CHAPTER 10

MAT FOUNDATIONS

10-1 INTRODUCTION

A mat foundation is a large concrete slab used to interface one column, or more than one column in several lines, with the base soil. It may encompass the entire foundation area or only a portion. A mat may be used to support on-grade storage tanks or several pieces of industrial equipment. Mats are commonly used beneath silo clusters, chimneys, and various tower structures. It becomes a matter of definition as to when the dimensions of a spread footing make the transition into being called a mat. Figure 10-1 illustrates several mat configurations as might be used for buildings. Those shown encompass the entire building plan, but this is not a requirement.

A mat foundation may be used where the base soil has a low bearing capacity and/or the column loads are so large that more than 50 percent of the area is covered by conventional spread footings. It is common to use mat foundations for deep basements both to spread the column loads to a more uniform pressure distribution and to provide the floor slab for the basement. A particular advantage for basements at or below the GWT is to provide a water barrier. Depending on local costs, and noting that a mat foundation requires both positive and negative reinforcing steel, one may find it more economical to use spread footings—even if the entire area is covered. Spread footings avoid the use of negative reinforcing steel and can be accomplished as in Fig. 10-2 by pouring alternate footings, to avoid formwork, and using fiber spacer boards to separate the footings poured later.

Mat foundations may be supported by piles in situations such as high groundwater (to control buoyancy) or where the base soil is susceptible to large settlements. We should note that the mat contact stresses will penetrate the ground to a greater depth or have greater relative intensity at a shallower depth (refer to Figs. 5-4 and 5-9). Both factors tend to increase settlements unless there is a stress compensation from excavated soil so that the *net* increase in pressure is controlled.

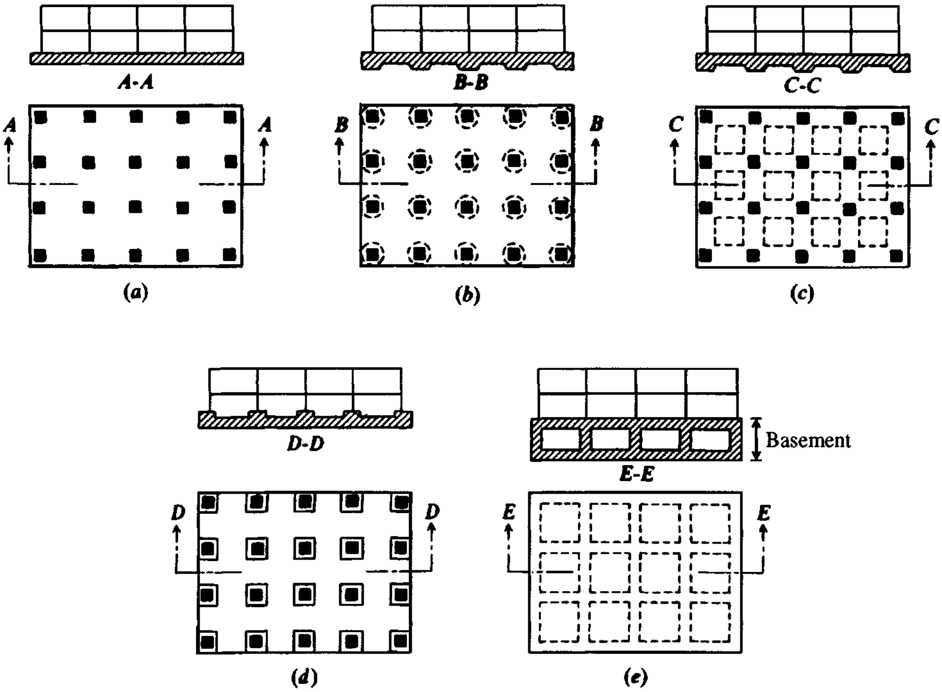

Figure 10-1 Common types of mat foundations. (*a*) Flat plate; (*b*) plate thickened under columns; (*c*) waffle-slab; (*d*) plate with pedestals; (*e*) basement walls as part of mat.

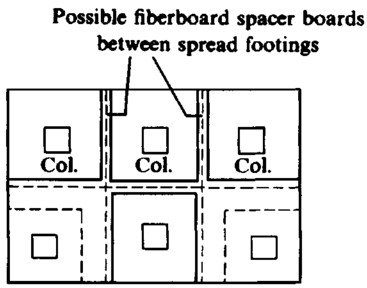

Figure 10-2 Mat versus possible use of spread footings to save labor, forming costs, and negative reinforcing steel.

10-2 TYPES OF MAT FOUNDATIONS

Figure 10-1 illustrates several possible mat-foundation configurations. Probably the most common mat design consists of a flat concrete slab 0.75 to 2 m thick and with continuous two-way reinforcing top and bottom. This type of foundation tends to be heavily overdesigned for three major reasons:

1. Additional cost of analysis methods, which are, however, not exact.
2. The extra cost of a reasonable overdesign of this element of the structure will generally be quite small relative to total project cost.
3. The extra margin of safety provided for the modest additional cost.

10-3 BEARING CAPACITY OF MAT FOUNDATIONS

The mat foundation must be designed to limit settlements to a tolerable amount. These settlements may include the following:

1. Consolidation—including any secondary effects
2. Immediate or elastic
3. A combination of consolidation and immediate amounts

A mat must be stable against a deep shear failure, which may result in either a rotational failure (see Fig. 4-1*a*), typified by the Transcona elevator failure (White, 1953), or a vertical (or punching) failure. A uniform vertical punching failure would not be particularly serious, as the effect would simply be a large settlement that could probably be landscaped; however, as the settlement is not likely to be uniform or predicted as such, this mode should be treated with concern equal to that for the deep-seated shear failure.

The bearing-capacity equations of Table 4-1 may be used to compute the soil capacity, e.g.,

$$q_{\text{ult}} = cN_c s_c i_c d_c + \gamma D N_q s_q i_q d_q + \frac{1}{2}\lambda B N_\gamma s_\gamma i_\gamma d_\gamma$$

or

$$q_{\text{ult}} = 5.14 s_u (1 + s'_c + d'_c - i'_c) + \bar{q}$$

Use B = least mat dimension and D = depth of mat (Fig. 10-3). The allowable soil pressure is obtained by applying a suitable factor of safety (see Table 4-9) and any applicable reduction for mat width B as suggested in Sec. 4-4.

When the bearing capacity is based on penetration tests (e.g., SPT, CPT) *in sands and sandy gravel,* one may use Eq. (4-13) rewritten [see Meyerhof (1965)] as follows:

$$q_a = \frac{N_{55}}{0.08}\left(\frac{\Delta H_a}{25.0}\right)K_d \qquad \text{(kPa)} \qquad (10\text{-}1)$$

where $K_d = 1 + 0.33D/B \leq 1.33$

ΔH_a = allowable settlement such as 25, 40, 50, 60 mm, etc.

The factor 0.08 converts Meyerhof's original equation to allow a 50 percent increase in bearing capacity and to produce kPa. The bracket ratio of ($\Delta H_a/25.0$) allows the reader to use any specified settlement, since the original equation was based on a settlement of 25 mm (1 inch). For a mat the ratio $((B + F_3)/B)^2 \approx 1.0$ and is neglected.

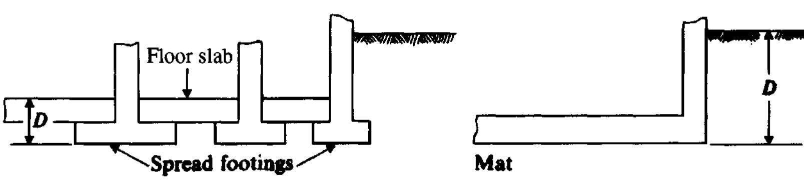

Figure 10-3 Increase in bearing capacity by using a mat foundation.

With q_c (in kPa) from a CPT we can use Fig. 3-23 or Eq. (4-20) to estimate an N_{55} value for use in Eq. (10-1). A typical computation for N_{55}, which you can use as a guide, is given in Fig. 3-23. For CPT in cohesive soil one can use Eq. (3-11) to obtain the undrained shear strength ($\phi = 0°$ case) s_u and use the bearing capacity equations (Meyerhof, Hansen, or Vesić) from Table 4-1 simplified to

$$q_{\text{ult}} = 5.14 s_u (1 + s'_c + d'_c - i_c) + \gamma D$$

Alternatively, use Eqs. (4-19) directly with q_c. In most cases the mat will be placed on cohesive soil, where q_u (or q_c) from standard penetration tests is the principal strength data available. In these cases SPT sampling is usually supplemented with several pushed thin-walled tube samples so that laboratory unconfined (or confined triaxial) compression tests can be performed to obtain what are generally considered more reliable strength parameters. Any triaxial laboratory tests may be CK_oXX, as indicated in Sec. 2-11, and either (or both) compression (case 1) and extension (case 3) type of Fig. 2-40. Alternatively, in situ tests may be performed, such as the pressuremeter or borehole shear, to obtain the design strength data.

10-4 MAT SETTLEMENTS

Mat foundations are commonly used where settlements may be a problem, for example, where a site contains erratic deposits or lenses of compressible materials, suspended boulders, etc. The settlement tends to be controlled via the following:

1. Use of a larger foundation to produce lower soil contact pressures.
2. Displaced volume of soil (flotation effect); theoretically if the weight of excavation equals the combined weight of the structure and mat, the system "floats" in the soil mass and no settlement occurs.
3. Bridging effects attributable to
   a. Mat rigidity.
   b. Contribution of superstructure rigidity to the mat.
4. Allowing somewhat larger settlements, say, 50 instead of 25 mm.

The flotation effect should enable most mat settlements, even where consolidation is a problem or piles are used, to be limited to 50 to 80 mm. A problem of more considerable concern is differential settlement. Again the mat tends to reduce this value. We can see in Fig. 10-4 that bending moments ($6EI\Delta/L^2$) and shear forces ($12EI\Delta/L^3$) induced in the superstructure depend on relative movement Δ between beam ends. Mat continuity results in a somewhat lower assumed amount of differential settlement relative to the total expected settlement versus a spread footing as follows:

Foundation type	Expected maximum settlement, mm	Expected differential settlement, mm
Spread	25	20
Mat	50	20

Computer methods that incorporate frame-foundation interaction can allow one to estimate both total and differential settlements. The total settlements will be only as good as the soil

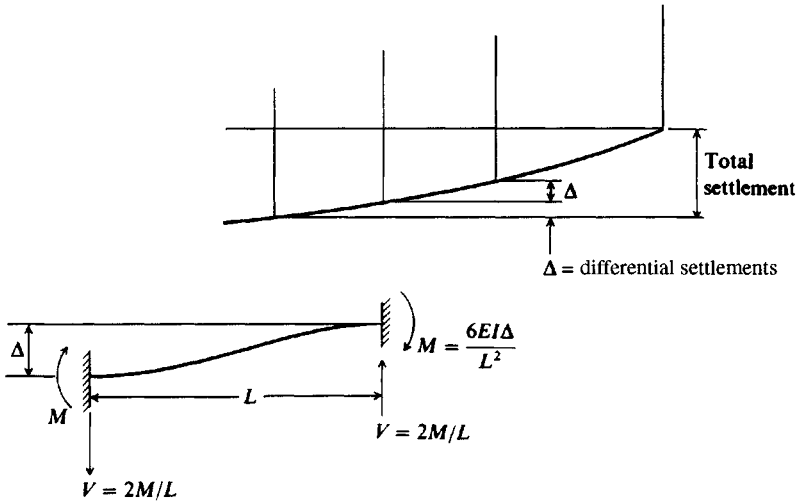

Figure 10-4 Reduction of bending moments in superstructure by using mat foundation. Bending moment M is based on differential settlement between columns and not on total settlement.

data, however, and if other than a strip from the mat is used as a beam-on-elastic foundation type of analysis, the computational effort is substantial.

The differential settlement may be arbitrarily taken as 20 mm (0.75 in.) if the total expected settlement ΔH is not more than 50 mm or may be approximated using a rigidity factor K_r [see ACI Committee 336 (1988)] defined as

$$K_r = \frac{EI_b}{E_s B^3} \tag{10-2}$$

EI_b may be taken as

$$EI_b = EI_f + \sum EI_{bi} + \sum \frac{Eah^3}{12} \tag{10-3}$$

where
EI_b = flexural rigidity of the superstructure and mat
E = composite modulus of elasticity of superstructure frame
EI_f = footing or mat flexural rigidity
E_s = modulus of elasticity of soil
$\sum \frac{Eah^3}{12}$ = effective rigidity of shear walls perpendicular to B; h = height; a = wall thickness
$\sum EI_{bi}$ = rigidity of the several members making up the frame resistance perpendicular to B
B = base width of foundation perpendicular to direction of interest

ACI Committee 336 suggests that mat differential settlements are related to both the total estimated foundation settlement ΔH and the structure rigidity factor K_r about as follows:

For K_r	Differential settlement expected	
0	$0.5 \times \Delta H$	for long base
	$0.35 \times \Delta H$	for square base
0.5	$0.1 \times \Delta H$	
> 0.5	Rigid mat; no differential settlement	

Analyses of settlements will have to be performed where the net increase in pressure exceeds the existing in situ pressure p'_o. These may be immediate and/or consolidation settlements adjusted for OCR and depending on the underlying soil stratification.

A major problem—particularly for deep excavations in clay—is expansion and/or lateral flow into the excavation base so that the base elevation rises. This phenomenon is termed *heave,* and values of 25 to 50 mm are very common. Values up to 200 mm (about 8 in.) are reported in the literature. It is difficult to compute settlements when heave has occurred. Theoretically, all the heave should be recovered if we reapply a mat pressure q_o equal to that previously existing. In practice this recovery does not occur, or at least it does not occur with the same rapidity as the heave. It should be expected that if part of the heave occurs from a deep-seated lateral flow (refer to Fig. 4-1 elements 1 and 2) it will be very difficult to predict either the total amount of heave or how much of this will be recovered by elastic recompression. In general, where heave is involved, considerable experience and engineering judgment are necessary in estimating probable soil response, for there are currently no reliable theories for the problem. There is some claim that a finite element of the elastic continuum computation can resolve the dilemma; however, this is a speculative procedure aided by hope of a happy outcome of computations and measurements. The reason is that a finite-element computation is only as good as the input parameters of E_s and μ. Even were we to be able to obtain a reliable initial E_s it will reduce during, and after excavation, as the loss of confining pressure p'_o and expansion produces heave.

Heave can also occur in deep excavations in sand but the amount is usually very small. Heave is usually not a consideration where the excavations are on the order of 2 to 3 m in depth in most soils, but it becomes a major problem for excavations of 10 to 20 m in clay.

Example 10-1. For the soil profile of Fig. E10-1 (a composite of the several site borings to save text space) estimate the allowable bearing capacity for a mat foundation to be located at $D \cong$ 1.5 m.

Solution. We will estimate an allowable bearing capacity based on q_u and adjust it so the settlement is approximately adequate.

These data are basically the type a geotechnical consultant would have on which to make an allowable pressure recommendation.

Step 1. Find a q_a based on strength alone with SF = 3 for clay. As in Example 4-4,

$$q_a = \frac{1.3N_c c}{\text{SF}} = 1.3(5.7)\frac{q_u}{2}\frac{1}{3}$$

$$= 1.3(5.14)\frac{300/2}{3} = 334.1 \text{ kPa}$$

Tentatively, q_a = **300** kPa = q_u (see Ex 4-4)

Step 2. Find q_a so mat settlement is on the order of about 50 mm.

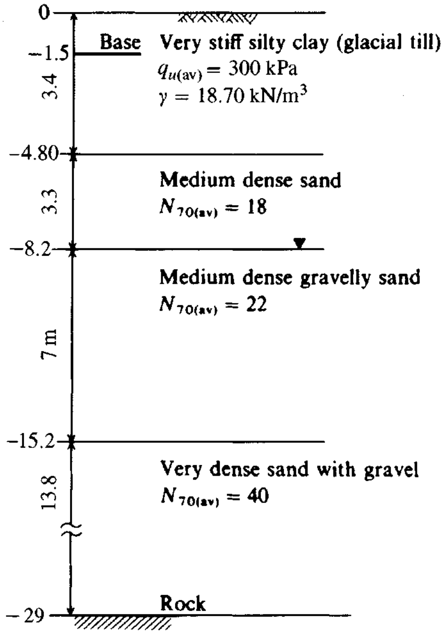

Figure E10-1

a. Find the average E_s.

The depth H from base of mat to rock is

$$H = (4.90 - 1.5) + 3.3 + 7 + 13.8 = 27.5 \text{ m}$$

The average E_s in this depth (and using Table 5-5) is

$$E_{s1} = \frac{1000q_u}{2} = 150\,000 \text{ kPa} \qquad \left(\text{average range of stiff clay and } s_u = \frac{q_u}{2}\right)$$

$$E_{s2} = 500(N_{55} + 15) \qquad \text{(the most conservative equation in Table 5-5)}$$

$$E_{s2} = 500\left[18\left(\frac{70}{55}\right) + 15\right] = 18\,950 \text{ kPa} \qquad \text{(converting } N_{70} \text{ to } N_{55} \text{ and rounding)}$$

$$E_{s3} = 500\left[22\left(\frac{70}{53}\right) + 15\right] = 22\,000 \text{ kPa} \qquad \text{(for 7 m stratum)}$$

$$E_{s4} = 500\left[40\left(\frac{70}{55}\right) + 15\right] = 32\,900 \text{ kPa}$$

The weighted average E_s is

$$E_{s(av)} = \frac{3.4(150\,000) + 3.3(18\,950) + 7.0(22\,000) + 13.8(32\,900)}{27.5}$$

$$= \frac{1\,180\,555}{27.5} = \mathbf{42\,930} \text{ kPa}$$

b. Estimate the mat will be on the order of 14 m, giving

$$\frac{H}{B'} = \frac{27.5}{7} \cong 4 \qquad \frac{L}{B} = 1$$

Estimate $\mu = 0.3$ for all the layers.

From Table 5-2 obtain I_i factors to compute I_s for use in Eq. (5-16*a*); thus,

$$I_s = 0.408 + \frac{1-2(0.3)}{1-0.3}(0.037) = 0.429$$

Estimating $D/B = 0.1$, we obtain $I_F \cong 0.95$ from your program FFACTOR. Using Eq. (5-16*a*), we write

$$\Delta H = q_o B'\left(\frac{1-\mu^2}{E_s}\right) m I_s I_F \qquad \text{[Eq. (5-16}a\text{)]}$$

$$\frac{\Delta H}{q_o} = 7\left(\frac{1-0.3}{42\,930}\right)^2 (4 \times 0.429)(0.9) = 0.000\,24 \text{ m}^3\text{/kN} \qquad \text{(at center)}$$

For a settlement $\Delta H = 50$ mm (0.050 m) we solve for the required $q_a(= q_o)$ to obtain

$$\frac{\Delta H}{q_a} = 0.000\,24 \rightarrow q_a = \frac{0.050}{0.0024} = \mathbf{208}\text{ kPa}$$

This q_a should limit the mat settlement to about 50 mm, which is a common allowable value for mat foundations.

Note: A qualifying statement should be included with this recommendation that if, as the design proceeds and B is found to be substantially different from 14 m, it may be necessary to revise q_a. Recommend q_a = **250** kPa.

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10-5 MODULUS OF SUBGRADE REACTION k_s FOR MATS AND PLATES

All three discrete element methods given in this chapter for mats/plates use the modulus of subgrade reaction k_s to support the plate. The modulus k_s is used to compute node springs based on the contributing plan area of an element to any node as in Fig. 10-5. From the figure we see the following:

Node	Contributing area
1 (corner)	$\frac{1}{4}$ of rectangle *abde*
2 (side)	$\frac{1}{4}$ of *abde* + $\frac{1}{4}$ of *bcef*
3 (interior)	$\frac{1}{4}$ of each rectangle framing to a common node (as node 3)

For a triangle one should arbitrarily use one-third of the triangle area to any corner node. For these area contributions the fraction of k_s node resistance from any element is

$$K_i = k_s, \text{ kN/m}^3, \times\text{Area, m}^2 = \text{units of kN/m (or kips/ft in Fps)}$$

Since this computation gives units of a "spring" it is common to call the effect a *node spring*.

In this form the springs are independent of each other, the system of springs supporting the plate is termed a "Winkler" foundation, and the springs are uncoupled. Uncoupling means that the deflection of any spring is not influenced by adjacent springs.

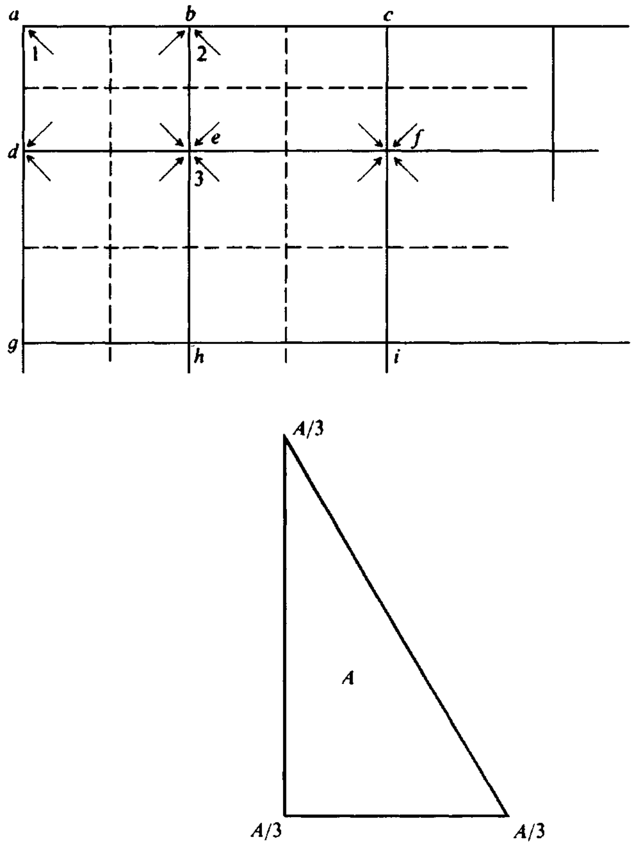

Figure 10-5 Method of prorating k_s to build node springs for rectangles and triangles.

Because the springs are uncoupled, some designers do not like to use the concept of k_s, preferring instead to use a FEM of the elastic continuum with E_s and μ as elastic parameters. This choice does somewhat couple the effects; however, the computations are extensive and only as good as one's estimate of E_s and μ. It has already been shown in Sec. 9-6 that there is a direct relationship between these parameters and k_s. In any case the use of k_s in analyzing mats is rather widespread because of the greater convenience of this parameter. There is actually little computational evidence that the FEM of the elastic continuum provides better solutions than using a "Winkler" foundation.

The author [Bowles (1986)], and as given in previous editions of this textbook, has approximately coupled the springs. In general, coupling can be done as follows:

1. Simply double the edge springs of a mat (we doubled the end springs of the beam-on-elastic foundation in Chap. 9). This should only be done under these conditions:
   a. The plate or mat is uniformly loaded except for edge moments as one would obtain from a tank base.
   b. The plate or mat has only one or at most two column loads.
   c. The computed node soil pressures q are in the range of mat load $\sum P/A_m$, where A_m = area of the mat. If there are large differences do not double the edge springs. How does

one ascertain this? Use computer program B-6 (FADMAT on your diskette), double the edge springs, and inspect the output. If the contact pressures q are questionable, copy the data file, edit the copy to remove the edge spring doubling controls, and rerun. Use the most reasonable output.

2. We can zone the mat area using softer springs in the innermost zone and transitioning to the outer edge. Zoning is computed as in Example 10-2 and usually the three zones computed there are sufficient. Refer to Example 10-6 and data file EXAM106B.DTA for the method.

   The simplest zoning (which effectively doubles the edge springs) is to use two zones—an interior one, which includes all of the nodes except the edge ones, and all the perimeter nodes for the second "zone." Use 1.5 to 2 × $k_{s,\text{interior}}$ for these edge (or perimeter) nodes. But be aware this will result in large computed edge pressures. Element data generator program B-18 (on your README.DOC file) is particularly well-suited to do these computations.

3. You should not both double the edge springs and zone the mat area for the same program execution. Use either one or the other, or simply use a constant k_s beneath the entire foundation. This latter may be the most nearly correct when there are a number of column loads. There has been some attempt at coupling by using the Boussinesq equation [Eq. (5-3) or (5-4)] in this fashion:
   a. Make a trial run and obtain the node pressures.
   b. Use these node pressures and compute the pressure increase profiles at adjacent nodes.
   c. Use these pressure increase profiles to modify the k_s around these several nodes. This approach requires a very massive amount of computing and is not recommended by the author. It does not make much sense to use an approximation to refine an "estimate."

Example 10-2. For the soil data of Example 10-1 recommend k_s for the 14 × 14 m mat foundation.

Solution. For many cases a single value of k_s is recommended that may be an average for the base. We will do this but also give the three zone values since very little additional effort is required.

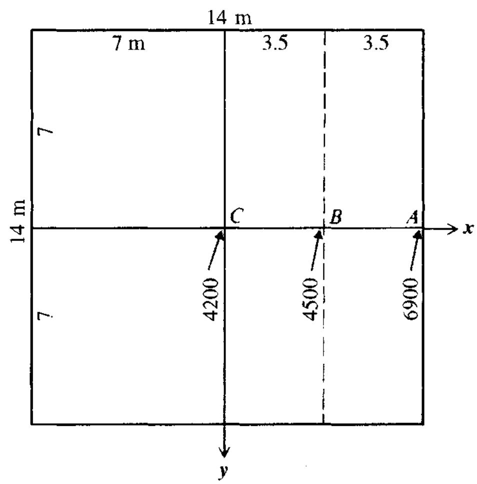

Figure E10-2

Step 1. The center k_s (point C of Fig. E10-2) is readily computed from the same data used to obtain q_a in Example 10-1. From Example 10-1 obtain

$$\frac{1}{k_s} = \frac{\Delta H}{\Delta q} = 0.000\,24 \rightarrow \frac{\Delta q}{\Delta H} = k_s = \frac{1}{0.000\,24}$$

and center $k_s = 4166.7 \rightarrow 4200\ \text{kN/m}^3$.

Step 2. The **edge** (point A of Fig. E10-2) value is obtained by dividing the 14 m mat into two parts of $B \times L = 7 \times 14$ with a common corner $H/B' = 4$ (as before) but $L/B = 2$, $D/B = 0.1$, $\mu = 0.3$, so

$$I_s = 0.476 + \frac{0.4}{0.7}(0.069) = 0.515 \qquad I_F = 0.95\ \text{(using FFACTOR)}$$

$$E_s' = \frac{1-\mu^2}{42\,930} = \frac{0.91}{42\,930} = 2.12 \times 10^{-5}$$

Rearranging Eq. (5-16*a*) into Eq. (9-7), we have

$$k_{se} = \frac{1}{B'E_s'mI_sI_f} = \frac{1}{7(2.12 \times 10^{-5})(2 \times 0.515)(0.95)} = 6887 \rightarrow \mathbf{6900}\ \text{kN/m}^3$$

(We used $m = 2$ since there are two common corners.)

Step 3. For the $\frac{1}{4}$ **point** (point B) we have midpoints of two sets of rectangles:

Set 1: $B' = 7$ $L' = 10.5\text{m}$ $L/B = 1.5$ $I_F = 0.95$ $H/B' \cong 4$
$I_s = \mathbf{0.483}$ (program FFACTOR direct)

Set 2: $B' = 3.5$ $L = 7.0\ \text{m}$ $L/B = 2$ $H/B = 27.5/3.5 = 7.86$
$I_s = \mathbf{0.630}$

$$k_{s(1/4)} = \frac{1}{7(2.12 \times 10^{-5})(2 \times 0.483)(0.95) + 35(2.12 \times 10^{-5})(2 \times 0.630)(0.95)}$$

(↑ for 2 rectangles ↑)

$$= \frac{1}{2.250 \times 10^{-4}} = 4444.9 \rightarrow \mathbf{4500}\ \text{kN/m}^3$$

The value of $k_{s(1/4)}$ is not the sum of set 1 and set 2 but is actually the reciprocal of the sum of the displacements as given by placing the sum under a single common numerator of 1 as here.

Step 4. If a single value of k_s is to be provided, one might use either

$$k_s = \frac{4200 + 4500 + 6900}{3} = 5200\ \text{kN/m}^3$$

or (weighting the center k_s)

$$k_s = \frac{4(4200) + 6900}{5} = 4740 \rightarrow \mathbf{4700}\ \text{kN/m}^3$$

Comments. This set of computations for k_s is theoretically exact since the displacements are theoretical and the definition of k_s has been strictly followed. The only approximations are in whether E_s and μ are correct.

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MODULUS OF SUBGRADE REACTION AND CONSOLIDATION SETTLEMENTS. It is not uncommon that a mat is placed on a soil that is analyzed by using k_s, but there are, in addition, consolidation settlements that will occur later.

It is a relatively simple exercise in using the definition of k_s to include the effect of consolidation settlements. This can be done as follows:

$$k_s = \frac{q_o}{\Delta H} \tag{a}$$

Although the base contact pressure q_o remains constant the total settlement is

$$\Delta H' = \Delta H + \Delta H_c$$

giving

$$k_s' = \frac{q_o}{\Delta H + \Delta H_c} \tag{b}$$

Dividing Eq. (*b*) by Eq. (*a*), we obtain

$$k_s' = \frac{k_s \Delta H}{\Delta H + \Delta H_c} \tag{c}$$

We can see that including the consolidation settlement reduces k_s to the lesser k_s' value of Eq. (*c*).

Example 10-3. What is the recommended k_s' (constant value) if the consolidation settlement is estimated to be 50 mm in Example 10-2? Use $k_s = 5200$ kN/m^3.

Solution. From Example 10-1 a contact pressure of $q_o = 205$ kPa produces $\Delta H = 50$ mm $=$ 0.050 m. From Example 10-2 we see the elastic k_s was independent of q_a; thus, using Eq. (*c*) we write

$$k_s' = \frac{5200(50)}{50 + 50} = \mathbf{2600}\ \text{kN/m}^3$$

Comments. It is presumed that the consolidation pressure is based on $q_0 = q_a = 200$ kPa. One would probably have to inspect the computer output to find out if the contact pressure in the zone of interest was much different from 200 kPa. If so, a new value of ΔH_c would have to be computed and the problem recycled.

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10-6 DESIGN OF MAT FOUNDATIONS

There are several methods to design a mat (or plate) foundation.

1. *An approximate method.* The mat is divided into strips loaded by a line of columns and resisted by soil pressure. This strip is then analyzed as a combined footing. This method can be used where the mat is very rigid and the column pattern is fairly uniform in both spacing and loads. This method is not recommended at present because of the substantial amount of approximations and the wide availability of computer programs that are relatively easy to use—the finite grid method (program B6 on your program diskette) in particular. A mat

is generally too expensive and important not to use the most refined analytical methods available.

2. *Approximate flexible method.* This method was suggested by ACI Committee 336 (1988) and is briefly described here, and the essential design aids are provided. If this method is used it should be programmed as for the AIRPAVE computer program noted in subsection 10-6.2 following.
3. *Discrete element methods.* In these the mat is divided into elements by gridding. These methods include the following:
   a. Finite-difference method (FDM)
   b. Finite-element method (FEM)
   c. Finite-grid method (FGM)

10-6.1 Approximate Flexible Method

The approximate flexible method of ACI Committee 336 requires the following steps:

2. Compute the plate rigidity D (unfortunately, same symbol as footing depth).
3. Compute the radius of effective stiffness L (Note: the approximate zone of any column influence is $\approx 4L$).
4. Compute the radial and tangential moments, the shear, and deflection using the following equations (the Z_i factors, from Hetenyi (1946), are not easy to compute) where load P acts:

$$M_r = -\frac{P}{4}\left[Z_4 - \frac{1-\mu_c}{x}Z_3'\right] \qquad (10\text{-}4)$$

$$M_t = -\frac{P}{4}\left[\mu_c Z_4 + \frac{1-\mu_c}{x}Z_3'\right] \qquad (10\text{-}5)$$

$$\Delta H = \frac{PL^2}{8D} \quad \text{(vertical displacement)} \qquad (10\text{-}6)$$

$$\Delta H = \frac{PL^2}{4D}Z_3 \quad \text{(at distance } r \text{ from load)} \qquad (10\text{-}6a)$$

$$V = -\frac{P}{4L}Z_4' \quad \text{(shear)} \qquad (10\text{-}7)$$

where P = column load, kN or kips

D = plate stiffness, as

$$D = \frac{E_c t^3}{12(1-\mu_c^2)} \quad \text{(units of moment)}$$

μ_c = Poisson's ratio for mat or plate (for concrete use 0.15)

x = distance ratio r/L shown on Fig. 10-6

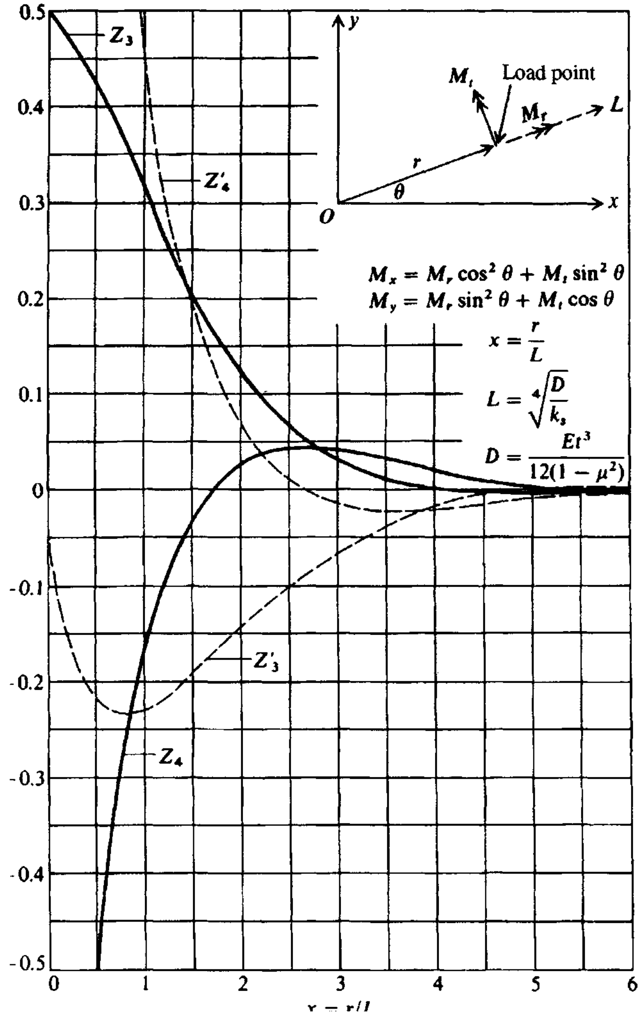


Figure 10-6 Z_i factors for computing deflections, moments, and shears in a flexible plate. [After Hetenyi (1946).]

Z_i = factors from Fig. 10-6 based on x (or from a computer program such as AIRPAVE)

L = influence radius defined as $\sqrt[4]{\frac{D}{k_s}}$

M_r, M_t = radial and tangential moments at the load point of Fig. 10-6, *per unit of width* in units of P, L

V = shear per unit of width of mat or plate in units of P

The radial (M_r) and tangential (M_t) moments in polar coordinates at the load point are converted to rectangular coordinates M_x, M_y, referenced to the origin, using the transfer equations shown in Fig. 10-6. For the several loads in the influence region L these M_x, M_y moment values are summed with attention to sign for design of the plate.

When the edge of the mat is within the radius of influence L, calculate the edge moment and shear. The parallel edge moment and shear are then applied as edge loads with opposite sign. When several columns overlap in the zone L, apply superposition to obtain the net effect.

An illustration of computations for a mat are given by Shukla (1984) using this procedure. The D calculated in this reference is in error, so that the resulting computations are not quite correct; but the general procedure gives an illustration of the method.

10-6.2 Mats or Slabs for Industrial Warehouses and Concrete Airstrips

Industrial floor slabs and concrete airport pavements are somewhat similar and can be designed using the procedure outlined here. One additional step is required. From Westergaard (1948) we can obtain an equation for the bending stresses in the bottom of a slab under a wheel load. This equation is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} = \frac{3P}{8\pi t^2}\left[(1+\mu_c)\ln\frac{E_c t^3}{k_s\left(\dfrac{a+b}{2}\right)^4} \mp 2(1-\mu_c)\frac{a-b}{a+b}\right] \tag{10-8}$$

Here terms not defined previously are t = plate thickness; a, b = axis dimensions of an ellipsoid used to model the tire footprint. Approximately [given by PCA (1955)] we have

$$\text{Area} = \text{tire load/tire pressure}$$

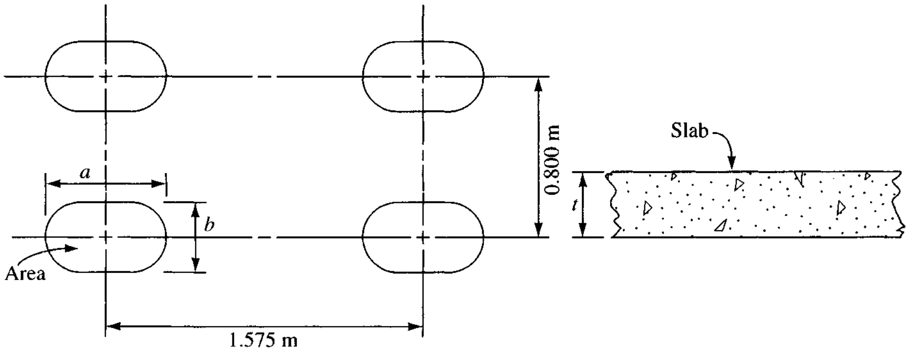
$$a = \sqrt{\frac{\text{Area}}{\pi(0.6655)}} \qquad \text{(long axis)}$$

$$b = 0.6655 \times a \qquad \text{(short axis)}$$

Use consistent units for t, a, b of meters or inches. If tire load is in kN use tire pressure in kPa; if load is in pounds use tire pressure in psi. As you can see, there is a sign convention involved with the several equations given here. To understand the significance of the signs you should solve a simple slab where you know there is tension (or compression) in the bottom and compare the result to the signs for stress or moment values given by the equations. Convert the moments of Eqs. (10-4) and (10-5) to stresses by using the conventional

$$f_c = \frac{Mc}{I} = \frac{6M}{t^2} \qquad (\text{since } M_i = \text{per unit of width})$$

In usage one would program Eq. (10-8) together with the Z_i factors and Eqs. (10-4) through Eq. (10-7). If the point where the stresses are wanted is under a wheel, include Eq. (10-8) in the analysis. If the point is not under a wheel, use only the equations containing the Z_i factors. You need to program the Z_i factors so that interpolation is not necessary (it is also difficult to obtain reliable values from Fig. 10-6). Figure 10-7 illustrates a set of wheels (these are one landing gear of an airplane) but could be from a wheeled loader in a warehouse. In using this procedure it may be worthwhile to provide suitable load transfer dowels into perimeter wall footings for jointed slabs. For airport runways the common procedure is to use continuously reinforced concrete (CRC) so joints are not used. The pavement edges are usually thickened such that edge formulas do not have to be considered. Floor slab edges in warehouses probably should be thickened as well, partly because of the difficulty in obtaining good compaction adjacent to the perimeter wall footings.



Dual-tandem landing gear
Total load = 667.2 kN (166.8 kN/wheel)

For $P = 166.8$ kN   Tire pressure = 690 kPa

$$\text{Area} = \frac{166.8}{690} = 0.2417\ \text{m}^2$$

$$a = \sqrt{\frac{0.2417}{3.14159(0.6655)}} = 0.340\ \text{m} = 340\ \text{mm}$$

$$b = 0.340(0.6655) = 226.3\ \text{mm}$$

Figure 10-7 One part of a landing gear set (nose wheel and other side gear not shown). Also shown are computations for tire load and the approximate ellipse dimensions for use in Eq. (10-8).

10-7 FINITE-DIFFERENCE METHOD FOR MATS

The finite-difference method uses the fourth-order differential equation found in any text on the theory of plates and shells [Timoshenko and Woinowsky-Krieger (1959)]:

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} + \frac{P}{D(\partial x\,\partial y)} \tag{10-9}$$

which can be transposed into a finite-difference equation when $r = 1$ (Fig. 10-8):

$$\begin{aligned} &20w_o - 8(w_T + w_B + w_R + w_L) + 2(w_{TL} + w_{TR} + w_{BL} + w_{BR}) \\ &\quad + (w_{TT} + w_{BB} + w_{LL} + w_{RR}) = \frac{qh^4}{D} + \frac{Ph^2}{D} \end{aligned} \tag{10-10}$$

When $r \neq 1$, this becomes (as in program B-19, but with much algebra and many steps not shown)

$$\begin{aligned} &\left(\frac{6}{r^4} + \frac{8}{r^2} + 6\right)w_o + \left(-\frac{4}{r^4} - \frac{4}{r^2}\right)(w_L + w_R) + \left(-\frac{4}{r^2} - 4\right)(w_T + w_B) \\ &+ \frac{2}{r^2}(w_{TL} + w_{TR} + w_{BL} + w_{BR}) + w_{TT} + w_{BB} + \frac{1}{r^4}(w_{LL} + w_{RR}) \\ &= \frac{qrh^4}{rD} + \frac{Ph^2}{rD} \end{aligned} \tag{10-11}$$

Since $q = -k_s w_o$ we must rearrange the w_o term of Eq. (10-11) to read

$$\left(\frac{6}{r^4} + \frac{8}{r^2} + 6 + \frac{k_s h^4}{D}\right)w_o \tag{a}$$

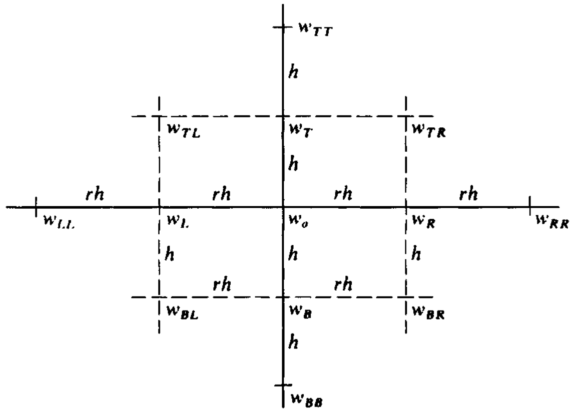


Figure 10-8 Finite-difference grid of elements of $rh \times h$ dimension.

The form shown for the k_s term results from computing a spring using $k_s rh^2$, dividing through by rh^2, and multiplying by h^4. Note that rD does not cancel in the P term.

When $r = 1$ we have the familiar deflection coefficient at any interior node of

$$\left(20 + k_s\frac{h^4}{D}\right)w_o \qquad (b)$$

Referring to Fig. 10-8, we see that the horizontal grid spacing rh can be different from the vertical grid spacing $h(1 \leq r$ or $r \leq 1)$. In a computer program, of course, one simply orients the mat so that the minimum grid points are horizontal with the origin of the grid at the lower left corner. The input then consists in the horizontal grid spacing and vertical grid spacing, which are constant, and the band width, which is $2 \times$ horizontal grid points $+ 1$ (thus, a minimum is obtained if the horizontal grid points are the minimum).

The finite-difference method has several advantages:

1. It has been widely used (and should be used as a check on alternative methods where it is practical).
2. It is reliable if the mat can be modeled using a finite-difference grid.
3. It is rapid since the input data are minimal compared with any other discrete method, and the computations to build the stiffness array are not so extensive as other methods. Usually only three to five lines of input data are needed compared with up to several hundred for the other methods.

There are also a number of disadvantages:

1. It is extremely difficult to model boundary conditions of column fixity.
2. It is very difficult to model notches, holes, or reentrant corners.
3. It is difficult to apply a concentrated moment (as from a column) since the difference model uses moment/unit of width.

The following example illustrates typical input and output from an FDM program (e.g., program FADMATFD, B-19).

Example 10-4. Do the Example 10-5 (p. 565) using the finite-difference method (FDM) to illustrate the small amount of input needed and typical output, at least output using program FADMATFD.

```
+++++++++++++ NAME OF DATA FILE USED FOR THIS EXECUTION:   EX104FDM.DTA

      EXAMPLE 10-4-SQUARE PLATE 3 X 3 M--AND NONLIN--FINITE DIFF METHOD--SI UNITS

        MAT FOUNDATION INPUT DATA:
                 NO OF COLS, M =  6
                 NO OF ROWS, N =  6
       NO OF NON-ZERO Q-VALUES =  4
     MAT GRID SPACING:  H =   .600   RH =   .600  M
           PLATE THICK, T =   .600  M
     MOD OF ELASTICITY, E =  22408000. KPA
     POISSON'S RATIO, XMU =  .150
           UNIT WT OF MAT =   .000 KN/M^3
          SOIL MODULUS SK =  15700.0 KN/M^3
     COMPUTED PARAMETERS:  FLEX RIGID D =  .41263E+06
                     FACTOR DD = H^2/R*D =  .87246E-06

         MAT DIMENSIONS ARE:  X =  3.000  M  HORIZ
                              Y =  3.000  M  VERT
     MAX NON-LIN SOIL DEF, XMAX =   .0200  M

     ++++++ BANDWIDTH OF MATRIX =  13  +++++++++++++++

     SOIL SPRING (SOK(I,J) CONSTANTS--EDGES DOUBLED IF IDBLK > 0--IDBLK =  1
                  1           2           3           4           5           6
       1       .00247      .00493      .00493      .00493      .00493      .00247
       2       .00493      .00493      .00493      .00493      .00493      .00493
       3       .00493      .00493      .00493      .00493      .00493      .00493
       4       .00493      .00493      .00493      .00493      .00493      .00493
       5       .00493      .00493      .00493      .00493      .00493      .00493
       6       .00247      .00493      .00493      .00493      .00493      .00247

     THE INPUT FOUNDATION LOADS AND COORDS ARE
    3    3      550.
    3    4      550.
    4    3      550.
    4    4      550.

          FOOTING WT =       .000    SUM OF INPUT LOADS =  2200.000 KN

          THE LOAD ARRAY--AND CORRECTED FOR NON-LINEAR EFFECTS IF CYCLE > 1
            THE CURRENT CYCLE =  1
                  1           2           3           4           5           6
       1       .00000      .00000      .00000      .00000      .00000      .00000
       2       .00000      .00000      .00000      .00000      .00000      .00000
       3       .00000      .00000   550.00000   550.00000      .00000      .00000
       4       .00000      .00000   550.00000   550.00000      .00000      .00000
       5       .00000      .00000      .00000      .00000      .00000      .00000
       6       .00000      .00000      .00000      .00000      .00000      .00000

                         NO OF STIFF(I) ENTRIES =   468
```

Figure E10-4

THE DEFLECTION MATRIX IS (M)

	1	2	3	4	5	6
1	.01107	.01126	.01139	.01139	.01126	.01107
2	.01126	.01146	.01161	.01161	.01146	.01126
3	.01139√	.01161√	.01181√	.01181√	.01161√	.01139√
4	.01139	.01161	.01181	.01181	.01161	.01139
5	.01126	.01146	.01161	.01161	.01146	.01126
6	.01107	.01126	.01139	.01139	.01126	.01107

CURRENT CYCLE = 1   CURRENT NON-LIN COUNT = 0   PREVIOUS COUNT = 0

THE BENDING MOMENTS IN SLAB IN KN-M ARE AS FOLLOWS

COORDS		X-AXIS	Y-AXIS	COORDS		X-AXIS	Y-AXIS
1	1	.0000	.0000	4	1	.0000	-154.0598
1	2	-74.0461	.0000	4	2	-60.4754	-180.3783 √
1	3	-154.0774	.0000	4	3	-259.3937	-259.3710 √
1	4	-154.0733	.0000	4	4	-259.3881	-259.3681 √
1	5	-74.0465	.0000	4	5	-60.4837	-180.3847
1	6	.0000	.0000	4	6	.0000	-154.0587
2	1	.0000	-74.0377	5	1	.0000	-74.0414
2	2	-66.8780	-66.8707	5	2	-66.8739 √	-66.8711
2	3	-180.4002	-60.4756	5	3	-180.4066 √	-60.4766
2	4	-180.4025	-60.4770	5	4	-180.4068 √	-60.4776
2	5	-66.8763	-66.8663	5	5	-66.8683	-66.8620
2	6	.0000	-74.0354	5	6	.0000	-74.0354
3	1	.0000	-154.0615	6	1	.0000	.0000
3	2	-60.4810	-180.3875	6	2	-74.0441	.0000
3	3	-259.3906	-259.3779	6	3	-154.0870	.0000
3	4	-259.3921	-259.3813	6	4	-154.0808	.0000
3	5	-60.4859	-180.3924	6	5	-74.0396	.0000
3	6	.0000	-154.0653	6	6	.0000	.0000

THE NODAL REACTIONS (KN) ARE AS FOLLOWS

	1	2	3	4	5	6
1	31.28732	63.63005	64.36333	64.36211	63.62643	31.2843
2	63.63162	64.78147	65.64455	65.64334	64.77783	63.6255
3	64.36652	65.64617	66.75721	66.75600	65.64252	64.3604
4	64.36701	65.64664	66.75768	66.75645	65.64296	64.3608
5	63.63311	64.78294	65.64601	65.64476	64.77919	63.6268
6	31.28856	63.63249	64.36577	64.36450	63.62873	31.2854

TOTAL SUM OF FOOTING LOADS = 2200.000 KN
SUM OF SOIL REACTIONS = 2200.396 KN

THE NODAL SOIL PRESSURE, KPA , IS

	1	2	3	4	5	6
1	173.81850	176.75010	178.78700	178.78360	176.74010	173.8017
2	176.75450	179.94850	182.34600	182.34260	179.93840	176.7376
3	178.79590	182.35040	185.43670	185.43330	182.34030	178.7789
4	178.79720	182.35180	185.43800	185.43460	182.34150	178.7801
5	176.75860	179.95260	182.35000	182.34650	179.94220	176.7413
6	173.82530	176.75690	178.79380	178.79030	176.74650	173.8080

Figure E10-4 (*continued*)

Solution. Refer to Fig. E10-5*a* for gridding, but for the FDM take the origin at the lower left corner (node 31). Count

$$M = 6 \text{ (nodes 31–36) and}$$

$$N = 6 \text{ (nodes 31, 25, 19, 13, 7, and 1)}$$

Prorate the column to four nodes as shown giving coordinates of

I	J	Load	I	J	Load
3	3	550 kN	4	3	550 kN
3	4	550 kN	4	4	550 kN

$$H = rH = 0.6 \text{ m (square grid)} \qquad \text{Mat concrete } \mu = 0.15$$

$$\text{Thickness } t = 0.6 \text{ m} \qquad E_c = 22\,408 \text{ MPa}$$

The input data set named EX104FDM.DTA is as follows:

```
EXAMPLE 10-4 SQUARE PLATE 3 × 3 M—AND NONLIN—FINITE DIFF METHOD—SI UNITS
6   6       4     0      13    0        0
0   1       1
.6000          .6000        .6000     22408.0     0.15     15700.     0.0     .02
3   3       550.000
                                         ↑          ↑         ↑                 ↑
3   4       550.000
4   3       550.000                   Ec, MPa       μ         ks             XMAX, m
4   4       550.000
```

The program computes

$$D = \frac{Et^3}{12(1-\mu^2)} = \frac{22\,408\,000 \times 0.6^3}{12(1-\mu^2)}$$
$$= 412\,628.1 \text{ kN} \cdot \text{m (computer output} = .412\,63\text{E}+06)$$

Select data marked with a ✓ from the computer output sheet (Fig. E10-4) is shown on Fig. E10-5*a*.

Note that the program FADMATFD allows a "nonlinear" displacement check, the inclusion of mat weight, the doubling of edge springs, and user input of node springs in the form of $SK \times H^4/D$ (gives $15\,700 \times 0.6^4/412\,628.1 = 0.004\,93$ for interior and doubled side nodes and $0.004\,93/2 = 0.002\,47$ for doubled corner nodes). Here the only option used was doubling of the edge springs so the output could be compared to the output from the other two methods shown on Fig. E10-5*a*. The program always checks for any soil-mat separation and recycles if any nodes separate from the soil regardless of the NONLIN input parameter.

Checking. Perform checks as follows since mat and load are symmetrical.

1. Displacement array is symmetrical. For example, corner nodes of 1,1, 1,6, 6,1, and 6,6 = **0.011 07** m.
2. Moments are symmetrical. For example, the x-moment at nodes 2,2, 2,5, 5,2, and 5, 5 = **−66.87** kN · m.
3. Since the displacements are symmetrical, the node reactions and soil pressures are also equal. The node soil pressure at node 2,2 = $SK \times X(2,2) = 15\,700 \times 0.011\,46 =$ **179.9** kPa.
4. Note the sum of the soil reactions = 2200.396 kN versus the sum of column loads = 2200.000 kN (slight computer round-off error).

////

10-8 FINITE-ELEMENT METHOD FOR MAT FOUNDATIONS

In the *finite-element* analysis, element continuity is maintained through use of displacement functions. The displacement function is of the form

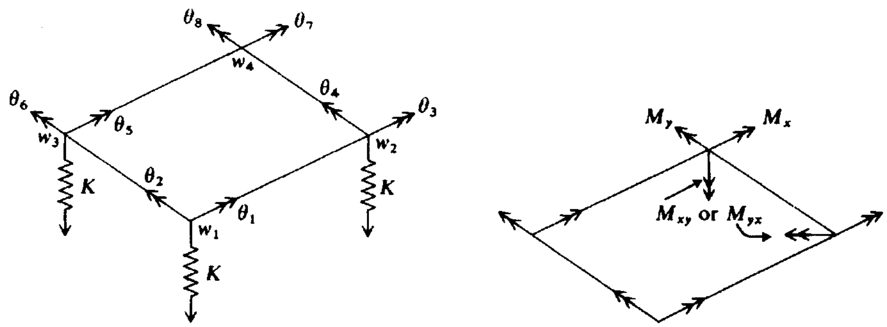
$$u = a_1 + a_2X + a_3Y + a_4X^2 + a_5XY + a_6Y^2 + a_7X^3 + a_8X^2Y + a_9XY^2 + a_{10}Y^3 + a_{11}X^4 + a_{12}X^3Y + a_{13}X^2Y^2 + a_{14}XY^3 + a_{15}Y^4 \quad (10\text{-}12)$$

With a rectangular plate and three general displacements at each corner node (Fig. 10-9) only 12 unknowns of Eq. (10-12) are necessary. This results in reducing the general displacement equation to one with 12 a_i coefficients instead of 15. Which three are best to discard becomes a considerable exercise in both engineering judgment and computational ability/tenacity. Various procedures have been and are being periodically proposed to reduce and solve the resulting matrix such as those proposed at the finite-element conferences at McGill University (1972), Wright Patterson AFB (1965, 1968, 1971), and regular papers in several journals including the *Journal of Structural Division,* ASCE.

One of the major advances in the FEM is using isoparametric element formulation so that a given element may have more nodes than an adjacent one. In any case, the FEM output is very difficult to interpret. Additionally the method is computationally intensive (about four times as long to run a problem of reasonable length as the FGM of the next section). The general methodology uses advanced mathematical concepts with which many civil and structural engineers are not familiar so that identification of incorrect output may be difficult.

Concentrated node moments can be readily input as part of the load array; however, a nodal statics check is difficult. The reason is that the output element node moments are in units of moment/unit of width, whereas the input is the moment at that node. A moment summation is not directly possible because of units incompatibility, and the situation is not helped from having to interpret and apply the twist moment M_{xy} of Fig. 10-9. Similarly a vertical force summation is not easy since element node shears are difficult to compute with the element moments obtained on a unit width basis.

For these several reasons, the author does not recommend use of the FEM for mat and plate problems. There are many design situations where the FEM is particularly suited; however, the FGM following is preferred for the more direct solution of foundation engineering problems.



(*a*) Displacements. (*b*) Element moments

Figure 10-9 Finite-element method using a rectangular plate element.

10-9 THE FINITE-GRID METHOD (FGM)

This method is particularly well-suited for use for the analysis of mats and plates. It has these distinct advantages:

1. The output is easy to interpret since beam-column type elements that have only bending and torsion are used. The moment/unit width is simply the node moment (from a node summation) divided by the element width.
2. It is easy to obtain design shears at the ends of the elements. The shear is simply the sum of the element end moments divided by element length. Then one divides the total element shear by the element width to get the shear/unit width.
3. It is easy to input concentrated column moments directly.
4. Boundary cases are as easily modeled as with the FEM.
5. It is relatively simple to extend the 3 degrees of freedom (d.o.f.) nodes of this method to use 6 d.o.f. nodes that are required for pile cap analysis [Bowles (1983)].

Although Table 10-1 (based on Fig. 10-10) established the general validity of the FGM, users have been the ultimate test. The program (your diskette program B-6 but often supplemented by professionals with the data generator program B-18) has been used for silo bases and liquid storage tank bases as well as mat foundations for buildings.[1] A recent comparison was made between the FEM and FGM for a full-scale mat foundation in Australia [Payne et al. (1992)]. This reference compared a modification of B-6 and the commercial computer program NASTRAN. They found a maximum difference of about 10 percent between the stresses computed by the two methods when analyzing a mat on expansive soil. It was not possible to identify the "correct" stress. About the best that could be done was to see if the programs predicted crack locations reasonably well.

The FGM is similar to the beam finite element used in Chap. 9 but extended to a beam column (which has torsion) and used for a plate. The same equations as in Sec. 9-8 are used, namely,

$$\mathbf{P} = \mathbf{AF} \qquad \mathbf{e} = \mathbf{A}^{\mathrm{T}}\mathbf{X} \qquad \mathbf{F} = \mathbf{Se} = \mathbf{SA}^{\mathrm{T}}\mathbf{X}$$
$$\mathbf{P} = \mathbf{ASA}^{\mathrm{T}}\mathbf{X} \qquad \mathbf{X} = (\mathbf{ASA}^{\mathrm{T}})^{-1}\mathbf{P}$$

As before, it is necessary to develop the element EA and ES matrices, with the computer taking care of the remainder of the work including the building of the global $\mathbf{ASA}^{\mathrm{T}}$ matrix.

Referring to Fig. 10-11, the element EA matrix is built by $\sum F$ at each node. For example, at node 1

$$P_1 = F_1 \sin\alpha + OF_2 - F_3 \cos\alpha$$
$$P_2 = F_1 \cos\alpha + OF_2 - F_3 \sin\alpha$$
$$P_3 = \frac{F_1}{L} + \frac{F_2}{L} + OF_3$$

---

[1]Since this textbook has been translated into several foreign languages and has been published in an international student edition the usage has been worldwide, not just the United States.

TABLE 10-1

Comparison of finite grid method (FGM) to the FEM using one-quarter of a symmetrical plate (Fig. 10-10) with edge supports indicated and plate *L/B* ratios shown [Bowles (1986*a*)].

Fps units from original source.

Ratio	Support	Element	For node 9: Deflections, ft			Moment, M_x, k • ft/ft			
L/B	type	type	FGM	FEM	Theory*	FGM	FEM	M_{xy}	Net FEM
1	Simple	Square	0.006 41	0.0068	0.006 45	9.0	10.4	−1.2	9.2
1	Simple	Triangle	0.006 25	0.0066	0.006 45				
1	Simple	Square†	0.007 04	0.0066	0.006 45	11.6	12.9	−1.1	11.8
1.2	Simple	Rectangle	0.007 42	0.0080	0.007 52	8.1	9.6	−1.2	8.4
2.0	Simple	Triangle	0.010 25	0.0086	0.009 18				
2.0	Simple	Rectangle	0.009 36	0.0097	0.009 18	5.7	7.2	−1.0	6.2
1.2	Simple	Mixed	0.007 56	—	0.007 52	[Fig. 10-10 with both diagonals (−)]			
1.2	Simple	Mixed	0.007 68	—	0.007 52	[Fig. 10-10 with one diag. (−) and other (+)]			
1.0	Fixed	Triangle	0.003 11	—	0.003 11				
1.0	Fixed	Square‡	0.003 29	0.003 24	0.003 11	7.4	8.6	−1.1	7.5
1.2	Fixed	Mixed	0.004 09	—	0.003 60				
1.4	Fixed	Triangle	0.004 11	—	0.003 84				
1.4	Fixed	Rectangle	0.003 93	0.0042	0.003 84	5.6	7.0	−1.0	6.0
2.0	Fixed	Triangle	0.004 72	—	0.004 01				
2.0	Fixed	Rectangle	0.003 89	0.0043	0.004 01	3.6	5.4	−0.9	4.5

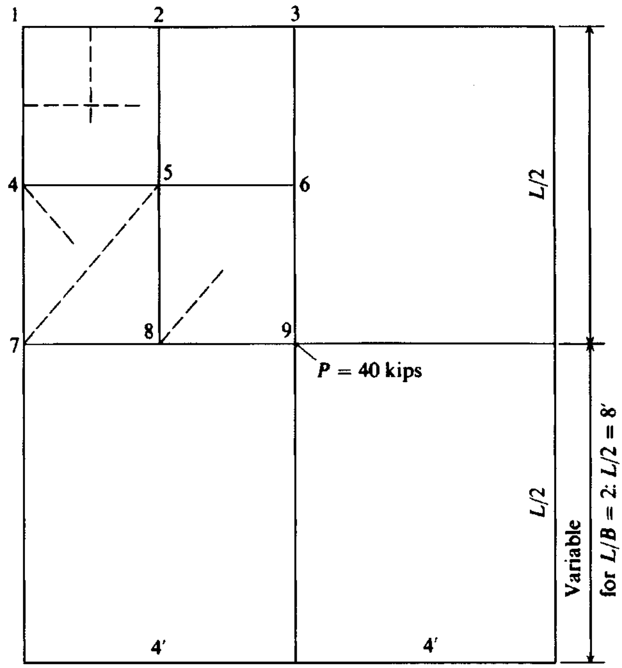
*From Timoshenko and Winowsky-Krieger (1959), pp. 143 and 206.

†Using 25 nodes instead of 9 (finer mesh).

‡Edge moment by FGM gives −4.9 k · ft/ft versus theoretical solution of 0.1257(40) = 5.0 k · ft/ft. The FEM value = −5.4 with M_{xy} = +0.7 to yield a comparable value of M = −4.7 k · ft/ft.

Notes: **1.** Triangle moments not shown since FEM centroid values require interpolation to node values.

2. Net FEM moments are obtained by adding FEM + M_{xy} as shown above.



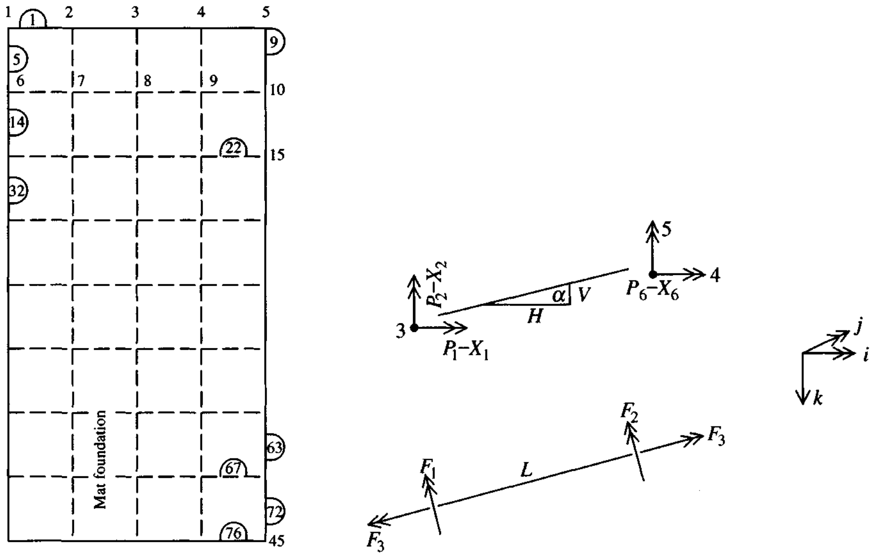
$P = 40$ kips ($P/4 = 10$ kips for 1/4 plate)
$T = 0.5'$ $\quad E_c = 432\,000$ ksf $\quad \mu = 0.15$
$D = 4603.6$ (computed to check theoretical solution)

Figure 10-10 Plate with simple and fixed edge supports to illustrate FGM versus FEM with select data given in Table 10-1. Only one-quarter of the plate is used with symmetry. Gridding for finer mesh and to use triangles are shown with dashed lines.

and the resulting matrix (Note: α makes program general but usually $\alpha = 0°$ or $\alpha = 90°$) is

EA =

P \ F	1	2	3
1	$-\sin\alpha$	0	$-\cos\alpha$
2	$\cos\alpha$	0	$-\sin\alpha$
3	$\frac{1}{L}$	$\frac{1}{L}$	0
4	0	$-\sin\alpha$	$\cos\alpha$
5	0	$\cos\alpha$	$\sin\alpha$
6	$-\frac{1}{L}$	$-\frac{1}{L}$	0



NNODES = 45 NP = 45 × 3 = **135**
NBAND = 6 × 3 − 1 + 1 = 18 (Elem 5)
NM = 76 (elements by direct count)
Stiffness matrix = NP × NBAND
= 135 × 18 = **2430**

(*a*) Plate gridding

(*b*) *P*–*X* (node) and *F*–*e* (element) coding

Figure 10-11 Method of finite-element (grid) analysis. Note that orientation of node numbers in (*a*) results in a banded stiffness matrix of minimum width of 18. Orient so origin is at upper left corner.

Similar to the ES of Sec. 9-8 but including a torsion adjustment factor Ω for F_3, the mat ES matrix is

ES =

F \ e	1	2	3
1	$\frac{4EI}{L}$	$\frac{2EI}{L}$	0
2	$\frac{2EI}{L}$	$\frac{4EI}{L}$	0
3	0	0	$\frac{\Omega GJ}{L}$

Node springs are built during element input based on node contributory area and saved in a "spring" array. After the global $\mathbf{ASA}^T$ is built (in band form to save computing effort and memory) the node springs are added at the appropriate NP location. All edge springs or, preferably, the perimeter k_s, should be doubled to approximate spring coupling.

The torsion factor J should be computed for a rectangle (see p. 528 unless a T or other shape is involved). The adjust factor Ω is used [along with the double area (see Fig. 10-12)] to make the solution better fit the theoretical solutions as found in Timoshenko and Woinowsky-Krieger (1959), usually used by others to verify FEM solutions. This step is

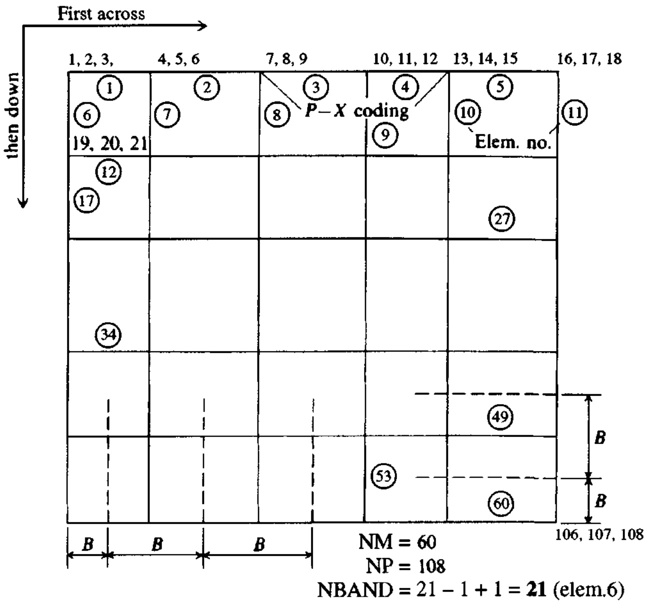


Figure 10-12 Typical coding for a mat. Program "sees" element widths B as shown above. For horizontal members $L = H$ and $V = 0$; vertical members $L = -V$ and $H = 0$. Note use of double the mat area since horizontal and vertical members overlap.

not greatly different from discarding terms in the FEM or using interpolation functions. At present the Ω factor for a best fit is

$$\Omega = \frac{0.75L}{B} \leq 1.1$$

where L, B = grid element length and width, respectively. This method has been extended to allow triangular elements (shown in Table 10-1) and described in more detail in Bowles (1986). As with the beam element, it is necessary to have access to the $P-X$ coding used to develop the EA array so the output can be interpreted.

The concept of subgrade reaction with spring contribution at nodes is easy to modify for soil separation, since the diagonal term is the only coefficient in the stiffness matrix with the soil spring K_i:

$$(A_{ii} + K_i)X_i = P_i$$

Thus, for footing separation we simply make $K_i = 0$, rebuild (or reuse a copy of) the stiffness matrix, and again solve for the displacements X_i.

Generally one should include the mat weight in the analysis. The mat does not cause internal bending moments due to self-weight since the concrete is poured directly on the subbase and in the fluid condition conforms to any surface irregularities prior to hardening. Should the loads cause separation, however, the mat weight tends to counter this. Deflections will be larger when including mat weight, since the soil springs react to all vertical loads.

Preliminary Work

Generally, the depth of the mat is established from shear requirements as in Example 10-6 following. This depth + clear steel cover D_c is used to compute the moment of inertia or D:

$$I = \frac{BD_c^3}{12}(\text{FEM}) \qquad D = \frac{E_c D_c^3}{12(1-\mu^2)}(\text{FEM})$$

The bending moments obtained from the later plate analysis are used to design the mat reinforcement in both directions.

Total deflections are sensitive to the value of k_s used. Bending moments are much less so, but the designer should try to use a realistic minimum and a probable maximum value of k_s and obtain at least two solutions. The design would be based on the best information available or the worst conditions obtained from either of the two solutions (generally when k_s is minimum).

Establishing Finite-Grid Elements (variables in brackets refer to diskette computer program B-6.)

Begin the design by drawing the mat plan to scale and locate all columns and walls. Next lay a grid on this plan such that grid intersections (nodes) occur at any points of zero rotation or displacements (at column faces, wall edges, fixed edges, and similar). Use any convenient gridding if no nodes have unknown rotations or displacements. Grid elements do not have to be the same size, but best results are obtained if very small members are not adjacent to large ones (e.g., a member 0.2 m long connecting to a 2-m-long member is not so satisfactory as a 1-m connecting to a 2-m member). For pinned columns the grid can be at convenient divisions. The load matrix is developed using both column locations and loads. Code the grid starting at the upper left corner, across and then down. Orient the grid so a minimum number of nodes are horizontal for minimum bandwidth.

Develop a data generator to produce element data including the member number (MEMNO) and the six NP values for each element [NPE(I)] and H, V, and B (refer to Fig. 10-12). A data generator (program B-18) is a necessity, since the element input data are enormous.

Develop the nonzero P-matrix entries for each load condition; use simple beam theory for pinned columns between nodes.

Establish if any changes in soil modulus are required. These may be accounted for in the data generator; however, for local soft spots, holes in the ground, hard spots, etc., it may be more convenient to hand-compute the node springs to input into the spring array.

Establish the number of zero boundary conditions (NZX).

Compute the number of NPs in the matrix: NP = 3 × number of nodes; also count the number of members (NM) to be used in the grid.

Compute the bandwidth of the matrix as follows:

1. Find the minimum NP value at various nodes.
2. Find the maximum NP value at adjacent nodes that are interconnected by grid lines.
3. Compute bandwidth as

$$\text{NBAND} = \text{NP}_{\text{max}} - \text{NP}_{\text{min}} + 1$$

As shown on Fig. 10-12, NBAND = 21 − 1 + 1 = 21 (element 6). The size of the resulting band matrix ISIZE is

$$\text{ISIZE} = \text{NBAND} \times \text{NP}$$

The Solution Procedure

With the displacements from $\mathbf{X} = (\mathbf{ASA}^{\mathrm{T}})^{-1}\mathbf{P}$ we can solve $\mathbf{F} = \mathbf{SA}^{\mathrm{T}}\mathbf{X}$ for each element in turn to find the element forces.

The computer program performs the necessary matrix multiplication to form the element $\mathbf{ESA}^{\mathrm{T}}$ (ESAT) and $\mathbf{EASA}^{\mathrm{T}}$ (EASAT). The element $\mathbf{EASA}^{\mathrm{T}}$ will be of size 6 × 6. The $\mathbf{EASA}^{\mathrm{T}}$ is then sorted for values to be placed at the appropriate locations in the global $\mathbf{ASA}^{\mathrm{T}}$ (ASAT) for later banding and solution. Normally the $\mathbf{ASA}^{\mathrm{T}}$ will have to be put on a disk or tape file capable of *random access.*

The computer routine next recalls the $\mathbf{ASA}^{\mathrm{T}}$ from disk (or tape) and stores the band in main memory (refer to Fig. 10-13), filling the lower right corner with zeros. Boundary conditions are applied if specified, which result in zeroing the appropriate *row* and *upper diagonal* of the band matrix and placing a 1.0 in the first column as shown here (typical):

$$\begin{matrix} & & & & & 0. \\ & & & & 0. & \\ & & & 0. & & \\ & & 0. & & & \\ & 0. & & & & \\ 1.0 & 0. & 0. & 0. & 0. & 0. \end{matrix}$$

With the band reduction method, the displacements are exchanged with the **P** matrix at the end of the reduction. If it is desired to save the original **P** matrix for any reason, it must be stored in some alternative location. The *X*s (or redefined *P*'s) are used to compute the

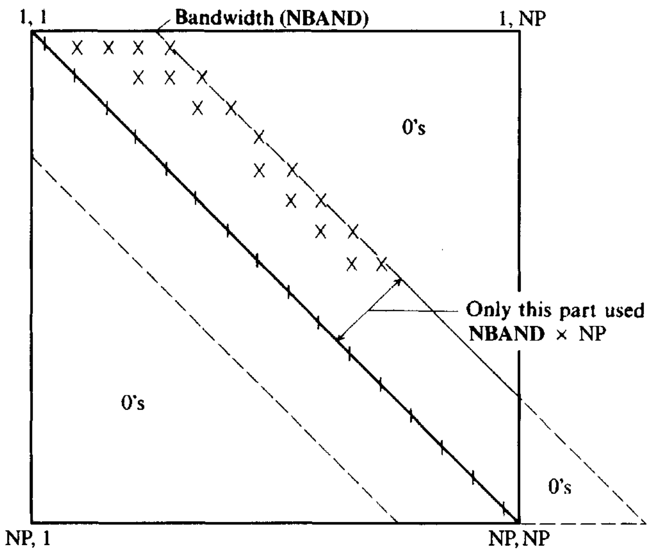
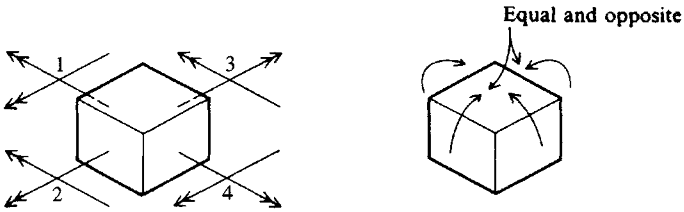


Figure 10-13 Symmetrical $\mathbf{ASA}^{\mathrm{T}}$ matrix. Part used in reduction is as shown.



(a) Node and moments.
(b) Node with moments summed for checking.

Figure 10-14 Checking moments in output for statics at a node.

F's. Also they should be scanned to see if mat-soil separation has occurred at any nodes. If negative deflections occur (tension soil springs), the stiffness matrix is rebuilt with no springs ($K = 0$) at those nodes, and the problem is cycled until the solution converges. Convergence is understood to occur when the current number of nodes with soil separation N_i is equal to the N_{i-1} number of nodes with soil separation just used, or

$$N_i = N_{i-1}$$

When convergence is achieved, the program then computes the element bending and torsion forces using

$$\mathbf{F} = \mathbf{ESA}^{\mathrm{T}}\mathbf{X}$$

Here some savings in computation time can be made if the $\mathbf{ESA}^{\mathrm{T}}$ was saved to a disk file when it was computed. But this can be done only if the node springs are added from a spring array; otherwise the element K_i values are included in the $\mathbf{ESA}^{\mathrm{T}}$.

It is helpful to have the program sum soil node forces $(X_i K_i)$ to compare with the input vertical forces as a quick statics check. It is also helpful if the program makes a moment summation (includes both bending and torsion moments) for the several elements framing into a node for a visual $\sum M_i = 0$ within computer round-off at any node i (see Fig. 10-14). These node moments can be directly used in design by dividing by the element width to obtain moment/unit width.

Design shear requires access to a listing of element forces so the two end moment values can be summed (with attention to sign) and divided by the element length and divided by the element width to obtain shear/unit width. In passing we note that at any node the sum of vertical applied force (from **P** matrix) + soil reaction + $\sum$ sum of element shears framing to node = 0 within computer round-off (and with attention to signs).

10-10 MAT FOUNDATION EXAMPLES USING THE FGM

The following several examples are used to illustrate mat analysis using the FGM.

Example 10-5. (*a*) Compare the Bowles finite-grid method (FGM) with the classical finite-element method (FEM) and the finite-difference method (FDM). Both the FDM and the FEM programs are from the author's program library. (*b*) Also compare the bending moments from these elastic