 1 as B. It presents to the screen the stability number based on $SF = q_{ult}/q_v$, where $q_v = \gamma H + q_{surcharge}$. Shown on the output, the SF = 3.985.
- 7. On the basis of the length and any surface surcharges, the program computes the overturning stability about point A of Fig. E12-3c (the toe). This is far from a rigid body, but conventional design makes a rigid body assumption. Here use block ADCB with a surcharge on DC. This gives a block of width = 3.0 m and height = 4.2 m. The overturning moment from the horizontal force is

$$P_h \bar{y} = M_o = 50.074(1.552) = 77.71 \text{ kN} \cdot \text{m}$$

The resisting moment consists of two parts—one is the block mass and the other is block friction. Block friction is based on the concept that the the block cannot turn over without developing a vertical friction force on its back face of $P_{ah} \tan \phi$ (it is soil-to-soil), and the block has a moment arm that equals block width (here 3.0 m):

$$M_r = W\bar{x} = [4.2(3.0)(17.1) + 3.0(10)]1.5 = 368.19 \text{ kN} \cdot \text{m}$$

The program asks whether this is satisfactory, and it is.

8. As a final step the program produces the last table shown. It uses the vertical spacing, assumes an overlap of 1 m, and obtains the length of fabric to be ordered. For example for layer 11 we have space = $0.40 \text{ m} + \text{lap} = 1.00 + \text{required } L_e = 3.00 \text{ m}$, or

$$L_{\text{tot}} = 0.40 + 1.00 + 3.00 = 4.40$$
 m (as shown in the table)

At the bottom, $L_{\text{tot}} = 0.30 + 1.00 + 3.00 = 4.30$ m (also as shown).

9. In the last column the actual geotextile stress f_r is shown, which varies with Rankine tension stress. The f_a is computed using the input partial SF values listed on output sheet 1 [Eq. (12-3), which is programmed into this program]. From the output sheet we find that the partial SF_i in combination gives SF = 2.265 and

$$f_a = \frac{f_{\text{ult}}}{\text{SF}} = \frac{34.4}{2.265} = 14.3 \text{ kPa} \text{ (shown)}$$

From inspection of f_r we see the following stresses for layers 1, 3, and 4:

Layer	f _r , kPa	f _a , kPa
1	14.44	14.30
3	16.84	14.30
4	15.23	14.30

What do we do? Use this fabric-soil combination, or a stronger fabric, or a closer spacing. We probably would not want to use a closer spacing, so that leaves either using this fabric or a

PARTIAL EXAMPLE OF REINFORCED EARTH WALL USING GEOTEXTILE SHEETS ++++++++ NAME OF DATA FILE USED FOR THIS EXECUTION: EXAM123.DTA NO OF CONC LOADS ON BACKFILL = 0 IMET (SI > 0) = 1WALL HEIGHT = 4.200 M BACKFILL SURCHARGE = 10.000 KPA BACKFILL SOIL: UNIT WEIGHT = 17.100 KN/M^3 ANGLE OF INT FRICT, PHI1 = 36.000 DEG .000 KPA BACKFILL COHESION = BACKFILL SLOPE, BETA1 = .000 DEG POISSON'S RATIO = .000 BASE SOTL: UNIT WEIGHT = 18.100 KN/M^3 ANGLE OF INT FRICT, PHI2 = 15.000 DEG 20.000 KPA BASE SOIL COHESION = EFF ANGLE OF INT FRIC TO FABRIC, EPHI2 = 12.000 EFF BASE SOIL COHESION TO FABRIC, ECOH2 = 16.000 KPA (.80) GEOTEXTILE TENSILE STRENGTH PERPENDICULAR TO WALL = 32.400 KN/M BASED ON THE INPUT ULTIMATE GEOTEXTILE TENSION. GSIG = 32.40 AND USING THE FOLLOWING SAFETY FACTORS: INSTALL DAMAGE, FSID = 1.10 CREEP, FSCR = 1.20CHEMICAL DEGRADATION, FSCD = 1.30 BIOLOGICAL DEGRADATION, FSBD = 1.20 SITE SPECIFIC FACTOR, FSSS = 1.10 COMBINED SF PRODUCT, FSCOMB = 2.265 THE ALLOWABLE FABRIC TENSION, ALLOWT = 14.3039 KN/M RANKINE HORIZ. FORCE RESULTANT, RFORC = 50.074 KN LOCATION ABOVE BASE, YBAR1 = 1.552 M .2597 (HORIZ FORCE BASED ON USING KA*COSB = .2597) THIS SET OF PRESSURES FOR EQUAL SPACINGS DOWN WALL BOUSQ QH TOT QH, KPA Ι DDY(I) OH(I) .0000 .0000 1 2.5969 2.5969 2 .6000 5.2614 .0000 5.2614 .9000 .0000 3 6.5936 6.5936 4 .0000 1.2000 7.9258 7.9258 5 9.2580 1.5000 .0000 9.2580 6 10.5903 .0000 1.8000 10.5903 7 2.1000 11.9225 .0000 11.9225 8 2.4000 13.2547 .0000 13.2547 .0000 9 2.7000 14.5869 14.5869 .0000 10 3.0000 15.9191 15.9191 .0000 3.3000 17.2514 11 17.2514 .0000 12 3.6000 18.5836 18.5836 .0000 13 3.9000 19.9158 19,9158 .0000 14 4.2000 21.2480 21.2480

FOR SLIDING STABILITY: REQUIRED BASE FABRIC LENGTH = 3.00 M BASED ON USING A SLIDING SF = 2.00 AND USING AVERAGE WALL HEIGHT, HAVGE = 4.20 M

Figure E12-3b (continued)

THIS	SET OF PRE	SSURES FOR	MODIFIED	VERTICAL SPACINGS				
I	DDY(I)	QH(I)	BOUSQ QH	I ТОТ QH, КРА				
1	.0000	2.5969	.0000	2.5969				
2	.4000	4.3732	. 0000	4.3732				
3	.8000	6.1495	.0000	6.1495				
4	1.2000	7.9258	.0000	7.9258				
5	1.6000	9.7021	.0000	9.7021				
6	2.0000	11.4784	.0000	11.4784				
7	2.4000	13.2547	.0000	13.2547				
8	2.8000	15.0310	.0000	15.0310				
9	3.2000	16.8073	. 0000	16.8073				
10	3.6000	18.5836	.0000	18.5836				
11	3.9000	19.9158	.0000	19.9158				
12	4.2000	21.2480	.0000	21.2480				
SOIL	SOIL-TO-FABRIC FRICTION FACTORS:							
	DELTA = 24	.UU DEG						

ALPHA = 1.00 (ON COHESION)

FABRIC LENGTH SUMMARY--ALL DIMENSIONS IN M

LAYER	DEPTH	VERT			LFILL
NO	DDY	SPACING	LE	LR	LE+LR
11	.40	.40	.500	1.936	2.436
10	. 80	.40	.500	1.733	2.233
9	1.20	. 40	.500	1.529	2.029
8	1.60	.40	.500	1.325	1.825
7	2.00	.40	.500	1.121	1.621
6	2.40	.40	.500	.917	1.417
5	2.80	.40	.500	.713	1.213
4	3.20	.40	.500	.510	1.010
3	3.60	.40	.500	.306	.806
2	3.90	. 30	.500	.153	.653
1	4.20	.30	.500	.000	3.000

COMPUTED BEARING CAPACITY = 326.04 KPA COMPUTED VERTICAL PRESSURE = 81.82 KPA GIVES COMPUTED SAFETY FACTOR SF = 3.985 ***BEARING CAPACITY BASED ON B = 3.00 M INITIAL BASE WIDTH = 3.00 M

EXTRA DATA FOR HAND CHECKING NC, NG = 12.9 2.5 FOR PHI-ANGLE = 15.00 DEG FOR VERTICAL PRESSURE USED AVERAGE WALL HEIGHT = 4.20 M

OVERTURNING STABILITY BASED ON USING: BASE FABRIC LENGTH = 3.00 M AVERAGE WALL HEIGHT = 4.20 M THE COMPUTED O.T. STABILITY = 6.14

E	ABRIC	LENGTH	I SUMMAR	RYALL DIM	ENSIONS IN	: M			
LAY	/ER	DEPTH	VERT	SPACING	OVERLAP	FILL	LE+LR*		REQ'D
ŧ	ŧ	DDY	ACTUAL	MAXIMUM	LO	ROUND	(REQ'D)	TOT L**	GSIG, KN/M
11	L	.40	.400	3.271	1.000	3.00	(2.44)	4.40	3.962
10)	.80	.400	2.326	1.000	3.00	(2.23)	4.40	5.572
5)	1.20	.400	1.805	1.000	3.00	(2.03)	4.40	7.181
8	3	1.60	.400	1.474	1.000	3.00	(1.82)	4.40	8.791
7	1	2.00	.400	1.246	1.000	3.00	(1.62)	4.40	10.400
e	5	2.40	.400	1.079	1.000	3.00	(1.42)	4.40	12.009
5	;	2.80	.400	.952	1.000	3.00	(1.21)	4.40	13.619
4		3.20	.400	.851	1.000	3.00	(1.01)	4.40	15.228
3	3	3.60	.400	. 770	1.000	3.00	(.81)	4.40	16.838
2	2	3.90	. 300	.718	1.000	3.00	(.65)	4.30	13.534
1		4.20	. 300	.673	1.000	3.00	(3.00)	4.30	14.439

* = ROUNDED FILL Le + Lr AND ACTUAL (REQ'D) LENGTHS

** = TOTAL REQUIRED FABRIC LENGTH = Le + Lr + Lo + SPACING



Figure E12-3c

stronger one (which will cost more). Let us look again at the partial SF_i. Near the base, chemical degradation could be 1.2 instead of 1.3—this change gives SF = 2.09 instead of 2.265 and an allowable $f_a = 34.4/2.09 = 16.4$ kN/m.

Since the required f_r is computed using the same SF as on the geotextile we have in general,

 f_r = vertical space $\times q_R \times SF$

and before adjusting the SF,

 $f_R = 0.4(18.58)(2.265) = 16.83$ kN/m (as on output sheet)

After adjusting SF,

$$f_R = 0.4(18.58)(2.09) = 15.53 \text{ kN/m} < 16.4$$
 (O.K.)

10. All that is left is to draw a neat sketch so the construction crew can build the wall. Next determine the wall length (we would use one width of 4.40 m) and determine the number of rolls of geotextile needed, and the project is designed.

Comment. This geotextile may not be available in a **4.40** m width. If there is a large enough quantity, the mill might set up a special run to produce the desired (or a slightly larger) width. Otherwise it will be necessary to search the catalog for another producer. Since part of the design depends on

available widths, it should be evident that a highly precise design is not called for. Also, the Rankine zone appears to be more of a segment of a log spiral than the wedge shown, so it may not exceed 0.3H in any case. The reason for this statement is that we would search for an available fabric of width between 4.1 and 4.6 m with a strength ≥ 32.4 kN/m as satisfactory.

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12-4 CONCRETE RETAINING WALLS

Figure 12-9 illustrates a number of types of walls of reinforced concrete or masonry. Of these, only the reinforced concrete cantilever wall (b) and the bridge abutment (f) are much used at present owing to the economics of reinforced earth.

The reinforced earth configuration produces essentially the gravity walls of Fig. 2-9a and the crib wall of Fig. 12-9d. The "stretcher" elements in the crib wall function similarly to the reinforcement strips in reinforced earth walls.

The counterfort wall (c) may be used when a cantilever wall has a height over about 7 m. Counterforts (called buttresses if located on the front face of the wall) are used to allow a reduction in stem thickness without excessive outward deflection. These walls have a high labor and material cost, so they do not compete economically with reinforced earth. They may be used on occasion in urban areas where aesthetics, space limitations, or vandalism is a concern.

There are prefabricated proprietary (patented) walls that may compete at certain sites with other types of walls. Generally the producer of the prefabricated wall provides the design procedures and enough other data so that a potential user can make a cost comparison from the several alternatives.

Cantilever and prefabricated retaining walls are analyzed similarly, so a basic understanding of the cantilever procedure will enable a design review of a prefabricated wall for those cases where a cost comparison is desired.

The focus of the rest of this chapter is on the design of reinforced concrete cantilever retaining walls (as shown in Fig. 12-9b).

For reinforced concrete, the concept of *Strength Design* (USD) was used in Chaps. 8 through 10 for foundations. In those chapters multiple load factors were used, but they did not overly complicate the design. In wall design the use of load factors is not so direct, and, further, the ACI 318- does not provide much guidance—that is, the Code user must do some interpretation of Code intent.

When the USD was first introduced in the mid-1960s, it was common to use a single load factor (1.7 to 2.0) applied to any load or pressure to obtain an "ultimate" value to use in the USD equations. However, there is some question whether the use of a single load factor is correct, and ACI 318- is of no help for this. Retaining wall design procedures are often covered in reinforced concrete (R/C) design textbooks and range from using a single load factor to using multiple load factors—but only with USD since R/C design textbooks are based on this method.

For these and other reasons stated later the author has decided there is considerable merit in using the *Alternate Design Method* (ADM). This was the only method used prior to the mid-1960s, but it is still considered quite acceptable by both ACI 318- and AASHTO.

The ACI 318- places more emphasis on the USD because of claimed economies in building construction, but the AASHTO bridge manuals (including the latest one) give about equal consideration to both methods.



Figure 12-9 Types of retaining walls. (a) Gravity walls of stone masonry, brick, or plain concrete—weight provides stability against overturning and sliding; (b) Cantilever wall; (c) Counterfort, or buttressed wall—if backfill covers the counterforts the wall is termed a counterfort; (d) Crib wall; (e) Semigravity wall (uses small amount of steel reinforcement); (f) Bridge abutment.

The ADM procedure will be used here so that we can avoid the use of multiple load factors and the associated problems of attempting to mix earth pressures (LF = 1.7) with vertical soil and wall loads (LF = 1.4) and surcharge loads (some with LF = 1.4 and others with LF = ?). For retaining walls the ADM has two advantages:

- 1. The resulting wall design may (in some cases) be slightly more conservative than *strength* design unless load factors larger than the minimum are used.
- 2. The design is much simpler since all LF = 1 and thus less prone to error than the *strength* design method. Aside from this, the equations for design depth d and required steel area A_s are also easier to use.

12-5 CANTILEVER RETAINING WALLS

Figure 12-10 identifies the parts and terms used in retaining wall design. Cantilever walls have these principal uses at present:

- 1. For low walls of fairly short length, "low" being in terms of an exposed height on the order of 1 to 3.0 m and lengths on the order of 100 m or less.
- 2. Where the backfill zone is limited and/or it is necessary to use the existing soil as backfill. This restriction usually produces the condition of Fig. 11-12*b*, where the principal wall pressures are from compaction of the backfill in the limited zone defined primarily by the heel dimension.
- 3. In urban areas where appearance and durability justify the increased cost.

In these cases if the existing ground stands without caving for the depth of vertical excavation in order to place (or pour) the wall footing and later the stem, theoretically there is no lateral earth pressure from the existing backfill. The lateral wall pressure produced by the limited backfill zone of width b can be estimated using Eqs. (11-18) or (11-19)—this latter is option 8 in your program FFACTOR. There is a larger lateral pressure from compacting the backfill (but of unknown magnitude), which may be accounted for by raising the location of the resultant from H/3 to 0.4 to 0.5H using Eq. (11-15). Alternatively, use K_o instead of K_a with the H/3 resultant location.



Figure 12-10 Principal terms used with retaining walls. Note that "toe" refers to both point O and the distance from front face of stem; similarly "heel" is point h or distance from backface of stem to h.

It is common for cantilever walls to use a constant wall thickness on the order of 250 mm to seldom over 300 mm. This reduces the labor cost of form setting, but some overdesign should be used so that the lateral pressure does not produce a tilt that is obvious—often even a few millimeters is noticeable.

You can use your program FADBEMLP to compute an estimate of the tilt by using fixity at the stem base and loading the several nodes down the wall with the computed pressure diagram converted to nodal forces using the average end area method. Of course, it is possible to build a parallel-face wall with an intentional back tilt, but there will be extra form-setting costs.

Figure 12-11 gives common dimensions of a cantilever wall that may be used as a guide in a hand solution. Since there is a substantial amount of busywork in designing a retaining wall because of the trial process, it is particularly suited to a computer analysis in which the critical data of γ , ϕ , H, and a small base width B are input and the computer program (for example, the author's B-24) iterates to a solution.

The dimensions of Fig. 12-11 are based heavily on experience accumulated with stable walls under Rankine conditions. Small walls designed for lateral pressures from compaction, and similar, may produce different dimensions.

It is common, however, for the base width to be on the order of about 0.5H, which depends somewhat on the toe distance (B/3 is shown, but it is actually not necessary to have any toe). The thickness of the stem and base must be adequate for wide-beam shear at their intersections. The stem top thickness must be adequate for temperature-caused spalls and impacts from equipment/automobiles so that if a piece chips off, the remainder appears safe and provides adequate clear reinforcement cover.

The reinforcement bars for bending moments in the stem back require 70 mm clear cover⁴ (against ground) as shown in Fig. 12-12*a*. This requirement means that, with some T and S bars on the front face requiring a clear cover of 50 mm + tension rebar diameter + 70 mm and some thickness to develop concrete compression for a moment, a **minimum** top thickness of about 200 mm is automatically mandated.



Figure 12-11 Tentative design dimensions for a cantilever retaining wall. Batter shown is optional.

⁴Actually the ACI Code Art. 7-7.1 allows 50 mm when the wall stem is built using forms—the usual case. The code requires 70 mm only when the stem (or base) is poured directly against the soil.





(a) Wall pressure to use for shear and bending moment in stem design.
Also shown is bearing capacity pressure diagram based on Fig. 4-4 using B'=b-2e and L=L'=1 unit.



Figure 12-12 General wall stability. It is common to use the Rankine K_a and $\delta = \beta$ in (a). For β' in (b) you may use β or ϕ since the "slip" along ab is soil-to-soil. In any case compute $P_{av} = P_{ah} \tan \phi$ as being most nearly correct.

Walls are designed for wide-beam shear with critical locations as indicated in ACI 318-. The author suggests, however, taking the wide-beam shear at the stem face (front and back) for the base slab as being more conservative and as requiring a negligible amount of extra concrete. For the stem one should take the critical *wide-beam* location at the top of the base slab. The reason is that the base is usually poured first with the stem reinforcement set. Later the stem forms are set and poured, producing a discontinuity at this location.

Formerly, a wood strip was placed into the base slab and then removed before the stem was placed. This slot or *key* provided additional shear resistance for the stem, but this is seldom done at present. Without the key at this discontinuity, the only shear resistance is the bonding that develops between the two pours + any friction from the stem weight + reliance on the stem reinforcement for shear. ACI Art. 11.7.5 with the reduction given in Art. A.7.6 gives a procedure for checking *shear friction* to see if shear reinforcement is required at this location. The required ACI equation seems to give adequate resistance unless the wall is quite high.

12-6 WALL STABILITY

Figure 12-12 illustrates the general considerations of wall stability. The wall must be structurally stable against the following:

1. Stem shear and bending due to lateral earth pressure on the stem. This is a separate analysis using the stem height.

2. Base shear and bending moments at the stem caused by the wall loads producing bearing pressure beneath the wall footing (or base). The critical section for shear should be at the stem faces for both toe and heel. Toe bending is seldom a concern but for heel bending the critical section should be taken at the approximate center of the stem reinforcement and not at the stem backface.

The author suggests that for base bending and shear one use the rectangular bearing pressure (block *abde*) given on Fig. 12-12*a* in order to be consistent with bearing-capacity computations (see Fig. 4-4) for q_a . A trapezoidal diagram (*acf*) is also used but the computations for shear and moment are somewhat more complicated.

12-6.1 Sliding and Overturning Wall Stability

The wall must be safe against sliding. That is, sufficient friction F_r must be developed between the base slab and the base soil that a safety factor SF or stability number N_s (see Fig. 12-12b) is

$$SF = N_s = \frac{F_r + P_p}{P_{ah}} \ge 1.25 \text{ to } 2.0$$
 (12-4)

All terms are illustrated in Fig. 12-12b. Note that for this computation the total vertical force R is

$$R = W_c + W_s + P'_{av}$$

These several vertical forces are shown on Fig. 12-12b. The heel force P'_{av} is sometimes not included for a more conservative stability number. The friction angle δ between base slab and soil can be taken as ϕ where the concrete is poured directly onto the compacted base soil. The base-to-soil adhesion is usually a fraction of the cohesion—values of 0.6 to 0.8 are commonly used. Use a passive force P_p if the base soil is in close contact with the face of the toe. One may choose not to use the full depth of D in computing the toe P_p if it is possible a portion may erode. For example, if a sidewalk or roadway is in front of the wall, use the full depth (but not the surcharge from the sidewalk or roadway, as that may be removed for replacement); for other cases one must make a site assessment.

The wall must be safe against overturning about the toe. If we define these terms:

- \bar{x} = location of R on the base slab from the toe or point O. It is usual to require this distance be within the middle $\frac{1}{3}$ of distance Ob—that is, $\bar{x} > B/3$ from the toe.
- P_{ah} = horizontal component of the Rankine or Coulomb lateral earth pressure against the vertical line *ab* of Fig. 12-12*b* (the "virtual" back).
 - \bar{y} = distance above the base *Ob* to P_{ah} .
- P_{av} = vertical shear resistance on virtual back that develops as the wall tends to turn over. This is the only computation that should use P_{av} . The δ angle used for P_{av} should be on the order of the residual angle ϕ_r since the Rankine wedge soil is in the state of Fig. 11-1c and "follows" the wall as it tends to rotate.

We can compute a stability number N_o against overturning as

$$N_o = \frac{M_r}{M_o} = \frac{\sum W_i \bar{x} + P_{av} B}{P_{ah} \bar{y}} \ge 1.5 \text{ to } 2.0$$
(12-5)

In both Eqs. (12-4) and (12-5) the stability number in the given range should reflect the importance factor and site location. That is, if a wall failure can result in danger to human life

or extensive damage to a major structure, values closer to 2.0 should be used. Equation (12-5) is a substantial simplification used to estimate overturning resistance. On-site overturning is accompanied by passive resistances at (1) the top region of the base slab at the toe, (2) a zone along the heel at cb that tends to lift a soil column along the virtual back face line ab, and (3) the slip of the Rankine wedge on both sides of ab. Few walls have ever overturned—failure is usually by sliding or by shearoff of the stem.

The $\sum (W_c + W_s)$ and location \bar{x} are best determined by dividing the wall and soil over the heel into rectangles and triangles so the areas (and masses) can be easily computed and the centroidal locations identified. Then it becomes a simple matter to obtain

$$(W_c + W_s + P'_{av})\bar{x} = P_{ah}\bar{y} - P_p\bar{y}_p$$
$$\bar{x} = \frac{M_o - P_p\bar{y}_p}{W_c + W_s + P'_{av}}$$

If there is no passive to resistance (and/or P'_{av} is ignored) the preceding equations are somewhat simplified.

12-6.2 Rotational Stability

In Fig. 12-13 we see that in certain cases a wall can rotate as shown—usually when there are lower strata that are of poorer quality than the base soil. This failure is similar to a slope stability analysis using trial circles. These computations can be done by hand. Where several circles (but all passing through the heel point) are tried for a minimum stability number N_r , though, the busywork becomes prohibitive; and a computer program (see author's B-22) for slope stability analysis—adjusted for this type of problem as an option—should be used. This procedure is illustrated later in Example 12-4.

12-6.3 General Comments on Wall Stability

It is common—particularly for low walls—to use the Rankine earth-pressure coefficients K_a and K_p (or Table 11-5), because these are somewhat conservative. If the wall angle α of Fig. 11-4 is greater than 90°, consider using the Coulomb equations with $\delta \ge 0$.



Figure 12-13 Wall-soil shear failure may be analyzed by the Swedish-circle method. A "shallow" failure occurs when base soil fails. A "deep" failure occurs if the poor soil stratum is underlying a better soil, as in the figure.

For stem analysis the friction angle δ of Fig. 12-12*a* is taken as the slope angle β in the Rankine analysis. The friction angle is taken as some fraction of ϕ in a Coulomb analysis, with 0.67 ϕ commonly estimated for a concrete wall formed using plywood or metal forms so the back face is fairly smooth.

For the overall wall stability of Fig. 12-12b the angle β' may be taken as β for the Rankine method, but for the Coulomb analysis take $\beta' = \phi$. This value then is used to obtain the horizontal component of P_a as shown. For the vertical friction component P_{av} resisting overturning take

$$P_{av} = P_{ah} \tan \phi_r \tag{12-6}$$

since the δ angle shown on Fig. 12-12b is always soil-to-soil, but the soil is more in a "residual" than a natural state.

The Rankine value for K_p (or see Table 11-5) is usually used if passive pressure is included. If there is uncertainty that the full base depth D is effective in resisting via passive pressure, it is permissible to use a reduced value of D' as

$$D' = D$$
 – potential loss of depth

The potential loss of depth may be to the top of the base or perhaps the top 0.3^+ m, depending on designer assessment of how much soil will remain in place over the toe. Note that some of this soil is backfill, which must be carefully compacted when it is being replaced. Otherwise full passive pressure resistance may not develop until the wall has slipped so far forward that it has "failed."

12-6.4 Base Key

Where sufficient sliding stability is not possible—usually for walls with large H—a base key, as illustrated in Fig. 12-14, has been used. There are different opinions on the best location for a key and on its value. It was common practice to put the key beneath the stem as in Fig. 12-14*a*, until it was noted that the conditions of Fig. 12-14*b* were possible. This approach was convenient from the view of simply extending the stem reinforcement through the base and into the key. Later it became apparent that the key was more effective located as in Fig. 12-14*c* and, if one must use a key, this location is recommended. The increase in *H* by the key depth may null its effect.

12-6.5 Wall Tilt

Concrete retaining walls have a tendency to tilt forward because of the lateral earth pressure (Fig. 12-15*a*), but they can also tilt from base slab rotation caused by differential settlement. Occasionally the base soil is of poor quality and with placement of sufficient backfill (typically, the approach fill at a bridge abutment) the backfill pressure produces a heel settlement that is greater than at the toe. This difference causes the wall to tilt into the backfill as shown in Fig. 12-15*b*.

If the Rankine active earth pressure is to form, it is necessary that the wall tilt forward as noted in Sec. 11-2. A wall with a forward tilt does not give an observer much confidence in its safety, regardless of stability numbers. Unless the wall has a front batter, however, it is difficult for it to tilt forward—even a small amount—without the tilt being noticeable. It may be possible to reduce the tilt by overdesigning the stem—say, use K_o instead of K_a pressures and raise the location of the resultant. When one makes this choice, use a finite-element program such as your B-5 to check the wall movements. Although this type of analysis may not be completely accurate, there is currently no better way of estimating wall tilt.



- (a) Base key near stem so that stem steel may be extended into the key without additional splicing or using anchor bends.
 - (b) Potential sliding surface using the key location of a. There may be little increase in sliding resistance
 from this key, if the slip surface develops as shown.



(c) Possible sliding modes when using a heel key.

Figure 12-14 Stability against sliding by using a base key.





material such as clay or peat.

(b) Excessive heel zone settlement (from back fill).

Figure 12-15 Causes of excessive wall tilting.

12-6.6 Other Considerations in Retaining Wall Design

When there is a limited space in which to place the wall base slab and the sliding stability number N_s is too small, what can be done? There are several possible solutions:

- 1. Look to see if you are using a slab-soil friction angle δ that is too small—for concrete poured on a compacted soil it can be $\delta = \phi$. Are you using any P'_{av} contribution? Can you?
- 2. Consider placing the base slab deeper into the ground. At the least, you gain some additional passive resistance.
- **3.** Consider using short piles, on the order of 2 to 2.5 m in length, spaced about 1.5 to 2.0 m along the wall length. These would be for shear, i.e., laterally loaded.
- 4. Consider improving the base soil by adding lime or cement to a depth of 0.3^+ m just beneath the base.
- 5. Consider sloping the base, but keep in mind that this is not much different from using a heel key. Considerable hand work may be needed to obtain the soil slope, and then there is a question of whether to maintain the top of the base horizontal or slope both the top and bottom. You may get about the same effect by increasing the base-to-soil δ angle 1 or 2°.
- 6. Sloping the heel as shown in Fig. 12-16 has been suggested. This solution looks elegant until one studies it in depth. What this configuration hopes to accomplish is a reduction in lateral pressure—the percentage being

$$R = 100.0 - \left(\frac{H_s}{H}\right)^2 100 \qquad (\%)$$

Note that because of the natural *minimum energy law* a soil wedge will form either as A'C'D' or as *BCDA*. A'C'D' is the Rankine wedge, so if this forms the heel slope *BA* is an unnecessary expense.

If the wedge BCDA forms, the net gain (or loss) is trivial. We can obtain the value from plotting two force diagrams—one for wedge AC'D, which is in combination with the force diagram from block BCC'A as done in the inset of Fig. 12-16.

Keep in mind that if this slope is deemed necessary, the reason is that the base slab is narrow to begin with. By being narrow, the overturning moment from P_{ah} may tend to lift the heel away from the underlying soil, so the value of R_2 may be close to zero. If the heel slope compresses the soil, friction may be so large that wedge A'C'D' is certain to form. Walls built using this procedure may be standing but likely have a lower than intended SF. Their current safety status may also be due to some initial overdesign.

7. It has been suggested that for high walls Fig. 12-17 is a possible solution—that is, use "relief shelves." This solution has some hidden traps. For example, the soil must be well compacted up to the relief shelf, the shelf constructed, soil placed and compacted, etc. In theory the vertical pressure on the shelf and the lateral pressure on the wall are as shown. We can see that the horizontal active pressure resultant P_{ah} is much less than for a top-down pressure profile—at least for the stem.

What is difficult to anticipate is the amount of consolidation that will occur beneath the shelves—and it will—regardless of the state of the compaction. This tends to cantilever the shelves down, shown as dashed lines in the pressure profile diagram. When this occurs, either the shelf breaks off or the wall above tends to move into the backfill and develop





passive pressure. The wall therefore must be well reinforced on both faces and of sufficient thickness to carry this unanticipated shear and moment.

There is also the possibility of a Rankine wedge forming on line GH (overall wall stability). In this case the relief shelves have only increased the design complexity of the project.

12-7 WALL JOINTS

Current practice is to provide vertical *contraction joints* at intervals of about 8 to 12 m. These are formed by placing narrow vertical strips on the outer stem face form so that a vertical groove is developed when the concrete hardens. The groove produces a plane of weakness to locate tension cracks (so they are less obvious) from tensile stresses developing as the concrete sets (cures) or from contraction in temperature extremes.

Joints between successive pours are not currently identified—the new concrete is simply poured over the old (usually the previous day's pour) and the wall continued. When the forms are stripped, any obvious discontinuities are removed in the wall finishing operation.

Very large walls previously tended to be made with vertical *expansion joints* at intervals of 16 to 25 m. Current practice discourages⁵ their use, since they require a neat vertical joint

⁵Formerly it was considered good practice to require expansion joints in concrete walls at a spacing not to exceed 27 m (about 90 ft).