CHAPTER 9

SPECIAL FOOTINGS AND BEAMS ON ELASTIC FOUNDATIONS

9-1 INTRODUCTION

This chapter will take up the design of several of the more complicated foundation members such as those required to support several columns in a line or from industrial loadings. Chapter 10 will be concerned with multiple lines of columns supported by mat or plate foundations.

When a footing supports a line of two or more columns, it is called a *combined footing*. A combined footing may have either rectangular or trapezoidal shape or be a series of pads connected by narrow rigid beams called a *strap footing*. We will also briefly consider footings for industrial applications, in particular the round (actually octagonal) footing widely used in the petrochemical industry. These several footing types are illustrated in Fig. 9-1.

Combined footings similar to that shown in Fig. 9-1*f* are fairly common in industrial applications using wide rectangular supports for horizontal tanks and other equipment. In these cases, operational loads, differential temperatures, cleaning operations, and the like can result in both vertical and horizontal loads. The horizontal loads at the equipment level produce support moments that must be resisted by the combined footing.

Both the conventional "rigid" and the beam-on-the-foundation method of combined footing analysis will be presented. The latter method requires a computer program for maximum design efficiency. A reasonably complete program for this type of analysis is included as B-5 (FADBEMLP) on your diskette.

9-2 RECTANGULAR COMBINED FOOTINGS

It may not be possible to place columns at the center of a spread footing if they are near the property line, near mechanical equipment locations, or irregularly spaced. Columns located off-center will usually result in a nonuniform soil pressure. To avoid the nonuniform soil pressure, an alternative is to enlarge the footing and place one or more of the adjacent columns in the same line on it (Fig. 9-2). The footing geometry is made such that the resultant of the

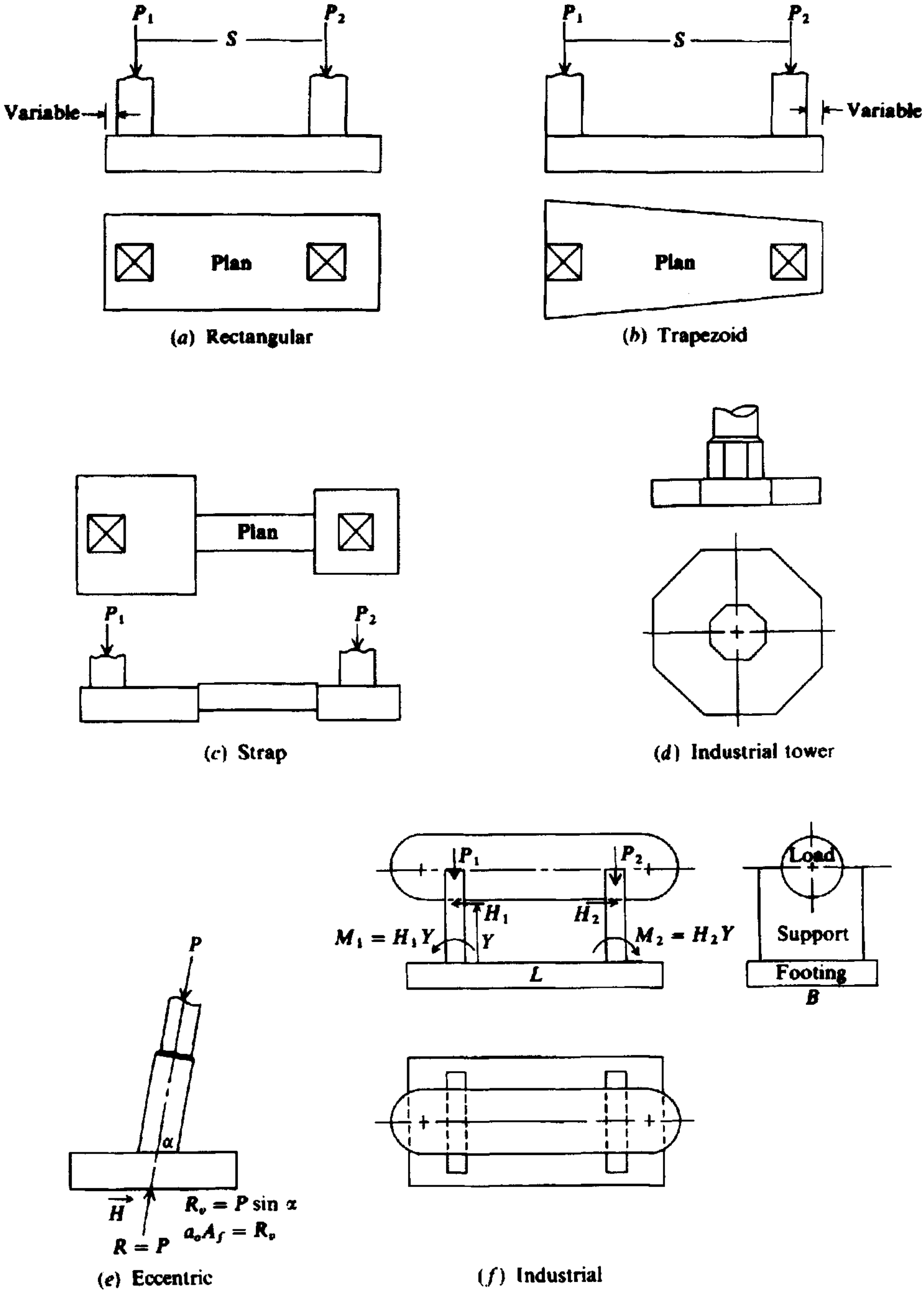


Figure 9-1 Typical special footings considered in this chapter.

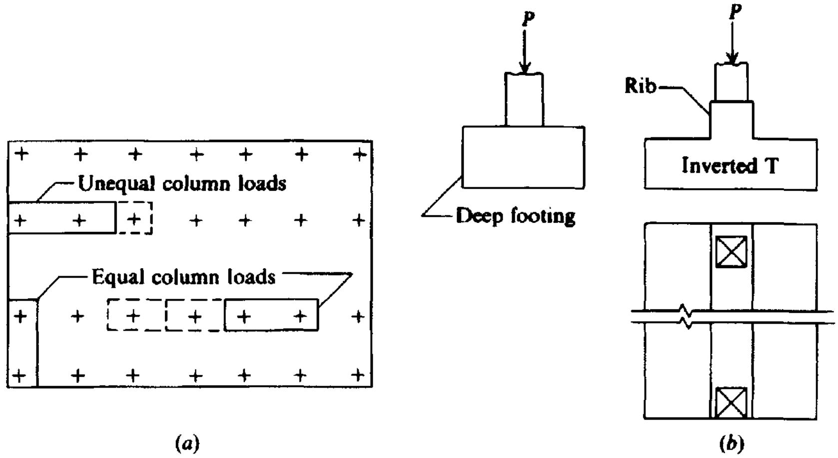


Figure 9-2 (*a*) Typical layout of combined footings for column loads as shown; more than two columns can be used. (*b*) Deep footings for heavy loads and the use of a rib or inverted T beam to reduce footing mass.

several columns is in the center of the footing area. This footing and load geometry allows the designer to assume a uniform soil pressure distribution. The footing can be rectangular if the column that is eccentric with respect to a spread footing carries a smaller load than the interior columns. Bridge piers are also founded on very rigid combined rectangular footings.

The basic assumption for the design of a rectangular combined footing is that it is a rigid member, so that the soil pressure is linear. The pressure will be uniform if the location of the load resultant (including column moments) coincides with the center of area. This assumption is approximately true if the soil is homogeneous and the footing is rigid. In actual practice it is very difficult to make a rigid footing, for the thickness would have to be great; nevertheless, the assumption of a rigid member has been successfully used for many foundation members. Success has probably resulted from a combination of soil creep, concrete stress transfer, and overdesign.

In recognition of the overdesign using the conventional (or "rigid") method, current practice tends to modify the design by a beam-on-elastic-foundation analysis. This produces smaller design moments than those obtained by the rigid method, as will be illustrated later.

The conventional (or rigid) design of a rectangular combined footing consists in determining the location of the center of footing area. Next the length and width can be found. With these dimensions the footing is treated as a beam supported by the two or more columns, and the shear and moment diagrams are drawn. The depth, based on the more critical of two-way action or wide-beam shear, is computed. Critical sections for two-way action and wide-beam shear are the same as for spread footings, i.e., at $d/2$ and d, respectively, from the column face. It is common practice not to use shear reinforcement, both for economy and so that a larger footing thickness is required for greater rigidity. The labor costs to bend and place the shear reinforcement are likely by far to exceed the small savings in concrete that would result from its use.

With the depth selected, the flexural steel can be designed using the critical moments from the moment diagram. Alternatively, the depth and loading can be used in a finite-element analysis to obtain modified moments for the flexural steel. These beam-type members usually have both positive and negative moments, resulting in reinforcing steel in both the top and bottom of the footing. The minimum percentage of steel should be taken as $1.4f_y$ since the footing is designed as a "beam" or flexural member. Footings with negative (or top) steel are not economical, so oversized spread footings should be used if possible.

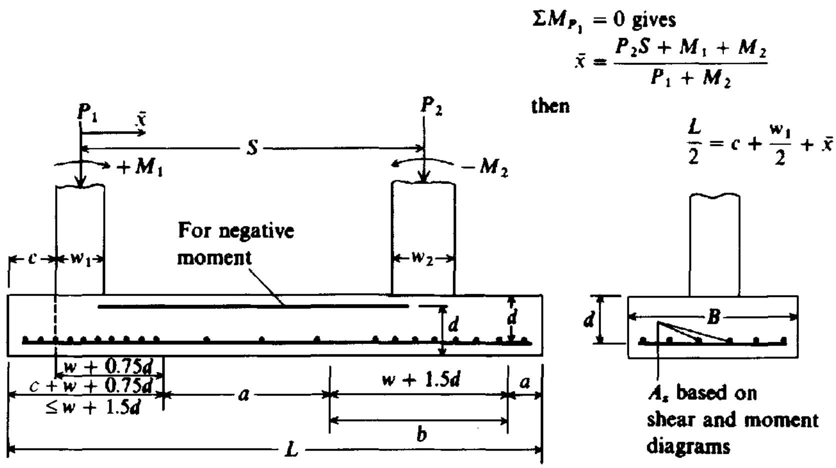
If we compute the short, or transverse, direction bending moments as for a rectangular spread footing, they will be in substantial error. The reason is the soil pressure is larger near the columns, from their stiffening effect on the footing, and lesser in the zone between columns. That zone closest to, and approximately centered on, the column is most effective and should be analyzed somewhat similarly to the ACI Code requirement for rectangular footings. The Code does not directly specify this effective column zone width, but based on inspection of a number of computer printouts using both the finite-difference and finite-element methods the author suggests that the effective zone should be about as shown in Fig. 9-3. Note that as the width of this zone decreases its rigidity increases from the additional reinforcing bars that are required. The increased rigidity will tend to attract moment from the zone between columns but would be difficult to predict since the moment of inertia based on D_c, rather than either the transformed section or effective moment of inertia, is commonly used in finite-element/difference analyses. Making the effective zone reasonably narrow should ensure adequate steel is used to take care of any additional "attracted" moment.

The conventional design method requires computing shears and moments at sufficient locations that a shear and moment diagram can be drawn. It is also standard practice to round computed dimensions to multiples of 75 mm or 0.25 ft. If this is done prior to computing shear and moment diagrams there will be a closure error that depends on the amount the length is changed; thus, it is recommended that footing dimensions be rounded as the final design step.

The column loads are actually distributed over the column width as shown in Fig. 9-4 but should always be taken as point loads. This assumption greatly simplifies the shear and moment computations, and the values at the critical locations are the same by either method.

It should be self-evident that combined footings are statically determinate for any number of columns. With the column loads known and assuming a rigid footing, the resulting soil pressure $q = \sum P/A$. The problem then becomes that of a uniformly loaded continuous beam with all the reactions (the columns) known.

Figure 9-3 Steel for rectangular combined footing. Note the several values of *d*. Steel in zone *a* satisfies minimum code requirements, in *b* satisfies both bending and minimum code requirements.



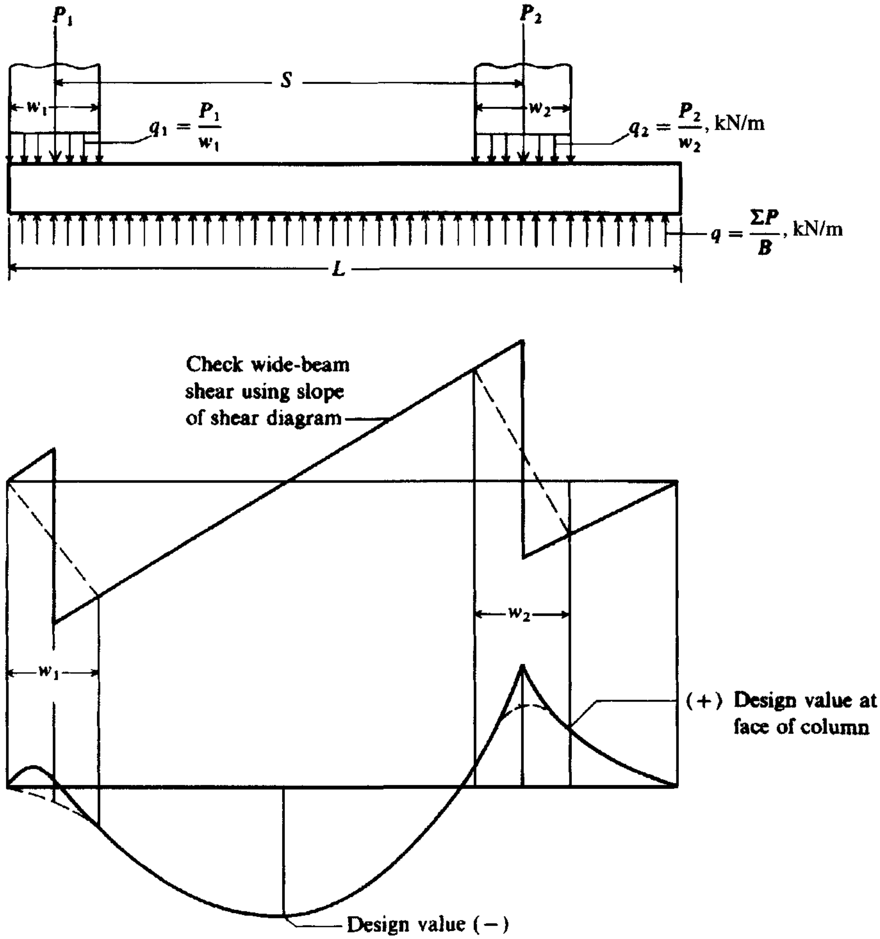


Figure 9-4 Shear and moment diagrams (qualitative) for a combined footing considering the column loads as point loads and as distributed loads (dashed line). It can be seen that in the design areas it makes no difference how the diagrams are drawn, and the point load case is much simpler.

Example 9-1. Design a rectangular combined footing using the conventional method.

Given. $f_c' = 21$ MPa (column and footing) $f_y =$ Grade 400 $q_a = 100$ kPa

Column number	Working loads						
	DL	*LL*, kN	M_D	M_L	*P*	P_u, kN	M_u, kN · m
1	270	270	28	28	540	837	86.8
2	490	400	408	40	890	1366	124
				Total	1430	2203	

Ultimate values = $1.4DL + 1.7LL$, etc.

$$\text{Soil:} \qquad q_{\text{ult}} = \frac{\sum P_u}{\sum P} q_a = \frac{2203}{1430}(100) = \mathbf{154.1} \text{ kPa}$$

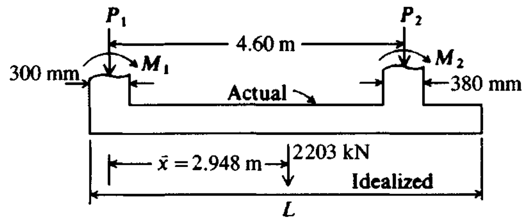


Figure E9-1*a*

It is necessary to use q_{ult} so base eccentricity is not introduced between computing L using q_a and L using q_{ult}.

Solution.

Step 1. Find footing dimensions.

$$\sum M_{col.1} = R\bar{x} \qquad \text{where} \qquad R = \sum P_u = 837 + 1366 = 2203 \text{ kN}$$

For uniform soil pressure R must be at the centroid of the base area (problem in elementary statics), so we compute

$$R\bar{x} = M_1 + M_2 + SP_{ult,2}$$

$$2203\bar{x} = 86.8 + 124.0 + 4.60(1366)$$

$$\bar{x} = \frac{6494.4}{2203} = 2.948 \text{ m}$$

It is evident that if $\bar{x}$ locates the center of pressure the footing length is

$$L = 2 \times (\tfrac{1}{2} \text{ width of col. 1} + \bar{x}) = 2 \times (0.150 + 2.948) = \mathbf{6.196} \text{ m}$$

Also for a uniform soil pressure $q_{ult} = 154.1$ kPa, the footing width B is computed as

$$BLq_{ult} = P_{ult}$$

$$B = \frac{2203}{6.196 \times 154.1} = \mathbf{2.307} \text{ m}$$

We will have to use these somewhat odd dimensions in subsequent computations so that shear and moment diagrams will close. We would, however, round the dimensions for site use to

$$L = \mathbf{6.200} \text{ m} \qquad B = \mathbf{2.310} \text{ m}$$

Step 2. Obtain data for shear and moment diagrams (or at critical locations). Use any convenient method, e.g., calculus, as

$$V = \int_{x_1}^{x_2} q(dx)$$

$$M = \int_{x_1}^{x_2} V(dx) \qquad \text{with attention to values at the limits}$$

Since calculations for the conventional design of a combined footing involve an enormous amount of busywork (with potential for errors) it is preferable to use a computer program such as B-15 (see supplemental program list on your diskette in file README.DOC). This has been done by the author to obtain the accompanying printout (Fig. E9-1*b*) to which reference will be made with the design steps following.

Step 3. From critical shear find the depth for wide-beam and two-way action. Note that columns may have either a four- (case 1) or three-side (case 2) two-way action perimeter. The computer

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++++++++++++ NAME OF DATA FILE USED FOR THIS EXECUTION:  EXAM91.DTA

 EXAMPLE 9-1 FOUND. ANALY. AND DESIGN--SI UNITS
FOOTING DESIGN INPUT DATA IS AS FOLLOWS:
COL NO    WIDTH x LEN,  M    LOAD,    KN    MOMENT, KN-M   COL SPAC, M
 1        .300 X   .300         837.0           86.8
                                                                4.600
 2        .380 X   .380        1366.0          124.0

      DIST END FTG TO LT FACE COL 1 =   .000  M
            INPUT FOOTING WIDTH, BF =   .000  M
              LENGTH INCREMENT, DX, =   .500  M
  THE FACTORED ALLOW SOIL PRESSURE = 154.10 KPA

CONCRETE AND STEEL STRESSES:    F1C =     21.0 MPA
                                 FY =    400.0 MPA

COMPUTED FOOTING DIMENSIONS:  WIDTH =   2.307  M
                             LENGTH =   6.196  M
                LENGTH/WIDTH RATIO =  2.685
            UNIFORM LOAD ALONG FTG =  355.554 KN/M

 MAX WIDE BEAM SHEAR AT LEFT  FACE COL  2 =    784.327    KN
      DEPTH OF CONCRETE FOR WIDE BEAM = 423.172 MM
                ALLOW WIDE BEAM SHEAR =     .649 MPA

 DEPTH OF CONCRETE FOR CASE 1 @ COL 1 =     .000 MM(  1.298 MPA)
DEPTH OF CONCRETE FOR CASE 1 @ COL  2 = 342.565 MM
 DEPTH OF CONCRETE FOR CASE 2 @ COL 1 = 369.707 MM(  1.298 MPA)
DEPTH OF CONCRETE FOR CASE 2 @ COL  2 = 225.748 MM

++++++++++++ DEPTH OF CONCRETE USED FOR DESIGN = 423.172 MM

+++ AS = TOTAL STEEL AREA FOR FTG IN WIDTH BF =   2.307  M
DISTANCE
FROM END                SHEAR              MOMENT,KN-M            AS, M**2

     .00LF                .00                  .00          .0000E+00*
     .15CL              53.33                 4.00          .2626E-04*
     .15CR            -783.67                90.80          .6002E-03*
     .30RF            -730.33               -22.75          .1496E-03*
     .50              -659.22              -161.71          .1075E-02*
    1.00              -481.45              -446.87          .3039E-02*
    1.50              -303.67              -643.15          .4449E-02
    2.00              -125.89              -750.54          .5242E-02
    2.35MM                .00              -772.83          .5409E-02 √
    2.50                51.89              -769.04          .5380E-02
    3.00               229.66              -698.66          .4857E-02
    3.50               407.44              -539.38          .3697E-02
    4.00               585.22              -291.22          .1955E-02*
    4.50               762.99                45.84          .3019E-03*
    4.56LF             784.33                92.26          .6099E-03*
    4.75CL             851.88               247.70          .1657E-02*
    4.75CR            -514.12               371.70          .2512E-02*
    4.94RF            -446.56               280.43          .1881E-02*
    4.50              -603.01               169.84          .1129E-02*
    5.00              -425.23               254.28          .1702E-02*
    5.50              -247.45                86.11          .5689E-03*
    6.00               -69.67                 6.83          .4483E-04*
    6.20                  .00                  .00          .0000E+00*

** = AS > ASMAX--INCREASE D;  * = AS < ASMIN--USE ASMIN

MAX % STEEL = .0171 %    MAX STEEL AREA = .1667E-01 M**2
MIN % STEEL = .0035 %    MIN STEEL AREA = .3417E-02 M**2

TRANSVERSE STEEL IN COLUMN ZONES OF WIDTH BPR FOR DEPTH DCP =  385.17 MM

COL #    PRESS,DQ    ARM, M  WIDTH,BPR, M   AS, M**2   ASMAX     ASMIN, M**2
 1       362.76      1.004       .617        .1412E-02 .4059E-02 .8323E-03
 2       592.04       .964      1.015        .2110E-02 .6672E-02 .1368E-02

   STEEL AREAS FOR WIDTH BPR--IF AS < ASMIN USE ASMIN
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Figure E9-1*b*

program routinely checks wide-beam and both cases 1 and 2 for each column with depths printed for checking and then selects the largest d for the design value. We see here wide-beam shear controls giving $d = 423.172$ mm on the computer printout.

When wide-beam shear controls d, it may not be necessary to check ACI Eq. (11-36) or (11-37) since the limiting value of two-way shear v_c equals the wide-beam value of $2\phi\sqrt{f_c'}$. It may, however, be necessary to compare the "wide-beam" distances. That is, which is the larger distance, the two-way perimeter of the end (or corner column) or the wide-beam width?

Here the perimeter p_O is calculated as

$$\begin{aligned} p_O &= 0.300 + 0.432 + 0.300 + 0.300 + 2(0.432)/2 \\ &= 1.764 \text{ m} < B = 2.307 \end{aligned}$$

By ACI Eq. (11-37) the allowable two-way shear stress is:

$$\left(\frac{\alpha_s d}{b_o} + 2\right)\phi\sqrt{f_c'} = \left(\frac{30(0.432)}{1.764} + 2\right)\phi\sqrt{f_c'} = 9.35\phi\sqrt{f_c'} \gg 4\phi\sqrt{f_c'}$$

With the column being square, the two-way shear stress is the smaller of these $= 4\phi\sqrt{f_c'}$. Since the column width $w \ll 4d[0.3 \ll 4(0.432)]$, it is evident that the depth will be controlled by wide-beam shear, with the allowable v_c obtained directly from Table 8-2 and using a beam width of $B = 2.307$ m. It is instructive, however, for the reader to make the two-way shear check at least one time.

Step 4. Find the steel for bending. There will be both (+)-bending steel in the bottom of the footing near columns and (−)-bending steel in the top near or in the center portion between columns. Note the signs in the computer printout. The required steel area at each moment location including the maximum (MM) is output. For convenience the program also computes the maximum allowable amount of steel based on p_b (here, 16,670 mm^2, which is far in excess of the A_s required for the largest moment location of 5409 mm^2) and the minimum ACI Code requirement based on $1.4/f_y$, giving 3417 mm^2. Notice that the minimum of 3417 mm^2 controls the bottom longitudinal reinforcing bars since it is larger than any of the A_s values computed for the (+) moments. For longitudinal steel we will use the following:

Top bars :	twenty No. 20 bars (20 × 300) = **6000** mm^2(5409 required)
Bottom bars :	twelve No. 20 bars (12 × 300) = **3600** mm^2(3417 required)

We should run all the (−) (or top) bars the full length of the footing, for trying to cut them and satisfy Code requirements for extra length beyond the theoretical is not worth the extra engineering and bar placing effort. We should run about one-half (six bars) of the (+) (or bottom) bars (in the right end zone) all the way as well so that the transverse bars can be supported.

Step 5. Design the transverse steel (refer to Fig. 9-3 for the effective base widths). We will adjust the depth $d = 0.4232 - 0.038$ (approximately 1.5 × No. 25 bar) giving for transverse steel a $d = 0.385$ m for bending. But we will use the initial d for column zone widths.

Column 1: $B_1' + w + 0.75d = 0.30 + 0.75(0.4232) = 0.6174 \rightarrow \mathbf{0.62}$ m

The soil pressure in this reduced zone (and rounding $B = 2.31$ m; $L = 0.620$ m) is

$$q_{\text{ult}} = \frac{P_{\text{ult}}}{B \times B'} = \frac{837}{2.31 \times 0.62} = \mathbf{584.4} \text{ kPa} \qquad (587.6 \text{ computer})$$

The effective moment arm is

$$L_1' = \frac{B - w}{2} = \frac{2.31 - 0.30}{2} = 1.01 \text{ m } (1.015)$$

The resulting M_u is

$$M_u = \frac{q_{\text{ult}}(L_1')^2}{2} = \frac{584.4(1.01)^2}{2} = \mathbf{298.1}\ \text{kN}\cdot\text{m}$$

For $f_c' = 21$ MPa and $f_y = 400$ MPa, we find that

$$a = \frac{f_y A_s}{0.85 b f_c'} = 22.4A_s$$

The required steel area in this zone, which is 0.62 m wide, is

$$A_s\left(d - \frac{a}{2}\right) = \frac{M_u}{\phi F_y}$$

$$A_s(0.385d - 22.4A_s/2) = 298.1/(0.9 \times 400 \times 1000)$$
$$11.2A_s^2 - 0.385A_s = 0.000\,828$$
$$A_s = 0.002\,30\ \text{m}^2/\text{m} = 2300\ \text{mm}^2/\text{m}$$

The zone width $= 0.62$ m, so the required A_s is

$$A_s = 2400(0.62/1)$$
$$= \mathbf{1488}\ \text{mm}^2 \text{ in zone}$$
$$= 1412\ \text{mm}^2 \text{ (by computer)}$$

Use five No. 20 bars, giving $A_s = 5(300) = 1500\ \text{mm}^2 > 1488$. With five bars there will be four spaces, so that

$$s = 620/4 = 155\ \text{mm} > d_b \text{ (Art.7.6)} \qquad \text{O.K.}$$

$$\textbf{Column 2:}\quad L_2' = \frac{2.31 - 0.380}{2} = \mathbf{0.965}\ \text{m}$$

The effective width $B' = w + 1.5d = 0.380 + 1.5(0.4232) = 1.01$ m.

$$q_{\text{ult}} = \frac{P_{\text{ult}}}{B \times B'} = \frac{1366}{1.01 \times 2.31} = 585.5\ \text{kPa} \qquad \text{(583.4 computer)}$$

$$M_u = \frac{585.5 \times 0.965^2}{2} = 272.6\ \text{kN}\cdot\text{m}$$

$$11.2A_s^2 - 0.385A_s = 272.6/(0.9 \times 400 \times 1000)$$
$$A_s^2 - 0.0344A_s + 0.000\,0676 = 0$$
$$A_s = 0.002\,17\ \text{m}^2/\text{m} \rightarrow 2100\ \text{mm}^2/\text{m}$$

Here we have a zone width of 1.01, so by proportion

$$A_s = 2100(1.01/1.) = \mathbf{2121}\ \text{mm}^2 \text{ for width (2110 computer)}$$

Use eight No. 20 bars $\rightarrow A_s = 8(300) = 2400\ \text{mm}^2 > 2121$. The spacing will satisfy ACI Art. 7.6.

Use T and S steel for the remainder of the short side [$p = 0.0018$ since $f_y = 400$ MPa (Art. 7.12.2.1)]. One might also consider using $1.4/f_y$ of Art. 10.5. The difference is

$$\text{T and S} = 0.0018 \qquad 1.4/400 = 0.0035$$

Compute the total depth (use 50 mm top and 70 mm bottom of clear cover), or

$$D_c = d + \text{bottom bar}/2 + 70 + \text{top bar}/2 + 50$$
$$= 432 + 10 + 70 + 10 + 50 = \mathbf{572}\ \text{mm}$$

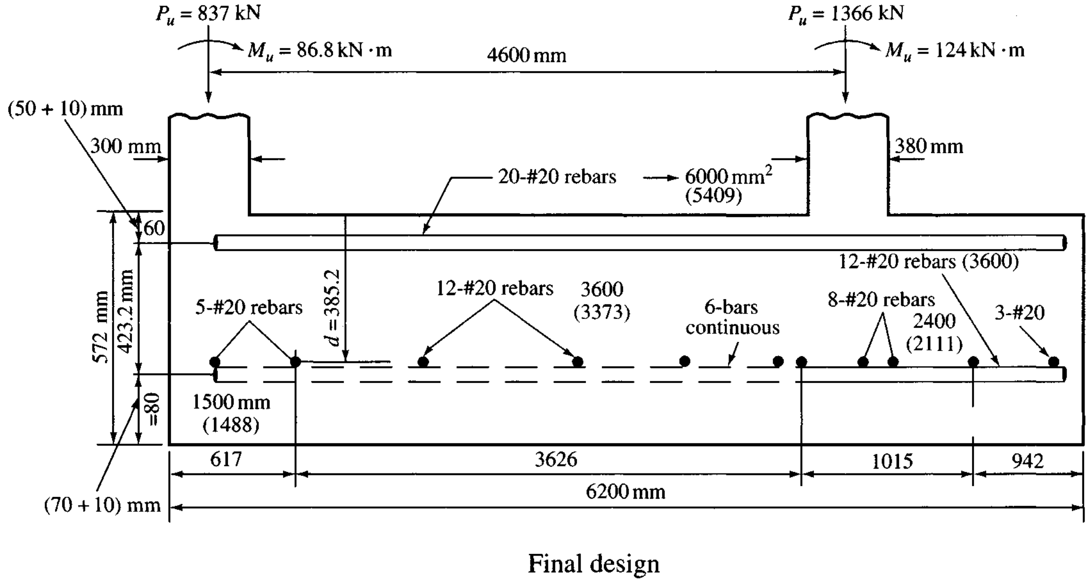

Figure E9-1c

Using 0.0018 × gross area

$$\text{Interior zone} = 0.0018(3626)(572) = 3373\ \text{mm}^2$$
$$\text{Right end zone} = 0.0018(942)(572) = 970\ \text{mm}^2$$

Note: The ACI code specifies the ratio × gross area. Some designers use gross area as width × d whereas others use width × D_c (the total depth). Clearly, using the total depth D_c is conservative.

Step 6. Check columns bearing on the base. At column 1 the area ratio $\sqrt{A_2/A_1} = 1$, and will probably require dowels to assist in transfer of column load into the footing. Column 2 has an area ratio = 2 and should only require the minimum of $0.005A_{col}$ to tie the column to the footing.

Step 7. Prepare a final design sketch as in Fig. E9-1c so that the final drawings can be made and the footing constructed. Note these features:

1. Transverse steel uses a different d from longitudinal steel. Here we used 38 mm less (actual reduction is 20 mm). The result is conservative. Note that except for the column zones, T and S steel controls.
2. Top (−) steel uses the same d as (+) bottom steel.
3. Bottom steel clear distance is 70 mm (concrete poured directly on ground).
4. Top steel clear distance is 50 mm (concrete not poured on ground but may be in contact with soil).

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9-3 DESIGN OF TRAPEZOID-SHAPED FOOTINGS

A combined footing will be trapezoid-shaped if the column that has too limited a space for a spread footing carries the larger load. In this case the resultant of the column loads (including moments) will be closer to the larger column load, and doubling the centroid distance as done for the rectangular footing will not provide sufficient length to reach the interior column. The

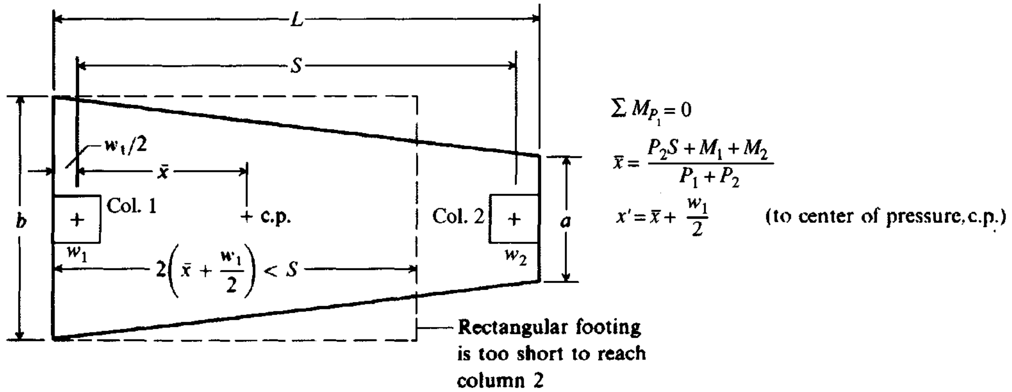

Figure 9-5 A trapezoidal footing is required in this case unless the distance S is so great that a cantilever (or strap) footing would be more economical.

footing geometry necessary for a two-column trapezoid-shaped footing is illustrated in Fig. 9-5 from which we obtain

$$A = \frac{a+b}{2}L \tag{9-1}$$

$$x' = \frac{L}{3}\frac{2a+b}{a+b} \tag{9-2}$$

From Eq. (9-2) and Fig. 9-5 we see that the solution for $a = 0$ is a triangle, and if $a = b$ we have a rectangle. Therefore, it follows that a trapezoid solution exists only for

$$\frac{L}{3} < x' < \frac{L}{2}$$

with the minimum value of L as out-to-out of the column faces. In most cases a trapezoid footing would be used with only two columns as illustrated, but the solution proceeds similarly for more than two columns. The forming and reinforcing steel for a trapezoid footing is somewhat awkward to place. For these reasons it may be preferable to use a strap footing (next section) where possible, since essentially the same goal of producing a computed uniform soil pressure is obtained.

With x' falling at a particular location and defining the center of area, the dimensions a and b have unique values that require a simultaneous solution of Eqs. (9-1) and (9-2). The value of L must be known, and the area A will be based on the soil pressure and column loads ($A = \sum P/q_o$ or $\sum P_u/q_{ult}$).

When the end dimensions a and b are found, the footing is treated similarly to the rectangular footing (as a beam) except that the "beam" pressure diagram will be linear-varying (first-degree) because a and b are not equal. The resulting shear diagram is a second-degree curve and the moment diagram is a third-degree curve. Calculus is a most efficient means to obtain critical ordinates for these diagrams and to treat the columns as point loads. A trapezoid-shaped footing can also be analyzed as a beam on an elastic foundation, only in this case the finite-element widths are average values.

Example 9-2. Proportion and partially design a trapezoidal footing for the given data:

$f'_c = 21$ MPa $\quad f_y = 400$ MPa (grade 400 rebars) $\quad q_a = 190$ kPa

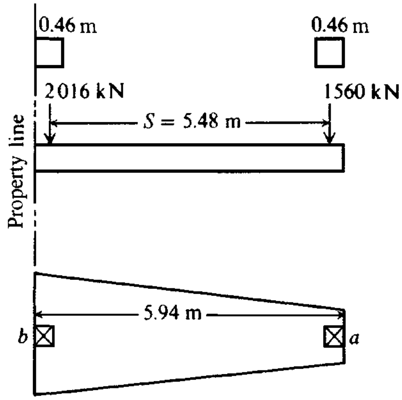

Figure E9-2*a*

Column	*DL*	*LL*	*P*, kN	P_{ult}, kN
1	1200	816	2016	3067.2 (1.4*D*+1.7*L*)
2	900	660	1560	2382.0
		Total	3576	5449.2

$$\text{Soil}: \quad q_{\text{ult}} = \frac{P_{\text{ult}}}{P}(q_a) = \frac{5449.2}{3576}(190) = \mathbf{289.5}\ \text{kPa}$$

There is much busywork with designing a trapezoid footing, so the only practical modern method is to use a computer program such as B-16.

Solution

Step 1. Find the end dimensions *a* and *b* of Fig. E9-2*a*.

First locate the center of area by taking moments through center of col. 1:

$$5449.2\bar{x} = 5.48[1.4(900) + 1.7(660)]$$

$$\bar{x} = \frac{13\,053.4}{5449.2} = 2.395\ \text{m} \qquad \text{and} \qquad x' = 2.395 + \frac{0.46}{2} = 2.625\ \text{m}$$

$$L = 5.48 + 2\frac{(0.46)}{2} = \mathbf{5.94}\ \text{m}$$

Since $L/2 > x' > L/3$ we have a trapezoid. From Eq. (9-1) the area is

$$A = \frac{a+b}{2}L = \frac{a+b}{2}(5.94)$$

but based on q_{ult} and the footing loads,

$$A = \frac{5449.2}{289.5} = 18.823\ \text{m}^2$$

Equating these two *A*-values, we have

$$\frac{a+b}{2}(5.94) = 18.823 \qquad a + b = \mathbf{6.338}\ \text{m}$$

From Eq. (9-2) and $x' = 2.625$ m,

$$x' = \frac{L}{3}\frac{2a+b}{a+b}$$

$$\frac{2a + b}{a + b} = \frac{3(2.625)}{5.94} = 1.326 \text{ m}$$

but $a + b = 6.338$, from which $b = 6.338 - a$ and substituting for both,

$$\frac{2a + 6.338 - a}{6.338} = 1.326 \text{ m}$$
$$a = \mathbf{2.065} \text{ m}$$
$$b = 6.338 - 2.065 = \mathbf{4.273} \text{ m}$$

One should routinely back-substitute a and b into Eq. (9-1) and compare A.

Step 2. Draw shear and moment diagrams:

$$\text{Pressure big end} = 4.273(289.5) = 1237.03 \text{ kN/m}$$
$$\text{Pressure small end} = 2.065(289.5) = 597.82 \text{ kN/m}$$
$$\text{Slope of the pressure line } s = (1237.0 - 598.0)5.94 = \mathbf{107.6} \text{ kN/m}^2$$
$$q = 1237 - 107.6x$$
$$V = \int_0^x q\,dx = 1237.0x - 107.6\frac{x^2}{2} + C$$

At $x = 0.23$ m, $C = 0$: $V = 1237.0(0.23) - 53.8(0.23)^2 = 282$ kN
At $x = 0.23 + dx$, $C = -3067$: $V = 282 - 3067 = -2785$ kN
At column 2, $x = 5.71$, $C = -3067$: $V = 2242$ kN
And at $x = 5.71 + dx$: $V = -140$ kN

Values of shear at the interior faces of columns 1 and 2 are 2509.4 and 2096.1 kN, respectively (rounded values shown in Fig. E9-2*b*). The maximum moment occurs where the shear diagram is zero (which should be somewhere between cols. 1 and 2), giving

$$V = \int_0^x q(dx) + C_1 = 0$$

Integrating, inserting q and using $P_u = -3067$ kN (col. 1) $= C_1$ we obtain

$$V = 1237.0x - 107.6x^2/2 - 3067 = 0$$

Solving, we find $x = 2.828$ m from left end. Moments are computed similarly,

$$M = \int_0^x V dx = 1237.0\frac{x^2}{2} - 107.6\frac{x^3}{6} - C_1 x''$$

At $x = 0.23$ and $x'' =$ distance from previous discontinuity $= 0$,

$$M = 32.0 \text{ kN} \cdot \text{m}$$

At the right face of column 1, $M = -576.0$ kN · m. Maximum m is at $x = 2.828$ m, so

$$M = 4946.5 - 405.6 - 3067(2.828 - 0.23) = -3429 \text{ kN} \cdot \text{m}$$

At the left face of column 2, $M = -479$ kN · m. These values are sufficient to draw the shear and moment diagrams of Fig. E9-2*b*.

Step 3. Find the depth for wide-beam shear at the small end and check two-way action at the large end. The reasoning is

$$\frac{V_b}{V_s} = \frac{2509}{2096} = 1.2 \qquad \frac{b}{a} = \frac{4.27}{2.06} = 2.07 \gg 1.2$$

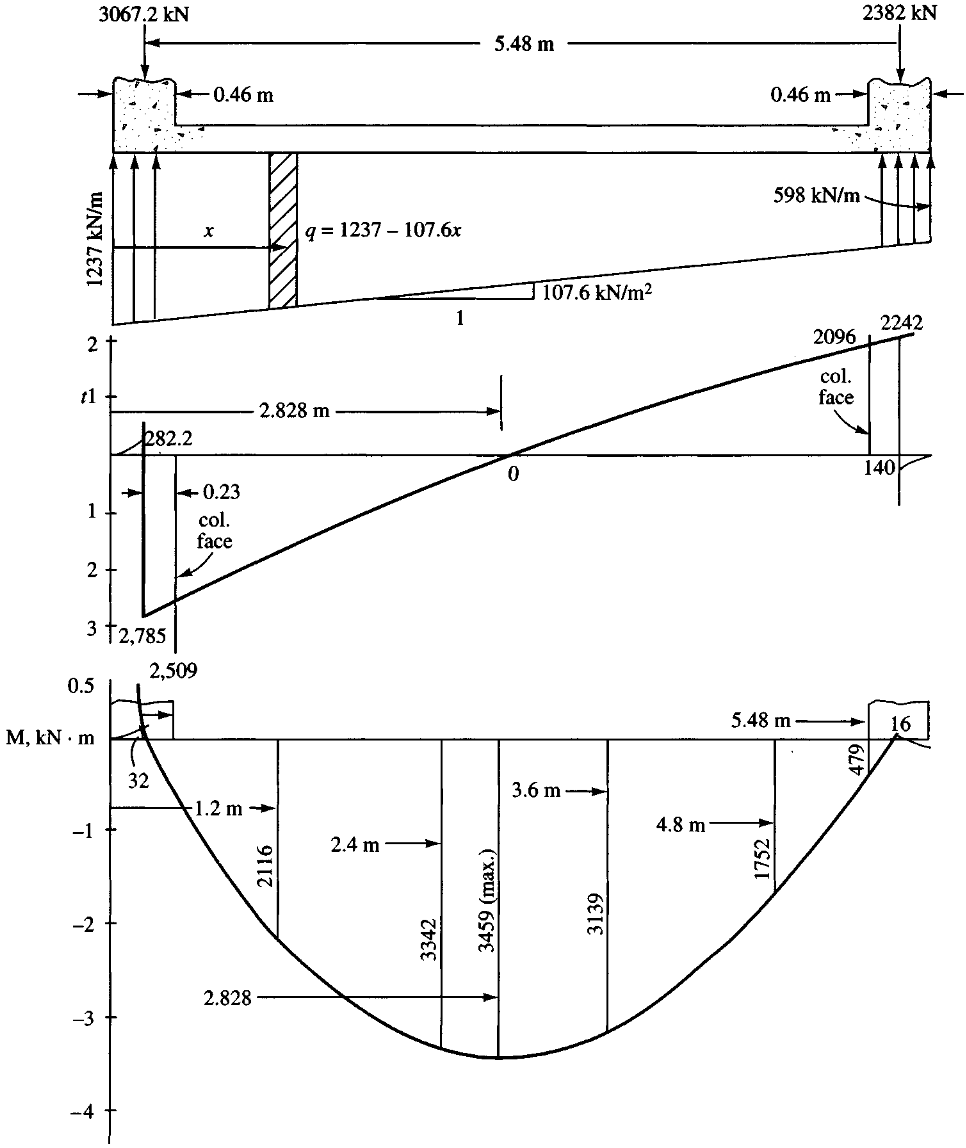

Figure E9-2*b*

Since the width ratio is much larger than the shear ratio, d will probably be based on wide-beam shear at the small end.

$$V = 1237.0x = 107.6\frac{x^2}{2} - 3067 \qquad \text{where} \qquad x = 5.48 - d \qquad \text{(from big end)}$$

$$= 2096 - 647d - 53.8d^2 \qquad \text{(net shear at section at } d \text{ from left face of col. 2)}$$

$$\text{Width} = 2.065 + \frac{4.27 - 2.06}{5.94}(d + 0.46)$$

$$= 2.065 + 0.372d + 0.17 = 2.24 + 0.372d$$

$$v_c = 0.649 \text{ MPa} = 649 \text{ kPa} \quad \text{(Table 8-2)}$$

Equating concrete shear to external shear $(2.24 + 0.372d)d(649) = 2096 - 647d - 53.8d^2$,

$$295d^2 + 2103d = 2096 \qquad d^2 + 7.1d = 7.1 \qquad d = \mathbf{0.89}\text{ m}$$

Two-way action at the large end (not possible to check at small end) requires $d = 0.75$ m. Actually, when wide-beam shear "d" is used it is not necessary to check ACI two-way action since the minimum two-way shear is $2\phi\sqrt{f'_c}$ = wide-beam v_c.

Step 4. Design the flexural steel. Since the width varies, one should check A_s for several locations, resulting in the following table. This table was obtained from a computer printout and there are slight discrepancies between hand and computer computations resulting from rounding for hand computations.

x	V, kN	M, kN · m	w, m	A_s, cm²/m
0	0	0	4.27	0.0
0.6	−2344.6	−916.1	4.05	6.9 × 100 = 690 mm²/m
1.2	−1660.6	−2115.8	3.83	17.0
1.8	−1015.4	−2916.6	3.60	25.2
2.4	−408.9	−3342.0	3.38	31.0
2.828 (max)	0.0	−3428.7	3.22	33.5 × 100 = 3350 mm²/m
3.0	+159.0	−3415.0	3.16	34.1
3.6	688.1	−3159.0	2.94	33.9
4.8	1630.3	−1752.4	2.49	21.8 × 100 = 2180 mm²/m
5.94	0.0	0.0	2.07	0.0

The max. steel = 144.2 cm²/m (based on Table 8-1 and computer printout)
The min. steel = 29.6 cm²/m based on $1.4/f_y$

Step 5. Steel in short direction. Treat same as rectangular footing using appropriate zone of $w + 0.75d$, since columns are at end of footing. Use the average width of footing in this zone for bending, for example, at large end:

$$w + 0.75d = 0.46 + 0.75(0.89) = 1.12 \text{ m}$$

$$B_1 = 4.27 \qquad B_2 = 4.27 - 1.12\frac{4.27 - 2.07}{5.94} = 3.85$$

$$\text{Average}: \quad w = \frac{4.27 + 3.85}{2} = \mathbf{4.06}\text{ m}$$

$$L' = \frac{4.06 - 0.46}{2} = \mathbf{1.8}\text{ m}$$

$$M = \frac{289.5}{2}1.8^2 = \mathbf{469}\text{ kN}\cdot\text{m}$$

The remainder of the problem is left for the reader.

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9-4 DESIGN OF STRAP (OR CANTILEVER) FOOTINGS

A strap footing is used to connect an eccentrically loaded column footing to an interior column as shown in Fig. 9-6. The strap is used to transmit the moment caused from eccentricity to the interior column footing so that a uniform soil pressure is computed beneath both footings. The strap serves the same purpose as the interior portion of a combined footing but is much narrower to save materials. Note again in Fig. 9-6 that the resultant soil pressure is assumed at the centers of both footings so that uniform soil pressure diagrams result. They may not be equal, however.

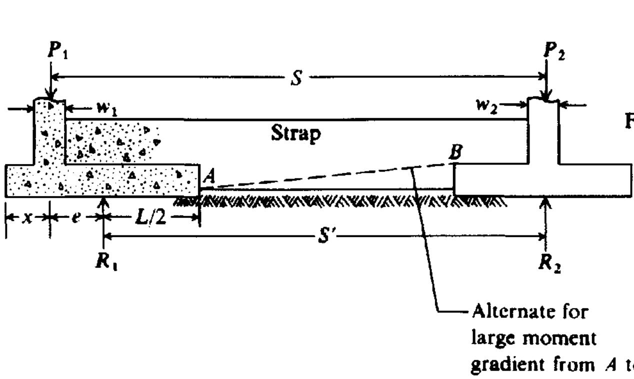

Figure 9-6 Assumed loading and reactions for a strap footing design. Make strap width about the same as the smallest column w.

The strap footing may be used in lieu of a combined rectangular or trapezoid footing if the distance between columns is large and/or the allowable soil pressure is relatively large so that the additional footing area is not needed. Three basic considerations for strap footing design are these:

1. Strap must be rigid—perhaps $I_{\text{strap}}/I_{\text{footing}} > 2$ (based on work by the author). This rigidity is necessary to control rotation of the exterior footing.
2. Footings should be proportioned for approximately equal soil pressures and avoidance of large differences in B to reduce differential settlement.
3. Strap should be out of contact with soil so that there are no soil reactions to modify the design assumptions shown on Fig. 9-6. It is common to neglect strap weight in the design. Check depth to span (between footing edges) to see if it is a deep beam (ACI Art. 10-7).

A strap footing should be considered only after a careful analysis shows that spread footings—even if oversize—will not work. The extra labor and forming costs for this type of footing make it one to use as a last resort. Again, it is not desirable to use shear reinforcement in either the two footings or the strap so that base rigidity is increased.

The strap may have a number of configurations; however, that shown in Fig. 9-6 should produce the greatest rigidity with the width at least equal to the smallest column width. If the depth is restricted, it may be necessary to increase the strap width to obtain the necessary rigidity. The strap should be securely attached to the column and footing by dowels so that the system acts as a unit.

The strap dimensions to provide adequate rigidity may be most conveniently determined using a beam-on-elastic-foundation computer program such as your diskette program B-5. One would input sufficient data to define the footing and strap stiffness (EI/L) and the program should have an option for no soil reactions against the strap. One then makes a solution and checks the displacement profiles of the two footings. If they are nearly constant across the footing, the strap is sufficiently thick. If there is a nearly linear variation of the displacements, the strap is not rigid enough and is allowing the footing to rotate.

The equations shown in Fig. 9-6 are used to proportion the footing dimensions. The length dimension of the eccentrically loaded footing is dependent upon the designer's arbitrarily selected value of e, so a unique solution is not likely.

Example 9-3. Proportion a strap footing for the column spacing and loading shown in Fig. E9-3a. The allowable pressure is 120 kPa. Both columns are 400 mm square.

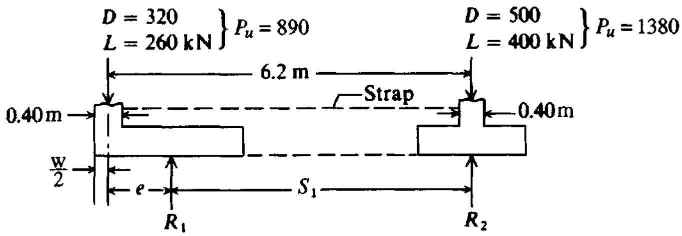

Figure E9-3*a*

Solution.

Step 1. Convert P_w to P_u and try $e = 1.20$ m.
Compute $S_1 = 6.2 - 1.2 = 5.0$ m.

$\sum M$ about column 2 $= 0$:

$$5R_1 - 6.2(890) = 0 \qquad R_1 = \frac{6.2(890)}{5} = 1103.6 \text{ kN}$$

$\sum M$ about $R_1 = 0$:

$$-1.2(890) + 1380(5) - R_2(5) = 0 \qquad R_2 = 1380 - 890\left(\frac{1.2}{5}\right) = 1166.4 \text{ kN}$$

Check by $\sum F_v = 0$ (note we are deriving equations shown in Fig. 9-6).

$$R_2 = P_1 + P_2 - R_1 = 890 + 1380 - 1103.6 = 1166.4 \text{ kN} \quad \text{(checks)}$$

Step 2. Find footing dimensions:

$$\text{UR} = \frac{P_u}{P_w} = \frac{2270}{1480} = 1.53 \qquad q_{\text{ult}} = q_a(\text{UR}) = 120(1.53) = \mathbf{183.6} \text{ kPa}$$

Footing dimensions for column 1:

$$L_1 = 2(e + w/2) = 2(1.2 + 0.2) = 2.8 \text{ m}$$

$$L_1 B_1 q_{\text{ult}} = R_1$$

$$B_1 = \frac{1103.6}{(2.80)(183.6)} = 2.147 \text{ m} \qquad \text{use } B = 2.15 \text{ m}$$

Footing dimensions for column 2 (use a square footing):

$$B^2 q_{\text{ult}} = R_2$$

$$B = \sqrt{\frac{1166.4}{183.6}} = 2.521 \text{ m} \qquad \text{use } B_2 = 2.52 \text{ m}$$

Use Column 1: $L = \mathbf{2.80}$ m $\quad B = \mathbf{2.15}$ m

Column 2: $B = \mathbf{2.52} \times \mathbf{2.52}$ m

Settlements should be nearly equal, since q is the same for both and the widths B are not greatly different. It is possible an $e = 1.1$ m could provide a closer agreement between B_1 and B_2, but this is left for the reader to verify.

Step 3. Draw shear and moment diagrams as in Fig. E9-3*b*.
Design footing depths for the worst case of two-way action and wide-beam shear; obtain wide-beam shear from V diagram.
Design strap for $V = 213$ kN and $M = 770$ kN · m.
Design footing reinforcing as a spread footing for both directions. Design strap as beam but check if it is a "deep" beam.

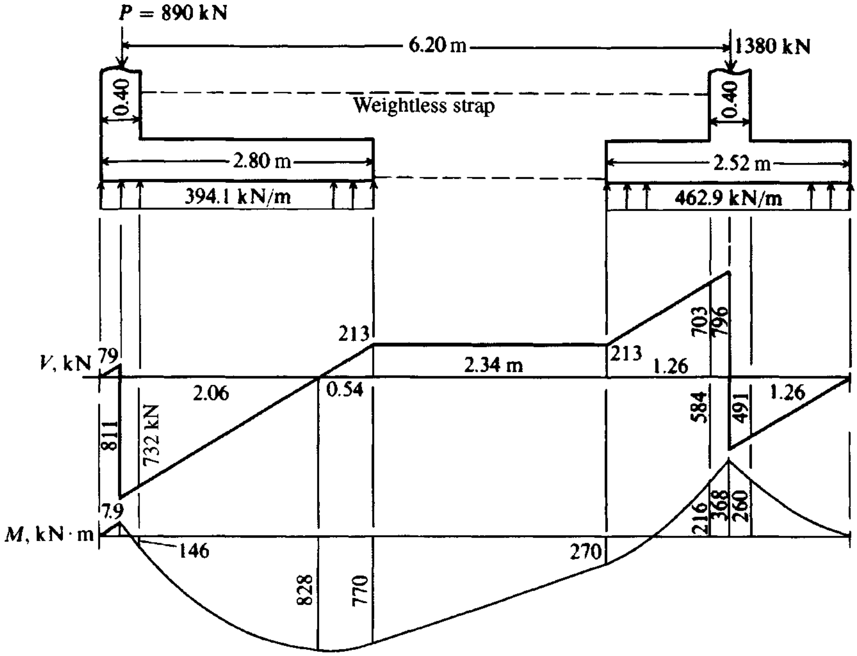

Figure E9-3*b*

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9-5 FOOTINGS FOR INDUSTRIAL EQUIPMENT

Footings for industrial applications are not directly covered by the ACI Code. On occasion local codes may include some guidance, and certain industries may have recommended standards of practice, but often the engineer has little guidance other than what in-house design experience may exist. These gaps in practice are sometimes filled by handbooks or by professional committees. (ACI, for example, has over 100 committees). ACI Committee 318 is responsible for the ACI "Building Code 318-"; ACI Committee 351 is concerned with foundations for industrial equipment. Professionals who have a mutual interest make up the membership of these committees.

Footings for industrial application are often one of a kind; the loadings are very difficult to define and, as a consequence, the footing is conservatively designed so that, one hopes, the worst possible load condition (or some loading not anticipated at design time) is covered.

Footings in industrial applications often have large horizontal forces and overturning moments in addition to vertical forces. These moments are primarily from wind but may also be from an earthquake analysis or from use. The geotechnical consultant would not know either the moment or horizontal force at this preliminary stage, so that the allowable bearing capacity q_a is not likely to be based on footing eccentricity or any of the refined methods of Chap. 4. (e.g., Fig. 4-4*b*). Rather the allowable bearing capacity is very probably a routine determination using the SPT and/or q_u with some possible reduction to allow for loading uncertainties.

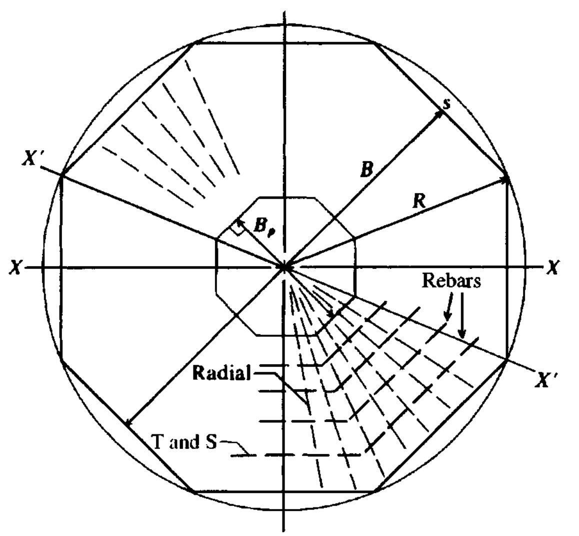
It would be up to the structural designer to accept the recommended q_a or discuss with the consultant whether the value should be further reduced. The designer may also wish to discuss whether an increase may be allowed for wind, and some recommendation for the backfill should be obtained, since this is a substantial contribution to overturning stability and might provide some sliding stability. Two factors usually allow this procedure to work:

1. The critical loading (wind or earthquake) is transitory and represents an upper bound in most cases.
2. The footings are usually embedded in the soil to a substantial depth so that the increase in bearing capacity, which may not be accounted for, more than offsets any reduction from eccentric loadings. If the center of footing area coincides with the resultant (refer to Fig. 9-1*e*) there would be no reduction for eccentricity.

Sliding stability is based on a combination of base adhesion, soil-to-concrete friction, and possibly passive earth pressure (see Chap. 11). Friction resistance depends on the total weight of the system above the base of the footing. Generally the friction factor is tan ϕ but the adhesion should be reduced, with values from 0.6 to 0.8c being commonly used. If the designer includes passive pressure resistance to sliding, great care in backfilling is required so that the perimeter zone soil can provide lateral resistance to translation.

A round base is more economical than other shapes for tall vessels, process towers, and stacks because the direction of overturning from wind or earthquake is not fixed. A pedestal is nearly always used to interface the metal superstructure to the embedded footing. The pedestal is often round to accommodate the base ring, or frame, of the equipment but may be rectangular, hexagonal, or octagonal.

In practice, however, it is difficult to form a round footing member, so an octagon is widely used since it closely fits a circle and can be formed easily. The geometry of an octagon is given in Fig. 9-7 together with a number of section property equations for design use.



Properties of octagon

$A' = 2B^2 \tan 22.5° = 0.828B^2$

$s = 2R \sin 22.5° = 0.765R$

$B = 2R \cos 22.5° = 1.848R$

$I_{x-x} = I_{x'-x'} = 0.6381R^4$

$r_x = r_{x'} = 0.475R$

$S_x = 0.6906R^3$

$R = 0.5412B$

Figure 9-7 Properties of an octagon. Also shown is the suggested method of placing reinforcement for radial moments and tangential steel either for T and S or for tangential moments. Additional bars may be required on outer radius to meet T and S requirements.

Generally the maximum eccentricity should be limited to about $B/8$ so that the full footing is effective for all but wind on the vessel during erection. If a turnover wind is anticipated during erection, temporary guying can be used.

The design of an octagon-shaped foundation involves sizing the pedestal (diameter and height) and the base. This sizing should take into account the following:

1. Empty condition with and without wind
2. Proof test condition with or without wind
3. Operating conditions with or without wind

The footing soil depth is then tentatively selected. The backfill over the footing has a considerable stabilizing effect and should be included when checking for overturning stability. The weights of the pedestal and footing slab are computed and used in combination with the overturning from wind or earthquake to find the soil pressures at the toe and heel for the several load cases. It is common but no longer recommended by the author to use

$$0 < \frac{P}{A} \pm \frac{Mc}{I} \leq q_a$$

Actually, one should use the equivalent rectangle of Fig. 4-4*b* with a *rectangular* soil pressure distribution and solve for the effective footing area by trial.

Wind and/or earthquake loads are obtained from local building codes, from the client, or from one of the national building codes such as (in the United States) the Uniform Building Code.

The footing is checked for wide-beam shear (most likely to control) and two-way action and for bending with sections as in Fig. 9-8. Noting that two-way action is very difficult to analyze unless one has available curves such as Brown (1968), one can make a rapid approximation by checking for wide-beam and then computing the resisting shear on the curved section, which is first converted into an equivalent square (see step 5 of Example 9-4). If the resisting shear is greater than 90 percent of the factored vertical loads, the depth is adequate. If the resisting shear is less, a more refined analysis is required. At this point one must make a decision either to increase the footing arbitrarily by 25 to 50 mm with some increase in material costs or to refine the analysis with the resulting increase in engineering costs and a possibility of still having to increase the depth. Also carefully note: Shear steel should not be used, for the footing weight has a stabilizing effect on overturning. Most importantly, the footing rigidity is needed to satisfy the linear soil-pressure assumption used in the design.

The most efficient method of round base design is to use a computer program such as B-20 (see your diskette README.DOC file), which uses a radial gridding scheme so that a grid line can be placed at the outer face of the pedestal, which is nearly always used. This program is set up to allow each circular grid line to have a different modulus of subgrade reaction and to allow doubling of the edge springs. This program can iterate to a valid solution by setting node springs that have soil-footing separation to zero. This makes it easy to locate the line of zero pressure without resort to tables or charts and to find bending moments and shear values at the various nodes. In passing, note that it is not a trivial task to compute critical moments by hand when the base supports a pedestal. Moments may be under-computed by close to 30 percent if the pedestal is not considered. By trying several depths a near optimum value can be found and the design continued.

When the footing depth has been fixed, the reinforcing steel is computed. In most cases the minimum amount controls, but note the minimum percent (as a decimal) can be either $1.4/f_y$

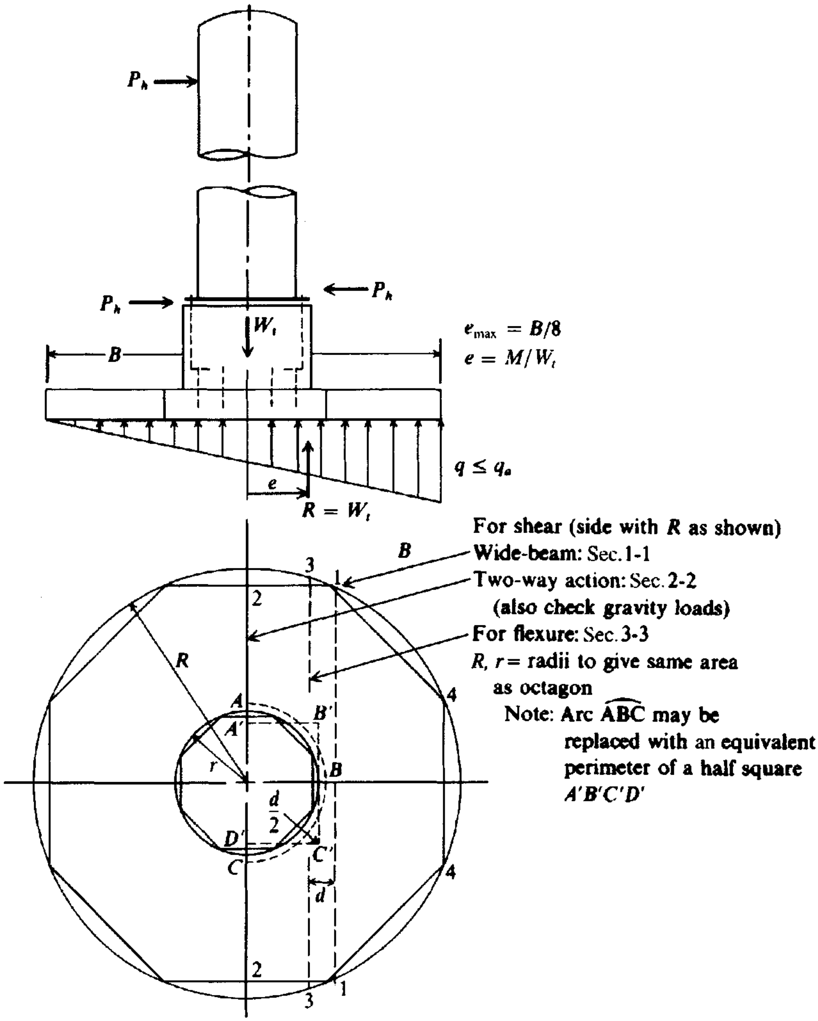


Figure 9-8 Layout of a vertical vessel foundation, critical soil pressure, and sections for shear and bending.

(or $200/f_y$) or a one-third increase in the actual computed amount (ACI Art. 10-5). This steel may be placed radially and distributed across each octagon face (Fig. 9-7). Tangential steel based on temperature and shrinkage (T and S) requirements should be placed parallel to each octagon face. Chu and Afandi (1966) suggest that tangential moments are not likely to exceed $0.05q_oR^2$, so that T and S steel will usually control. Steel requirements for bending moments are computed both for the bottom (with computations based on eccentric soil pressure) and for the top, based on no soil pressure and the weight of backfill and footing acting with full loss of soil pressure.

The pedestal may be hollow but is commonly solid to increase overturning stability. The bearing between base ring and pedestal is checked using the method of Sec. 8-6 for allowable bearing. Depending on base ring dimensions and pedestal configuration this check may set f_c' (which does not have to be same as footing) of the pedestal. The pedestal steel is designed to provide enough steel to resist the overturning moment at the base of the pedestal. This steel may be computed on the basis of using the section modulus of a *line circle* with r = radius; t = width and is very small compared to r of the reinforcing bar circle. This is obtained as

$$A_{\text{ring}} = 4\int_0^{\pi/2} rt\,d\theta \rightarrow \mathbf{2\pi rt}$$

Similarly the moment of inertia about an axis through the diameter is

$$I_x = 4t\int_0^{\pi/2} r^2 \sin^2\theta d\theta = \boldsymbol{\pi tr^3}$$

and the section modulus $\boldsymbol{S_x} = \pi tr^2$. The line area is also the number of bolts × bolt area as

$$A = 2\pi rt = N_b A_s$$

and multiplying $\boldsymbol{S_x}$ by $A_{\text{ring}}/A_{\text{ring}} = N_b A_s / 2\pi rt$, we obtain (with $r = \frac{D_b}{2}$)

$$\boldsymbol{S_x} = \frac{N_b A_s D_b}{4}$$

For combined stresses and with the vertical compressive force W reducing the overturning stresses we obtain

$$T = A_s f_s = \left(\frac{M}{\boldsymbol{S_x}} - \frac{W}{N_b A_s}\right) A_s$$

Substituting and simplifying, we obtain

$$A_s = \frac{1}{f_s}\left(\frac{4M}{N_b D_b} - \frac{W}{N_b}\right) \qquad (a)$$

where A_s = area of a rebar bar or anchor bolt
D_b = diameter of rebar or anchor bolt circle
f_s = allowable steel stresses of bolts or bars in units consistent with A_s and W
M = overturning moment in units consistent with D_b
N_b = number of bars or anchor bolts in circle
W = weight of vessel + pedestal

The pedestal seldom requires reinforcement; however, some designers routinely use a minimum percent steel ($A_s = 0.01A_{\text{ped}}$). A *cracked section* analysis using reinforcement may be required if unreinforced concrete tension stresses exceed some maximum value [given as $f_t \le 0.4\phi\sqrt{f_c'}$ ($5\phi\sqrt{f_c'}$, psi), $\phi = 0.65$ in ACI 318-1M, Art. 6.2.1]. If a cracked section analysis is necessary, it involves finding the neutral axis (using statics) of the pedestal; the resulting moment of inertia of the composite section and tension stresses in the rebars.

The anchor bolts are designed to resist the tension force from the overturning moment at the base of the vessel or stack. Equation (*a*) may also be used to approximate the anchor bolts.

A general overview of the design of an industrial footing is given in Example 9-4. There is some diversity of opinion on how these designs should be made and what is too conservative a design. One must weigh doubling or tripling engineering costs for a refined design using estimated loads against material savings of perhaps 50 to 150 mm of concrete depth or diameter change. A computer program such as B-20 is particularly useful in analyzing this type of base for node shear, moment, and soil pressures.

Example 9-4.

Given. The following data for the design of a vertical refining vessel:

Diameter (less insulation) = 1.85 m
Insulation thickness = 0.075 m
Height of vessel above pedestal = 33.5 m
Diameter of bolt circle of base ring = 2.00 m
Weights (including anchor or base ring): Shipping = 290 kN
Operating = 580 kN
Test (proofing) = 1160 kN
Allowable net soil pressure q_a = 150 kPa
Unit weight γ of backfill = 16.50 kN/m^3
Materials f'_c = 21 MPa f_y = 400 MPa
γ_c = 23.6 kN/m^3
Vessel location: southern Illinois

Obtain from the Uniform Building Code (UBC, 1994 edition):

Exposure B
Importance factor (hazardous materials), I = 1.15
q_s = 1.80 kPa (wind v = 190 kph and using UBC Table 23-F)

Required. Make a tentative design for this system using both a round base and round pedestal and for the given UBC requirements.

Solution. Some initial computations (not shown) are used to approximate a set of dimensions for the base, pedestal and base thickness. Clearly the pedestal will have to be about 0.15 m larger than tower diameter to provide adequate side cover so the anchor bolts do not split out. The base will have to be large enough to carry the tower load based on allowable soil pressure and the thickness (of 0.70 m) is estimated based on the base diameter (refer to Fig. E9-4*a*).

Step 1. We will only check wind moments (although earthquake moments should also be checked, as this site is in a zone that has an above average earthquake potential). From the Uniform Building Code (UBC) Sec. 2316,[1] we obtain the following equation for wind pressure:

$$p_w = C_e C_q q_s I$$

where C_e = exposure, height and gust factor (use average of (1.13 + 1.20)/2 = 1.17 (using UBC Table 23-G))

C_q = pressure coefficient for structure and for round and elliptical shapes = 0.8 (using UBC Table 23-H)

q_s = wind pressure, at the standard height of 10 m (C_e adjusts for greater heights) and based on the anticipated wind velocity in kph (using UBC Table 23-F). For 190 kph use[2] q_s = **1.80 kPa**

I = Importance factor (1.15 for hazardous materials, UBC Table 23-L)

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[1]The UBC method is quite similar to the ANSI A58-1 standard, available from ASCE as ANSI/ASCE 7-88.

[2]At the time this textbook was being prepared, the several available building codes had not converted to SI. The values used by the author are soft conversions from the source and rounded.

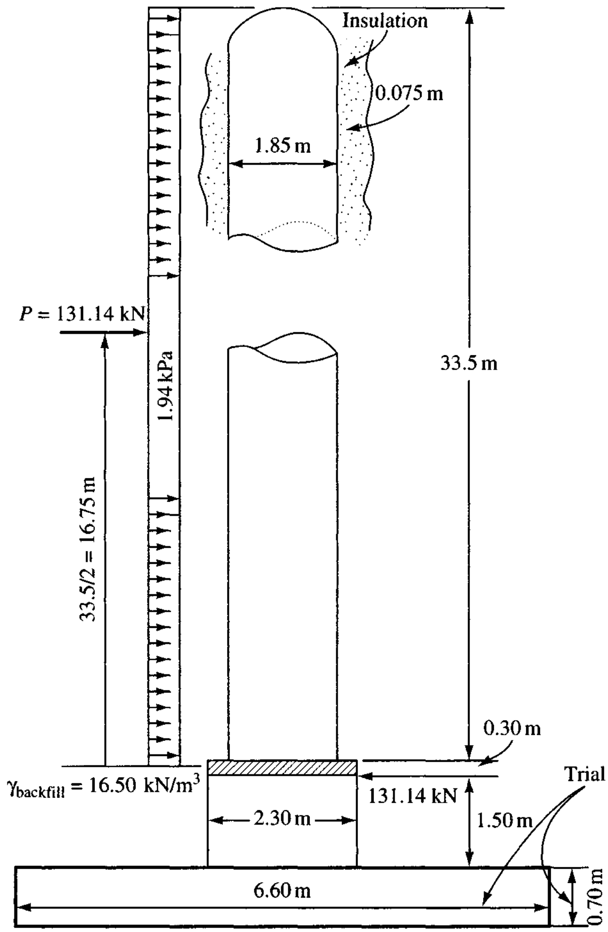

Figure E9-4*a*

Making substitutions, we have the average wind pressure for the tower height as

$$p_w = (1.17)(0.80)(1.80)(1.15) = \mathbf{1.94}\text{ kPa}$$

The total horizontal wind force is computed as the projected area $\times$ q_s with an *increase* factor to account for tower projections of various types. The increase factor may be 1.0, 1.1, 1.2, etc.; we will use a value of 1.0. The general equation format is

$$P_w = \text{height(diam.)(increase factor)}(p_w)$$

Substituting, we obtain the horizontal wind force as

$$\begin{aligned} P_w &= (33.5 + 0.3)[1.85 + 2(0.075)](1)(1.94) \\ &= 33.8(2.00)(1)(1.94) = \mathbf{131.14}\text{ kN} \end{aligned}$$

This force acts at midheight of the tower and produces a horizontal shear at the anchor ring, as shown in Fig. E9-4*a*. Strictly, the shear is at the top of the ground, but the small height of 0.3 m is negligible, especially since the anchor ring may be between 100 and 150 mm thick.

The 131.14-kN wind load produces an overturning moment, at the top of the anchor ring, of

$$M_{o,r} = 131.14(33.5/2) = \mathbf{2197}\text{ kN}\cdot\text{m (rounded)}$$

and, about the base (and using initial trial dimensions), of

$$M_{o,b} = 131.14(33.5/2 + 0.30 + 1.50 + 0.70)$$
$$= 131.14(19.25) = \mathbf{2524}\ \text{kN}\cdot\text{m (rounded slightly)}$$

Step 2. Estimate the gravity weights of the several elements in the system that contribute to foundation load. Take pedestal B_p = bolt ring diameter + 0.3 m = 2.30 m and base slab dimensions shown on Fig. E9-4*a*. Concrete $\gamma_c = 23.6$, soil $\gamma_s = 16.5\ \text{kN/m}^3$.

$$\text{Base area (from Fig. 9-7)} = A = 0.828B^2 = 0.828(6.60)^2 = 36.1\ \text{m}^2$$
$$\text{Pedestal weight} = 1.50(0.828)(2.30^2)(23.6) = 155.1\ \text{kN}$$
$$\text{Footing weight} = 36.1(0.70)(23.6) = 596.4\ \text{kN}$$
$$\text{Backfill weight (excluding pedestal zone)} = (36.1 - 4.38)(1.50)(16.5) = 785.1\ \text{kN}$$
$$\text{Total base weight} = \mathbf{1536.6}\ \text{kN}$$

The following load conditions are checked:

1. Erection weight = pedestal + footing + shipping
   = 155.1 + 596.4 + 290 = 1042 kN
2. Test weight = Total base + Test weight
   = 1536.6 + 1160 = 2697 kN
3. Operating weight = Total base + Operating
   = 1536.6 + 580 = 2117 kN

Step 3. Check overturning stability by taking moments about the toe or leading edge (line 4-4 of Fig. 9-8). For all case 1 gravity loads the resisting moment is at $B/2$ from edge to give

$$M_r = (290.0 + 155.1 + 596.4) \times 6.60/2 = \mathbf{3437}\ \text{kN}\cdot\text{m}$$

The worst case for overturning will be **case 1** of tower erection onto a base without backfill. The other two load cases are computed similarly.

$$M_O = \text{wind moment about base} = 2524\ \text{kN}\cdot\text{m (from Step 1)}$$
$$\text{SF} = \text{stability number} = M_r/M_O = 3437/2524 = 1.36$$

The SF is small but > 1. One might consider using some temporary guying during the erection phase.

For working conditions (**case 3**) we find

$$M_r = (580.0 + 1537) \times 6.60/2 = 6986\ \text{kN}\cdot\text{m} \qquad M_O = 2524\ \text{kN}\cdot\text{m as before}$$
$$\text{SF} = 6986/2524 = 2.77 > 1.5 \qquad \text{(O.K.)}$$

Step 4. Find soil pressures beneath toe and heel for cases 2 and 3.

For **case 3**:

$$e = M/P = 2524/(1537 + 580) = 2524/2117 = 1.19\ \text{m}$$
$$B/8 = 6.60/8 = 0.825 < 1.19.$$

Thus, part of the base under operating conditions appears to have soil-base separation. We will continue (in practice I would use program B-20, described on your diskette, and check the toe for q to see if $q > q_a$). Here, to prevent soil-base separation would require $B = 1.19 \times 8 = 9.52$ m—clearly an overdesign.

The effective radius R of the base is (see the equations on Fig. 9-7)

$$R = 0.5412B = 0.5412(6.60) = 3.57\ \text{m}$$

The section modulus about a diameter is (also see Fig. 9-7)

$$S_x = 0.6906R^3 = 0.6906(3.57^3) = 31.42 \text{ m}^3$$

We will compute soil pressures as $q = P/A \pm M/S_x \leq q_a$. Also the base and backfill weight will be neglected, since q_a is a *net* allowable pressure. The resulting error is the difference between the unit weight of concrete and soil and base thickness, $(\gamma_c - \gamma_s)D_c$.

For load test **case 2**, and including only the test load + pedestal weight, we have

$$q = \frac{1160 + 155.1}{36.1} \pm \frac{2524}{31.42}$$

$$= 36.4 \pm 80.3 = \mathbf{116.7} < 150 \quad \text{O.K.}$$

$$= \mathbf{-43.9} < 0 \quad \text{may be O.K.}$$

Since the test load is temporary, any small overstresses would probably not be critical.

At operating conditions (**case 3**) we have the operating load + pedestal weight, giving:

$$q = \frac{580 + 155.1}{36.1} \pm \frac{2524}{31.42}$$

$$= 20.4 \pm 80.3 = \mathbf{100.7} < 150 \quad \text{O.K.}$$

$$= \mathbf{-59.9} \text{ kPa} \quad \text{(base only partly effective)}$$

At this point we have the problem that with the base only partly effective, the section modulus $\mathbf{S}_x$ should be revised. We will not do this, for two reasons:

1. These pressures are only for base bending moment.
2. The actual soil pressure cannot be computed this simply; that is, when the heel begins to lift from the soil, the weight of that part of the base and overlying soil provides a resistance to soil separation. As previously shown it would require an extremely large base diameter to reduce the (−) heel pressure to zero using simple computations of the type used here.

Step 5. Check depth for shear.

We will find the shear value and arbitrarily apply the ACI Code LF = 1.4 (for dead loads) to the working design loads to make them "ultimate." Alternatively, we could recompute the pressures using some ACI factors such as $0.75(1.4D + 1.7W)$ or $0.9D + 1.3W$, but this single factor for the types of loads we have should be an adequate computation.

a. Check wide-beam shear: Take a 1-m wide strip at section 1-1 of Fig. 9-8 (refer also to Fig. E9-4b) as adequate. Take $d = 700 - 70 - 25$ (estimated 25 mm rebar diam both radial and tangential) to obtain a nominal design depth

$$d = 700 - 70 - 25 = 605 \text{ mm} \rightarrow \mathbf{0.605} \text{ m}$$

The shear to be resisted is the area *abcd* of Fig. E9-4*b* under the toe. The slope *s* of the pressure diagram for case 2 (appears worst case) is

$$s = (q_{\text{toe}} - q_{\text{heel}})/B = [116.7 - (-43.9)]/6.6 = 24.3$$

$$q_{ad} = 116.7 - s(X) = 116.7 - 24.3(1.545) = 79.2 \text{ kPa}$$

For a trapezoid pressure diagram using LF = 1.4, $L = X = 1.545$ m, and a width of 1 m obtain the shear along line *ad* as

$$V_{ad} = 1.4 \times \frac{(116.7 + 79.2)}{2} \times 1.545 = \mathbf{211.9} \text{ kN/m (with LF} = 1.4)$$

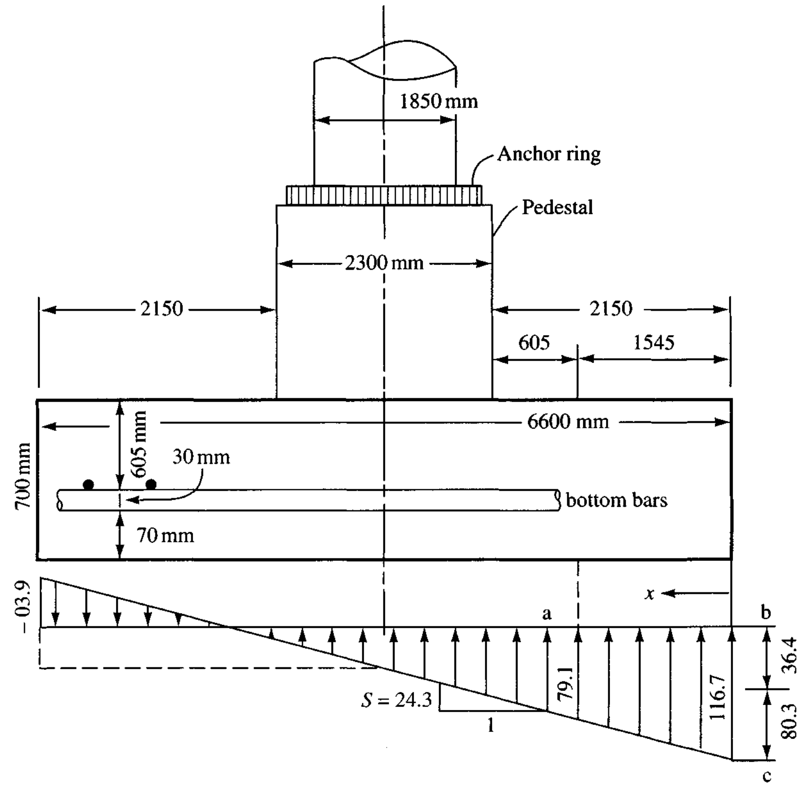

Figure E9-4*b*

Concrete wide-beam resistance is $v_c = 0.649$ MPa (Table 8-2)

$$V'_c = v_c bd = 0.649(1)(0.605)(1000)$$
$$= \mathbf{393}\ \text{kN} \gg 211.9 \qquad \text{O.K. wide-beam}$$

b. Check *two-way* action: We should check perimeter shear around arc ABC of Fig. 9-8. This arc is often converted into an equivalent one-half square with the same area. The shear on this perimeter is very hard to compute for the overturning case. In many cases its precise value is not necessary. For example, if the resisting shear is larger than about 90 percent of the total vertical load, the precise value is not needed. Let us compute the resisting two-way action shear (allowable v_c = **1.298** MPa from Table 8-2). We will use an equivalent square based on a diameter of

$$B_p + d = 2.300 + 0.605 = 2.905 \text{ m}$$

The equivalent side of a square with this diameter is

$$s_s = \sqrt{0.7854(2.905^2)} = 2.57 \text{ m}$$
$$p = 2.57 + 2 \times 2.57/2 = 5.14 \text{ m (one-half of two-way shear perimeter)}$$

Two-way action shear will be based (using LF = 1.4) on

$$V_l = 1.4 \text{ (operating load + pedestal weight)}$$
$$= 1.4(580 + 155.1) = \mathbf{1029}\ \text{kN}$$

The resisting shear (include 1000 to convert MPa to kPa) is

$$V_r = v_c p d = 1.298(5.14)(0.605)(1000) = \mathbf{4036}\text{ kN} \gg 1029$$

It appears the base is quite adequate for both wide-beam and two-way shear for all three load cases. Several comments are worthwhile at this point:

1. One could make the footing thinner, but the weight gives additional stability against overturning; thickness gives additional rigidity for satisfying the condition of linear soil pressure distribution.
2. One might consider using $f'_c = 18$ MPa, but when concrete strengths are much less than 21 MPa (3 ksi) the extra quality control needed might cost more than the extra sack or so of cement required for higher strength.
3. Reducing the footing thickness 0.150 m would save about 5.4 m^3 of concrete but would be likely to take an extra day to redesign the footing (especially to check two-way action shear). Obviously the "safety" would be somewhat less with a thinner base slab.

Step 6. Find the required area of bottom reinforcing steel for bending: Take a 1-m strip to the face of the pedestal perpendicular to line 3-3 of Fig. 9-8 (refer also to Fig. E9-4*b*).

$$\text{Cantilever arm } L = 2.15 \text{ m}$$

$$q = q_{\max} - sx = 116.7 - 24.3x$$

$$M = \int_0^L \int_0^L q\,dx = \frac{116.7L^2}{2} - \frac{24.3L^3}{6} \quad \text{(both integration constants = 0)}$$

$$= \frac{116.7 \times 2.15^2}{2} - \frac{24.3 \times 2.15^3}{6} = \mathbf{229.5}\text{ kN}\cdot\text{m/m}$$

For $f'_c = 21$ MPa, $f_y = 400$ MPa $\rightarrow a = 22.4A_s$. Using Eq. (8-2), we have

$$\phi f_y A_s\left(d - \frac{a}{2}\right) = M_u = 1.4M$$

Making substitutions, we have

$$A_s\left(0.605 - \frac{22.4A_s}{2}\right) = \frac{1.4(229.5)}{0.9(400)(1000)}$$

from which

$$A_s^2 - 0.0540A_s = 0.000\,0797$$

$$A_s = \mathbf{1519}\text{ mm}^2\text{/m}$$

Arbitrarily check the following:

$$\text{T \& S: } A_s = 0.0018(1000)(605) = 1089 \text{ mm}^2\text{/m} < 1519$$

Check Min A_s of $1.4/f_y$ (or $200/f_y$) (Art. 10.5.1):

$$A_s = \frac{1.4}{f_y}(0.605)(1)(10^6) = \mathbf{2118}\text{ mm}^2\text{/m} > 1519$$

Check Min A_s (Art. 10.5.2), since $1.4/f_y > 1519$ for bending

$$A_s = 1.33(1519) = \mathbf{2020}\text{ mm}^2\text{/m}$$

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From these we see that $1.4f_y$ controls, so use either $A_s \geq 2118$ mm²/m or $A_s \geq 2020$ mm²/m. Use four No. 30 bars ($4 \times 700 = 2800$ mm²/m) and place radially.

The pedestal produces a "fixed-end" rigidity such that the moment computed at the pedestal face of 239.1 kN · m/m could be as much as 30 percent low. ACI Art. 10.5.2 was used in this analysis to provide the required amount of steel. The Code commentary for Art. 10.5.3 states that for slabs supported by soil the one-third increase does not apply unless superstructure loads are transmitted by the slab to the soil. In this case the pedestal transmits the tower load to the footing, so the one-third increase is applicable. It is preferable, of course, to use a computer program and directly obtain the moment at the pedestal face—although the Art. 10.5.2 check would still have to be done.

Step 7. Top steel requirements (side opposite high toe pressure) are based on footing weight + backfill and full loss of soil pressure: Moment arm is same as used in step 6 = 2.15 m, LF = 1.4, and

$$M_u = 1.4(0.7 \times 23.6 + 1.5 \times 16.5) \times \frac{2.15^2}{2} = 133.6 \text{ kN} \cdot \text{m}$$

Based on this small moment and from step 6 it is evident that the minimum $A_s = 1.4/f_y$ will control. Therefore, use $A_s = \mathbf{2118}$ mm²/m → seven No. 20 bars ($7 \times 300 = 2100$ mm²/m). This steel is required in any case, as the top steel requirements result from wind, which can come from any direction.

Step 8. Find vertical steel for the pedestal, assuming that the rebars will carry all of the tension stresses. Take pedestal rebar diameter $B_p \approx 2.30 - 0.30 = \mathbf{2.0}$ m.

Find wind moment at top of footing (refer to Fig. E9-4*a*):

$$M_u = 131.14(33.5/2 + 0.3 + 1.5) = 2433 \text{ kN} \cdot \text{m}$$

Using Eq. (*a*) previously given, including the LF = 1.4 and rearranging (the 1000 converts m² to mm²) we have:

$$NA_s = \frac{1.4}{f_s}\left[\frac{4 \times M_o}{B_p} - W\right] = \frac{1.4 \times 1000}{0.9 \times 400}\left[\frac{4 \times 2433}{2.0} - (155.1 + 580)\right]$$

$$= \mathbf{16\,064} \text{ mm}^2 \qquad \text{(total in pedestal)}$$

$$0.01A_g = 0.01(0.828 \times 2.30^2)10^6 = \mathbf{43\,801} \text{ mm}^2$$

Using load factors from ACI Art. 9.2.2, $1.3W + 0.9D$, gives

$$A_s = 22\,027 \text{ mm}^2 > 16\,064 \text{ just computed}$$

Use 24 No. 35 bars ($A_s = 24 \times 1000 = 24\,000$ mm²) in the pedestal as follows (for octagon shape):

1 at each corner (uses 8)

2 at 1/3 points of each side (uses 16)

These rebars would have to be placed symmetrically, since wind can come from any direction.

The anchor bolts and tangential rebars (probably just T & S) are still to be designed but will be left as a reader exercise. For the anchor bolts the designer would require a plan of the ring support so that the anchorage hole positions are located.

Comment. What should one use for load factors in this type problem? Because the tower is fixed in dimension and volume, there is not an uncertainty factor of 1.7 and probably not of 1.4. The wind load could have a load factor of 1, because it is already estimated from a building code, and it does not make much sense to say, "The wind load is uncertain and may have an additional uncertainty of 30 (1.3), 40 (1.4), or 70 (1.7) percent."

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