CHAPTER 5

FOUNDATION SETTLEMENTS

5-1 THE SETTLEMENT PROBLEM

Foundation settlements must be estimated with great care for buildings, bridges, towers, power plants, and similar high-cost structures. For structures such as fills, earth dams, levees, braced sheeting, and retaining walls a greater margin of error in the settlements can usually be tolerated.

Except for occasional happy coincidences, soil settlement computations are only best estimates of the deformation to expect when a load is applied. During settlement the soil transitions from the current body (or self-weight) stress state to a new one under the additional applied load. The stress change Δq from this added load produces a time-dependent accumulation of particle rolling, sliding, crushing, and elastic distortions in a limited influence zone beneath the loaded area. *The statistical accumulation of movements in the direction of interest is the settlement.* In the vertical direction the settlement will be defined as ΔH.

The principal components of ΔH are particle rolling and sliding, which produce a change in the void ratio, and grain crushing, which alters the material slightly. Only a very small fraction of ΔH is due to elastic deformation of the soil grains. As a consequence, if the applied stress is removed, very little of the settlement ΔH is recovered. Even though ΔH has only a very small elastic component, it is convenient to treat the soil as a pseudo-elastic material with "elastic" parameters E_s, G', μ, and k_s to estimate settlements. This would appear reasonable because a stress change causes the settlement, and larger stress changes produce larger settlements. Also experience indicates that this methodology provides satisfactory solutions.

There are two major problems with soil settlement analyses:

1. *Obtaining reliable values of the "elastic" parameters.* Problems of recovering "undisturbed" soil samples mean that laboratory values are often in error by 50 percent or more. There is now a greater tendency to use in situ tests, but a major drawback is they tend to obtain horizontal values. *Anisotropy* is a common occurrence, making vertical elastic

values (usually needed) different from horizontal ones. Often the difference is substantial. Because of these problems, correlations are commonly used, particularly for preliminary design studies. More than one set of elastic parameters must be obtained (or estimated) if there is *stratification* in the zone of influence H.

2. *Obtaining a reliable stress profile from the applied load.* We have the problem of computing both the correct numerical values and the effective depth H of the influence zone. Theory of Elasticity equations are usually used for the stress computations, with the influence depth H below the loaded area taken from $H = 0$ to $H \to \infty$ (but more correctly from 0 to about $4B$ or $5B$). Since the Theory of Elasticity usually assumes an isotropic, homogeneous soil, agreement between computations and reality is often a happy coincidence.

The values from these two problem areas are then used in an equation of the general form

$$\Delta H = \int_0^H \epsilon \, dH$$

where ϵ = strain = $\Delta q/E_s$; but $\Delta q = f(H, \text{load})$, $E_s = f(H, \text{soil variation})$, and H (as previously noted) is the *estimated* depth of stress change caused by the foundation load. The principal focus in this chapter will be on obtaining Δq, E_s and H.

It is not uncommon for the ratio of measured to computed ΔH to range as $0.5 \leftarrow \frac{\Delta H_{\text{meas}}}{\Delta H_{\text{comp}}} \rightarrow 2$. Current methodology tends to minimize "estimation" somewhat so that most ratios are in the 0.8 to 1.2 range. Note too that a small computed ΔH of, say, 10 mm, where the measured value is 5 or 20 mm, has a large "error," but most practical structures can tolerate either the predicted or measured value. What we do not want is an estimate of 25 mm and a subsequent settlement of 100 mm. If we err in settlement computations it is preferable to have computed values larger than the actual (or measured) ones—but we must be careful that the "large" value is not so conservative that expensive (but unneeded) remedial action is required.

Settlements are usually classified as follows:

1. *Immediate,* or those that take place as the load is applied or within a time period of about 7 days.
2. *Consolidation,* or those that are time-dependent and take months to years to develop. The Leaning Tower of Pisa in Italy has been undergoing consolidation settlement for over 700 years. The lean is caused by the consolidation settlement being greater on one side. This, however, is an extreme case with the principal settlements for most projects occurring in 3 to 10 years.

Immediate settlement analyses are used for all fine-grained soils including silts and clays with a degree of saturation $S \lessapprox 90$ percent and for all coarse-grained soils with a large coefficient of permeability [say, above 10^{-3} m/s (see Table 2-3)].

Consolidation settlement analyses are used for all saturated, or nearly saturated, fine-grained soils where the consolidation theory of Sec. 2-10 applies. For these soils we want estimates of both settlement ΔH and how long a time it will take for most of the settlement to occur.

Both types of settlement analyses are in the form of

$$\Delta H = \epsilon H = \sum_{H_i}^{H_{i+1}} \frac{\Delta q_i}{E_{si}} \qquad (i = 1 \text{ to } n) \tag{5-1}$$

where the reader may note that the left part of this equation is also Eq. (2-43*a*). In practice the summation form shown on the right may be used where the soil is subdivided into layers of thickness H_i and stresses and properties of that layer used. The total settlement is the sum obtained from all n layers. The reader should also note that E_S used in this equation is the constrained modulus defined from a consolidation test as $1/m_v$ or from a triaxial test using Eq. (*e*) of Sec. 2-14, written as

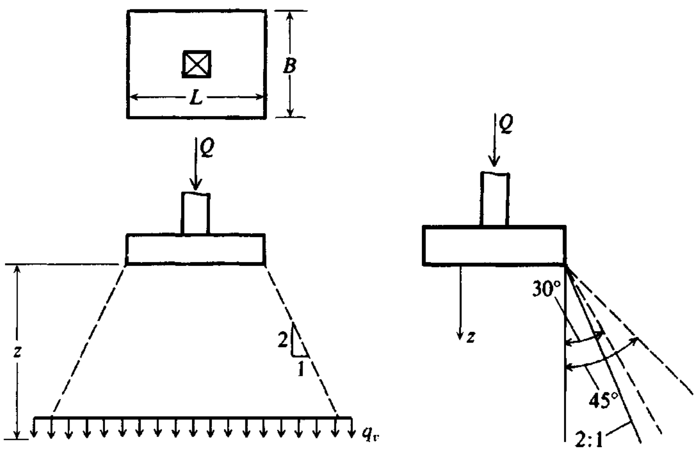
$$E_s = \frac{1}{m_v} = \frac{(1-\mu)E_{s,\text{tr}}}{(1+\mu)(1-2\mu)} \tag{5-1a}$$

where $E_{s,\text{tr}}$ = triaxial value [also used in Eq. (5-16)]. Note, however, that if the triaxial cell confining pressure σ_3 approximates that developed in situ when the load is applied, the triaxial E_s will approximate $1/m_v$. In most cases the actual settlements will be somewhere between settlements computed using the equivalent of $1/m_v$ as from a consolidation test [see Eq. (5-1*a*)] and E_s from a triaxial test. Unfortunately the use of Eq. (5-1*a*) also requires estimating a value of Poisson's ratio μ.

5-2 STRESSES IN SOIL MASS DUE TO FOOTING PRESSURE

As we see from Eq. (5-1), we need an estimate of the pressure increase Δq from the applied load. Several methods can be used to estimate the increased pressure at some depth in the strata below the loaded area. An early method (not much used at present) is to use a 2 : 1 slope as shown in Fig. 5-1. This had a great advantage of simplicity. Others have proposed the slope angle be anywhere from 30° to 45°. If the stress zone is defined by a 2 : 1 slope, the

Figure 5-1 Approximate methods of obtaining the stress increase q_v in the soil at a depth z beneath the footing.



pressure increase $q_v = \Delta q$ at a depth z beneath the loaded area due to base load[1] Q is

$$\Delta q = q_v = \frac{Q}{(B+z)(L+z)} \tag{5-2}$$

which simplifies for a square base ($B \times B$) to

$$q_v = \frac{Q}{(B+z)^2} \tag{5-2a}$$

where terms are identified on Fig. 5-1. This 2 : 1 method compares reasonably well with more theoretical methods [see Eq. (5-4)] from $z_1 = B$ to about $z_2 = 4B$ but should not be used in the depth zone from $z = 0$ to B. The *average* stress increase in a stratum ($H = z_2 - z_1$) is

$$\Delta q_v H = \int_{z_1}^{z_2} \frac{Q}{(B+z)^2}\, dz \rightarrow q_v = \frac{1}{H}\left[-\frac{Q}{B+z}\right]_{z_1}^{z_2} \tag{5-2b}$$

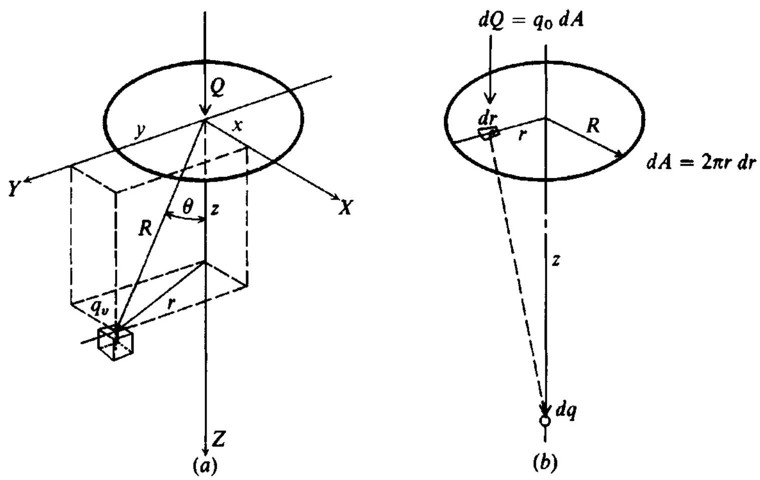
5-3 THE BOUSSINESQ METHOD FOR q_v

One of the most common methods for obtaining q_v is the Boussinesq (ca. 1885) equation based on the Theory of Elasticity. Boussinesq's equation considers a point load on the surface of a semi-infinite, homogeneous, isotropic, weightless, elastic half-space to obtain

$$q_v = \frac{3Q}{2\pi z^2}\cos^5\theta \tag{5-3}$$

where symbols are identified on Fig. 5-2*a*. From this figure we can also write $\tan\theta = r/z$, define a new term $R^2 = r^2 + z^2$, and take $\cos^5\theta = (z/R)^5$. With these terms inserted in Eq.

Figure 5-2 (*a*) Intensity of pressure q based on Boussinesq approach; (*b*) pressure at point of depth z below the center of the circular area acted on by intensity of pressure q_o.



[1]The vertical base load uses P, V, and Q in this textbook and in the published literature; similarly, stress increases from the base load are q_v, Δq_v, p, and Δp.

(5-3) we obtain

$$q_v = \frac{3Qz^3}{2\pi R^5} \tag{5-4}$$

which is commonly written as

$$q_v = \frac{3Q}{2\pi z^2} 1/[1 + (r/z)^2]^{5/2} = \frac{Q}{z^2} A_b \tag{5-5}$$

Since the A_b term is a function only of the r/z ratio we may tabulate several values as follows:

$\pm r/z$	0.000	0.100	0.200	0.300	0.400	0.500	0.750	1.000	1.500	2.000
A_b	0.477	0.466	0.433	0.385	0.329	0.273	0.156	0.084	0.025	0.008

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These values may be used to compute the vertical stress in the stratum as in the following two examples.

Example 5-1. What is the vertical stress beneath a point load $Q = 225$ kN at depths of $z = 0$ m, 0.6 m, 1.2 m, and 3.0 m?

Solution. We may write $q_v = (Q/z^2)A_b = 0.477Q/z^2$ (directly beneath Q we have $r/z = 0$). Substituting z-values, we obtain the following:

z, m	$q_v = 0.477(225)/z^2$, in kPa
0	∞
0.6	298 kPa
1.2	74.5
3.0	11.9

Example 5-2. What is the vertical stress q_v at point A of Fig. E5-2 for the two surface loads Q_1 and Q_2?

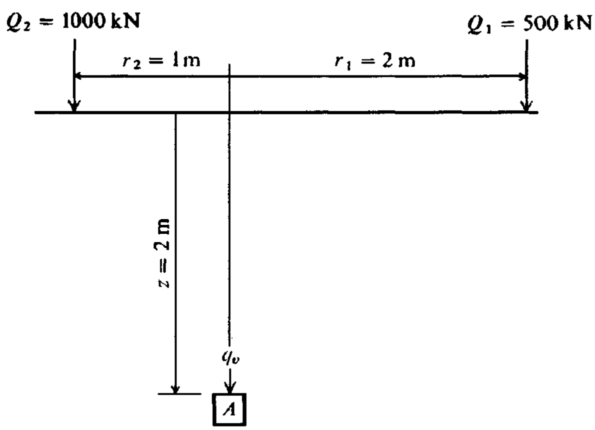

Figure E5-2

Solution.

$$q_v = \text{sum of stresses from the two loads}$$

$$Q_1\text{: } \frac{r}{z} = \frac{2}{2} = 1 \qquad A_b = 0.084$$

$$Q_2\text{: } \frac{r}{z} = -\frac{1}{2} = -0.5 \qquad A_b = 0.273$$

$$q_v = \frac{Q_1}{z^2}A_{b1} + \frac{Q_2}{z^2}A_{b2} = \frac{500(0.084)}{2\times 2} + \frac{1000(0.273)}{2\times 2} = \mathbf{78.8}\text{ kPa}$$

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Chart Methods

The purpose of foundations is to spread loads so that "point" loads with the accompanying very high stresses at the contact point ($z = 0$ of Example 5-1) are avoided. Thus, direct use of the Boussinesq equation is somewhat impractical until z is at a greater depth where computations indicate the point and spread load stress effects converge. We can avoid this by considering the contact pressure q_o to be applied to a circular area as shown in Fig. 5-2*b* so the load Q can be written as

$$Q = \int_0^A q_o\, dA$$

The stress on the soil element from the contact pressure q_o on the surface area dA of Fig. 5-2*b* is

$$dq = \frac{3q_o}{2\pi z^2}\frac{1}{[1 + (r/z)^2]^{5/2}}\,dA \qquad (a)$$

but $dA = 2\pi r\,dr$, and Eq. (*a*) becomes

$$q_v = \int_0^r \frac{3q_o}{2\pi z^2}\frac{1}{[1 + (r/z)^2]^{5/2}}2\pi r\,dr \qquad (b)$$

Performing the integration and inserting limits, we have

$$q_v = q_o\left\{1.0 - \frac{1}{[1 + (r/z)^2]^{3/2}}\right\} \qquad (5\text{-}6)$$

This equation can be used to obtain the stress q_v directly at depth z for a round footing of radius r (now r/z is a depth ratio measured along the base center). If we rearrange this equation, solve for r/z, and take the positive root,

$$\frac{r}{z} = \sqrt{\left(1 - \frac{q_v}{q_o}\right)^{-2/3} - 1} \qquad (c)$$

The interpretation of Eq. (*c*) is that the r/z ratio is also the relative size of a circular bearing area such that, when loaded, it gives a unique pressure ratio q_v/q_o on the soil element at a depth z in the stratum. If values of the q_v/q_o ratio are put into the equation, corresponding

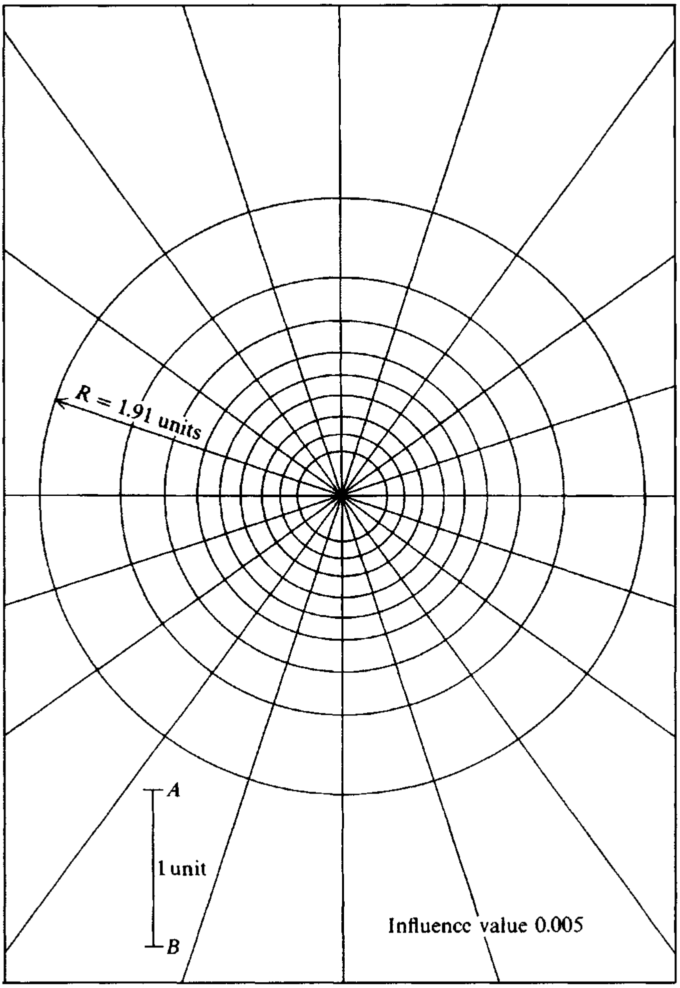


Figure 5-3 Influence chart for vertical pressure. *[After Newmark (1942).]*

values of r/z may be obtained as follows:

q_v/q_o = 0.0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.00
$\pm r/z$ = 0.0	0.270	0.400	0.518	0.637	0.766	0.918	1.110	1.387	1.908	∞

These values may be used to draw the Newmark (1942) chart in Fig. 5-3. The use of the chart is based on a factor termed the *influence value*, determined from the number of units into which the chart is subdivided. For example, if the series of rings is subdivided so that there are 400 units, often made approximate squares, the influence value is 1/400 = 0.0025. In making a chart it is necessary that the sum of the units between two concentric circles multiplied by the influence value be equal to the change in the q_v/q_o of the two rings (i.e., if the change in two rings is 0.1 q_v/q_o, then the influence value I multiplied by the number of units M should equal 0.1). This concept enables one to construct a chart of any influence value. Figure 5-3 is subdivided into 200 units; therefore, the influence value is 1/200 = 0.005. Smaller influence values increase the number of squares and the amount of work involved, since the sum of the squares used in a problem is merely a mechanical integration of Eq. (*a*). It is doubtful if much accuracy is gained using very small influence values, although the amount of work is increased considerably.

The influence chart may be used to compute the pressure on an element of soil beneath a footing, or from pattern of footings, and for any depth z below the footing. It is only necessary to draw the footing pattern to a scale of z = length AB of the chart. Thus, if $z = 5$ m, the length AB becomes 5 m; if $z = 6$ m, the length AB becomes 6 m; etc. Now if AB is 20 mm, scales of 1 : 250 and 1 : 300, respectively, will be used to draw the footing plans. These footing plans will be placed on the influence chart with the point for which the stress $\Delta q (< q_v)$ is desired at the center of the circles. The units (segments or partial segments) enclosed by the footing or footings are counted, and the increase in stress at the depth z is computed as

$$\Delta q = q_o M I \tag{5-7}$$

where Δq = increased intensity of soil pressure due to foundation loading at depth z in units of q_o

q_o = foundation contact pressure

M = number of units counted (partial units are estimated)

I = influence factor of the particular chart used

The influence chart is difficult to use, primarily because the depth z results in using an odd scale factor based on line AB in the figure. It has some value, however, in cases where access to a computer is not practical and there are several footings with different contact pressures or where the footing is irregular-shaped and Δq (or q_v) is desired for some point.

For single circular footings, a vertical center pressure profile can be efficiently obtained by using Eq. (5-6) on a personal computer. For square or rectangular footings the concept of the pressure bulb as shown in Fig. 5-4 is useful. The pressure bulbs are isobars (lines of constant pressure) obtained by constructing vertical pressure profiles (using similar to that of Fig. 1-1*a*) at selected points across the footing width B and interpolating points of equal pressure intensity (0.9, 0.8, 0.7q_o, etc.).

Numerical Methods for Solving the Boussinesq Equation

There are two readily available methods to obtain a vertical pressure profile using the Boussinesq equation and a computer. The first method is that used in program SMBWVP on your diskette (also applicable to the Westergaard equation of Sec. 5-5) as follows:

a. The square or rectangular base (for a round base convert to an equivalent square as $B = \sqrt{\pi r^2}$) with a contact pressure of q_o is divided into small square (or unit) areas of side dimension a so a series of "point" loads of $Q = q_o a^2$ can be used. Use side dimensions a on the order of 0.3 × 0.3 m (1 × 1 ft). Using very small a dimensions does not improve the result. The vertical pressure contributions from several bases can be obtained. The pressure at a point beneath a base such as the center, mid-side, or corner can be obtained from that footing as well as contributions from adjacent footings.

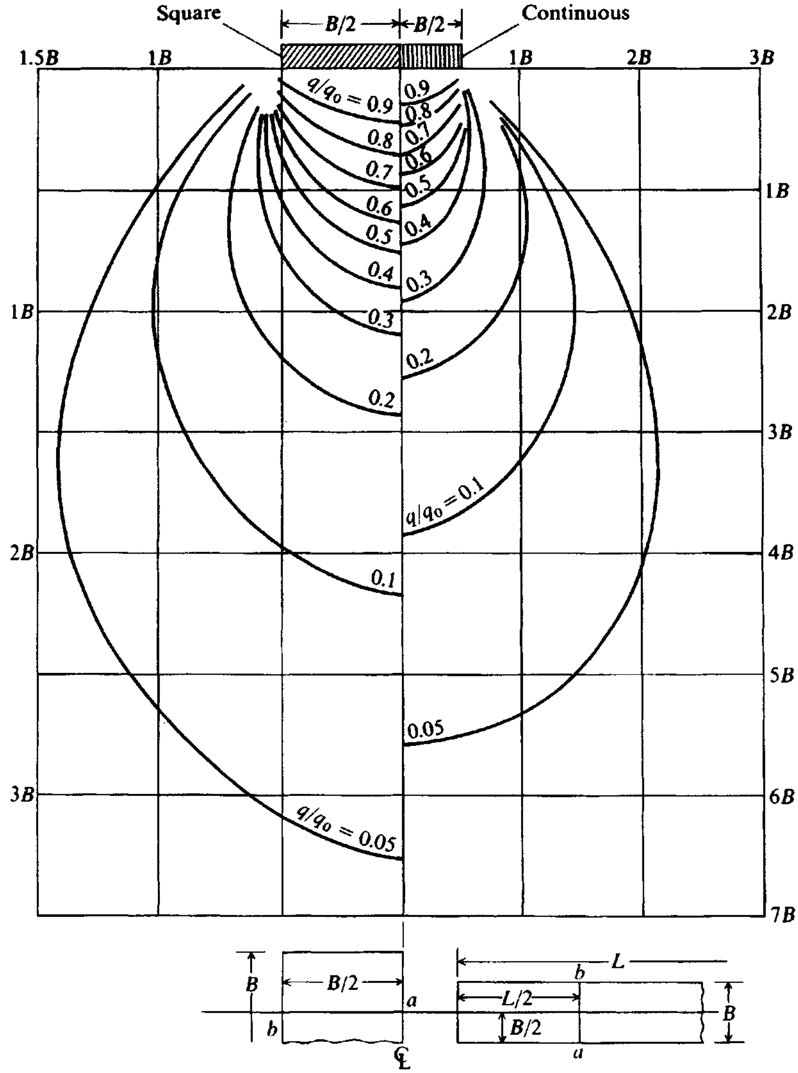


Figure 5-4 Pressure isobars (also called pressure bulbs) based on the Boussinesq equation for square and long footings. Applicable only along line *ab* from center to edge of base.

b. Input the location where the vertical pressure is wanted. Usually the *x*, *z* coordinates of this point are taken as the origin. Other bases (and this one if the point is under it) are referenced to the point where the vertical pressure is to be computed by distance DIST (see DTWAL of Fig. 11-19*a*) to the *far* side of the base and a perpendicular distance DOP [(+) to right side of DIST] to the base edge. Other bases that may contribute pressure are similarly referenced but in most cases bases not directly over the point can be treated as *point loads*. The pressures may be computed at any starting input depth Y_o; this may be at the ground surface or some point below. You can obtain a pressure profile using equally spaced depth increments *DY* or the vertical pressure at a single depth ($DY = 0$). For five depth increments input number of vertical points NVERT = 6; for 10 input NVERT = 11, etc.

c. The program computes the center x, y coordinates of each unit area making up a base. The program recognizes the base dimensions in terms of the number of unit squares in each direction NSQL, NSQW that is input for that base. In normal operation you would input both DIST and DOP as (+) values along with the side dimensions of the square SIZE and contact pressure q_o (QO). The program then locates the x, z coordinates of the center of the first square (farthest from point and to right) and so on. These would be used with a point load of $Q = q_o a^2$ in Eq. (5-4) to obtain one pressure contribution. There would be NSQW × NSQL total contributions for this footing.

A point load would use a single unit (NSQW = 1; NSQL = 1) area of $a = 0.3$ m. For example, if we have a point load at a distance of $z = 1.1$ m from the pressure point, we would input NSQW = 1, NSQL = 1, DIST = 1.1 + 0.3/2 = 1.25, and DOP = 0 + 0.3/2 = 0.15 m. The program would locate the point load correctly on the DIST line at $z = 1.1$ m and $x = 0.15 - 0.3/2 = 0$ using a single unit area (0.3 × 0.3 m). These values of 1.1, 0.0, YO and pressure $q_o = QO = Q_{act}/a^2$ would give the vertical pressure at the point of interest; i.e.,if $Q = 90$ kN, input $QO = 90/(0.3 \times 0.3) = 1000$ kPa.

For several contributing footings this process would be repeated as necessary to get the total increase in vertical pressure Δq at this depth YO.

d. The depth is incremented if more vertical points are required to a new YO = YO + DY, the process repeated, and so on.

The program has an option to output the pressure (and some checking data) for each depth increment and to output the pressure profile in compact form. It also gives the average pressure increase in the stratum (sum of pressures divided by number of points) for direct use in settlement computations.

Another method that is applicable to square or rectangular bases (and round ones converted to equivalent squares) is to use the Boussinesq equation integrated over a rectangle of dimensions $B \times L$. This is not a simple integration, but it was done by a number of investigators in Europe in the 1920s, although the most readily available version is in Newmark (1935) and commonly seen as in the charts by Fadum (1948). The equation given by Newmark—*applicable beneath the corner of an area $B \times L$*—is

$$q_v = q_o \frac{1}{4\pi}\left[\frac{2MN\sqrt{V}}{V+V_1}\frac{V+1}{V} + \tan^{-1}\left(\frac{2MN\sqrt{V}}{V-V_1}\right)\right] \tag{5-8}$$

where $M = \dfrac{B}{z} \quad N = \dfrac{L}{z} \qquad (q_v = q_o \text{ for } z = 0)$

$V = M^2 + N^2 + 1$

$V_1 = (MN)^2$

When $V_1 > V$ the $\tan^{-1}$ term is (−) and it is necessary to add π. In passing, note that $\sin^{-1}$ is an alternate form of Eq. (5-8) (with changes in V) that is sometimes seen. This equation is in program B-3 (SMNMWEST) on your diskette and is generally more convenient to use than Fadum's charts or Table 5-1, which usually requires interpolation for influence factors. The vertical stress at any depth z can be obtained for any reasonable proximity to or beneath the base as illustrated in Fig. 5-5 and the following examples.

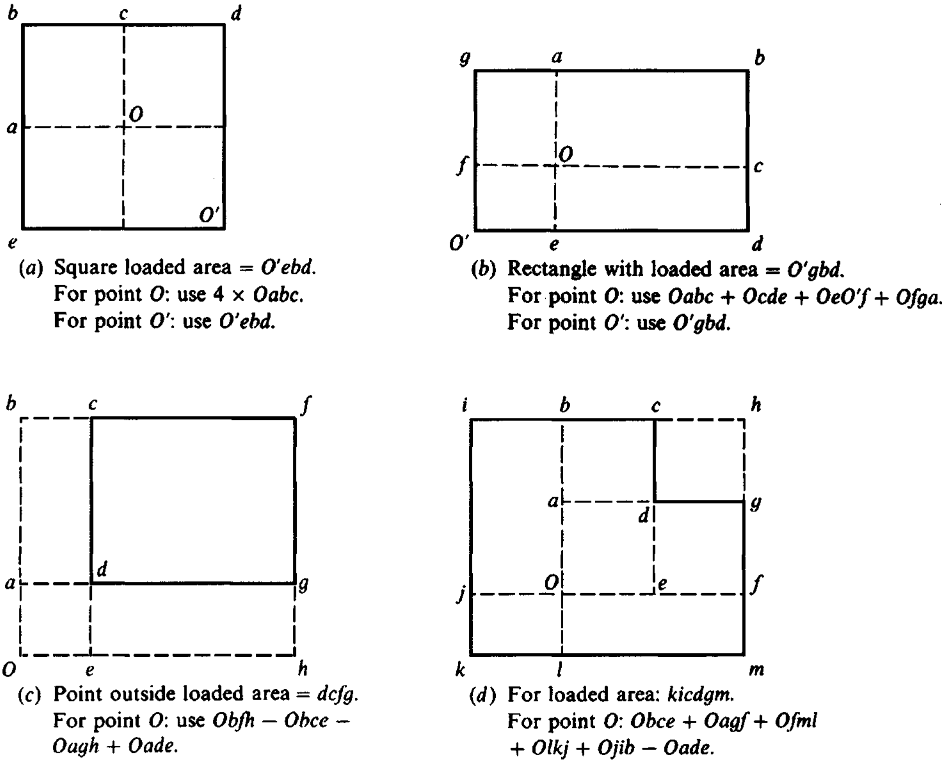
TABLE 5-1

Stress influence values I_σ from Eq. (5-8) to use in Eq. (5-8*a*) to compute stresses at depth ratios $M = B/z$; $N = L/z$ beneath the *corner* of a base $B \times L$.

M and *N* are interchangeable.

N \ M	.100	.200	.300	.400	.500	.600	.700	.800	.900	1.000
.1	.005	.009	.013	.017	.020	.022	.024	.026	.027	.028
.2	.009	.018	.026	.033	.039	.043	.047	.050	.053	.055
.3	.013	.026	.037	.047	.056	.063	.069	.073	.077	.079
.4	.017	.033	.047	.060	.071	.080	.087	.093	.098	.101
.5	.020	.039	.056	.071	.084	.095	.103	.110	.116	.120
.6	.022	.043	.063	.080	.095	.107	.117	.125	.131	.136
.7	.024	.047	.069	.087	.103	.117	.128	.137	.144	.149
.8	.026	.050	.073	.093	.110	.125	.137	.146	.154	.160
.9	.027	.053	.077	.098	.116	.131	.144	.154	.162	.168
1.0	.028	.055	.079	.101	.120	.136	.149	.160	.168	.175
1.1	.029	.056	.082	.104	.124	.140	.154	.165	.174	.181
1.2	.029	.057	.083	.106	.126	.143	.157	.168	.178	.185
1.3	.030	.058	.085	.108	.128	.146	.160	.171	.181	.189
1.4	.030	.059	.086	.109	.130	.147	.162	.174	.184	.191
1.5	.030	.059	.086	.110	.131	.149	.164	.176	.186	.194
2.0	.031	.061	.089	.113	.135	.153	.169	.181	.192	.200
2.5	.031	.062	.089	.114	.136	.155	.170	.183	.194	.202
3.0	.031	.062	.090	.115	.137	.155	.171	.184	.195	.203
5.0	.032	.062	.090	.115	.137	.156	.172	.185	.196	.204
10.0	.032	.062	.090	.115	.137	.156	.172	.185	.196	.205

N \ M	1.100	1.200	1.300	1.400	1.500	2.000	2.500	3.000	5.000	10.000
.1	.029	.029	.030	.030	.030	.031	.031	.031	.032	.032
.2	.056	.057	.058	.059	.059	.061	.062	.062	.062	.062
.3	.082	.083	.085	.086	.086	.089	.089	.090	.090	.090
.4	.104	.106	.108	.109	.110	.113	.114	.115	.115	.115
.5	.124	.126	.128	.130	.131	.135	.136	.137	.137	.137
.6	.140	.143	.146	.147	.149	.153	.155	.155	.156	.156
.7	.154	.157	.160	.162	.164	.169	.170	.171	.172	.172
.8	.165	.168	.171	.174	.176	.181	.183	.184	.185	.185
.9	.174	.178	.181	.184	.186	.192	.194	.195	.196	.196
1.0	.181	.185	.189	.191	.194	.200	.202	.203	.204	.205
1.1	.186	.191	.195	.198	.200	.207	.209	.211	.212	.212
1.2	.191	.196	.200	.203	.205	.212	.215	.216	.217	.218
1.3	.195	.200	.204	.207	.209	.217	.220	.221	.222	.223
1.4	.198	.203	.207	.210	.213	.221	.224	.225	.226	.227
1.5	.200	.205	.209	.213	.216	.224	.227	.228	.230	.230
2.0	.207	.212	.217	.221	.224	.232	.236	.238	.240	.240
2.5	.209	.215	.220	.224	.227	.236	.240	.242	.244	.244
3.0	.211	.216	.221	.225	.228	.238	.242	.244	.246	.247
5.0	.212	.217	.222	.226	.230	.240	.244	.246	.249	.249
10.0	.212	.218	.223	.227	.230	.240	.244	.247	.249	.250



(*a*) Square loaded area = *O′ebd*.
For point *O*: use 4 × *Oabc*.
For point *O′*: use *O′ebd*.

(*b*) Rectangle with loaded area = *O′gbd*.
For point *O*: use *Oabc* + *Ocde* + *OeO′f* + *Ofga*.
For point *O′*: use *O′gbd*.

(*c*) Point outside loaded area = *dcfg*.
For point *O*: use *Obfh* − *Obce* − *Oagh* + *Oade*.

(*d*) For loaded area: *kicdgm*.
For point *O*: *Obce* + *Oagf* + *Ofml* + *Olkj* + *Ojib* − *Oade*.

Figure 5-5 Method of using Eq. (5-8) to obtain vertical stress at point indicated.

In general use, and as in the following examples, it is convenient to rewrite Eq. (5-8) as

$$\Delta q = q_o m I_\sigma \qquad (5\text{-}8a)$$

where I_σ is all terms to the right of q_o in Eq. (5-8) as tabulated for selected values of M and N in Table 5-1.

The Boussinesq method for obtaining the stress increase for foundation loads is very widely used for all types of soil masses (layered, etc.) despite it being specifically developed for a semi-infinite, isotropic, homogeneous half-space. Computed stresses have been found to be in reasonable agreement with those few measured values that have been obtained to date.

Example 5-3. Find the stress beneath the center (point *O*) and corner of Fig. 5-5*a* for the following data:

$$B \times B = 2\text{ m} \times 2\text{ m} \qquad Q = 800\text{ kN}$$
$$\text{At corner} \quad z = 2\text{ m}$$
$$\text{At center for} \quad z = 0, 1, 2, 3, \text{ and } 4\text{ m}$$

Solution. It is possible to use Table 5-1; however, program SMNMWEST (B-3) on your diskette is used here for convenience (Table 5-1 is used to check the programming).

1. For the corner at $z = 2$ m

$$M = 2/2 = N = 1 \qquad \text{giving the table factor } 0.175 = I_\sigma$$

$$\Delta q = q_o m(0.175) = \frac{800}{2 \times 2} \times 1 \times 0.175 = \mathbf{35}\ \text{kPa}$$

2. For the center $B' = 2/2 = 1$; $L' = 2/2 = 1$ and with $m = 4$ contributions; for $M = N = \infty$ use 10.

z	M	N	Δq, kPa
0	∞	∞	$200 \times 0.250 \times 4 =$ **200 kPa***
1	1	1	$200 \times 0.175 \times 4 =$ **140**
2	0.5	0.5	$200 \times 0.084 \times 4 =$ **67**
3	0.333	0.333	$200 \times 0.045 \times 4 =$ **36**
4	0.25	0.25	$200 \times 0.027 \times 4 =$ **22**

*at $z = 0$, $\Delta q = 800/(2 \times 2) = 200$ kPa

////

Example 5-4. Find the stress at point O of Fig. 5-5c if the loaded area is square, with $dg = dc = 4$ m, $ad = 1$ m, and $ed = 3$ m for $q_o = 400$ kPa and depth $z = 2$ m.

Solution. From the figure the stress I_σ is the sum of $Obfh - Obce - Oagh + Oade$, and $m = 1$.

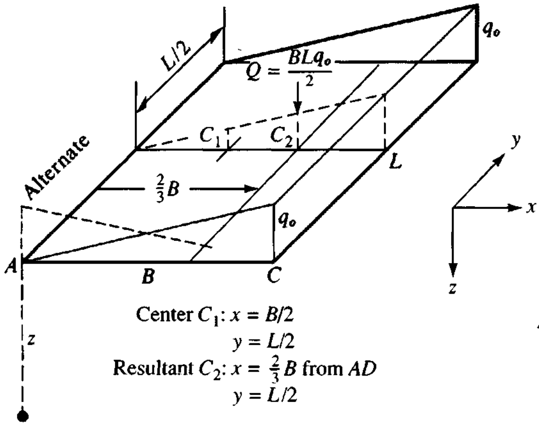
For	M	N	I_σ
Obfh	5/2	7/2	+0.243
Obce	1/2	7/2	−0.137
Oagh	3/2	5/2	−0.227
Oade	1/2	3/2	+0.131
			$I_\sigma =$ **+0.010**

$$q_v = 400(1)(0.010) = \mathbf{4}\ \text{kPa}$$

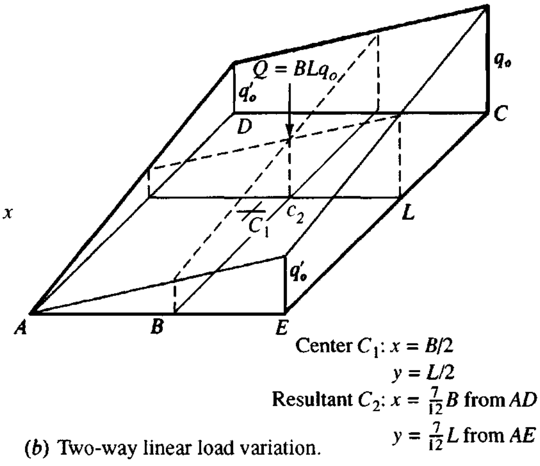
////

5-4 SPECIAL LOADING CASES FOR BOUSSINESQ SOLUTIONS

On occasion the base may be loaded with a triangular or other type of load intensity. A number of solutions exist in the literature for these cases but should generally be used with caution if the integration is complicated. The integration to obtain Eq. (5-8) is substantial; however, that equation has been adequately checked (and with numerical integration using program SMBWVP on your program diskette) so it can be taken as correct. Pressure equations for triangular loadings (both vertical and lateral) are commonly in error so that using numerical procedures and superposition effects is generally recommended where possible. Equations for the cases of Fig. 5-6 have been presented by Vitone and Valsangkar (1986) seem to be correct since they give the same results as from numerical methods. For Fig. 5-6a we have



(*a*) One-way linear load variation. intensity = q_o



(*b*) Two-way linear load variation. intensity = q_o; $q'_o = q_o/2$

Figure 5-6 Special Boussinesq loading cases. Always orient footing for B and L as shown (B may be > or < L).

At point A,

$$\Delta q = \frac{q_o L}{2\pi B}\left(\frac{z}{R_L} - \frac{z^3}{R_B^2 R_D}\right) \tag{5-9}$$

At point C,

$$\Delta q = \frac{q_o L}{2\pi B}\left\{\frac{zR_D}{R_L^2} - \frac{z}{R_L} + \frac{B}{L}\sin^{-1}\left(\frac{BL}{(B^2L^2 + R_D^2 z^2)^{1/2}}\right)\right\} \tag{5-10}$$

For Fig. 5-6*b* (there is a limitation on the intermediate corners that $q'_o = q_o/2$), we have

At point A,

$$\Delta q = \frac{q_o}{4\pi}\left\{\frac{L}{B}\left(\frac{z}{R_L} - \frac{z^3}{R_D R_B^2}\right) + \frac{B}{L}\left(\frac{z}{R_B} - \frac{z^3}{R_D R_L^2}\right)\right\} \tag{5-11}$$

At point C,

$$q = \frac{q_o}{4\pi}\left\{\frac{L}{B}\left(\frac{zR_D}{R_L^2} - \frac{z}{R_L}\right) + \frac{B}{L}\left(\frac{zR_D}{R_B^2} - \frac{z}{R_B}\right) + 2\sin^{-1}\left(\frac{BL}{(B^2L^2 + R_D^2 z^2)^{1/2}}\right)\right\} \tag{5-12}$$

where $R_B^2 = B^2 + z^2$

$R_L^2 = L^2 + z^2$

$R_D^2 = B^2 + L^2 + z^2$

These equations can be checked by computing the stresses at A and C and summing. The sum should equal that at any depth z for a rectangular uniformly loaded base. This check is illustrated in Example 5-5.

Example 5-5. Given the footing example in the ASCE Journal of Geotechnical Engineering Division, vol. 110, No. 1, January 1984, p. 75 (which has an error), find the vertical pressure beneath the corners A and C at $z = 10$ ft. This footing is $L = 8$ ft $\times$ $B = 6$ ft with a linearly varying load from 0 at A to 1 ksf at C across the 6-ft width.

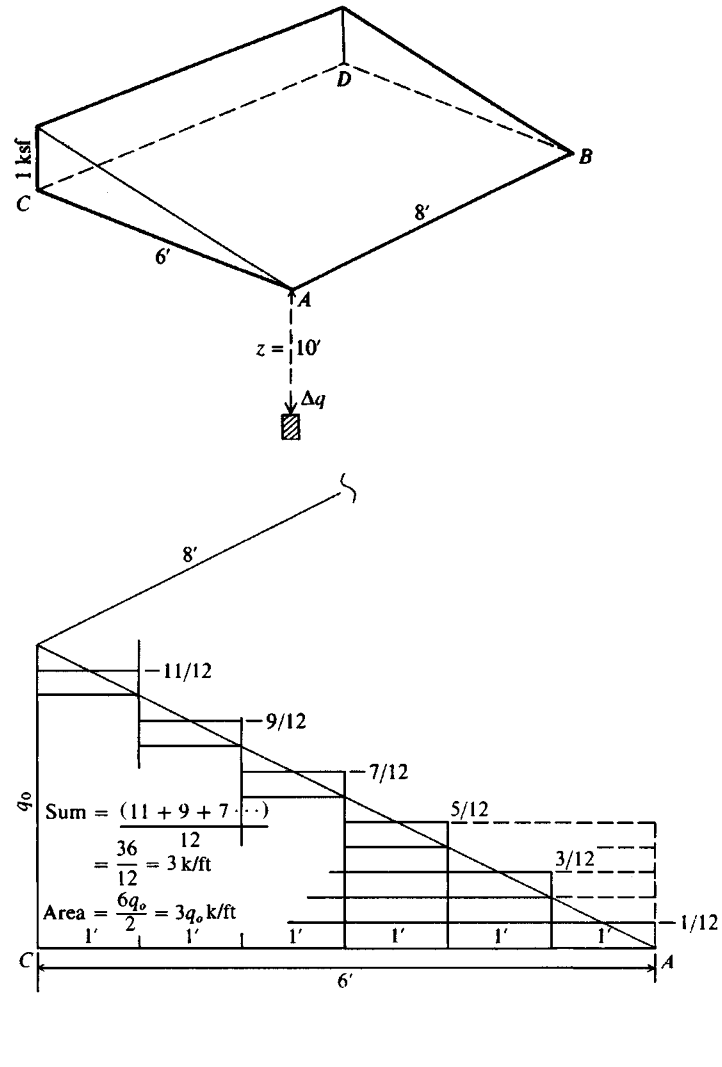


Figure E5-5

Solution. We will use the Newmark Eq. (5-8) and check it using Eqs. (5-9) and (5-10). For the Newmark method, draw the side view of the footing as shown in Fig. E5-5 and step the load intensity, so we have a series of strips loaded uniformly with the intensity fraction shown. The first strip is 1 ft $\times$ 8 ft, the second 2 ft $\times$ 8 ft, etc., so that we will have to subtract from strips after the first a fraction of the previous strip load to obtain the net strip contribution to the point at depth $z = 10$ ft. We will find the stresses at both A and C and use the sum as a check since it can be readily seen that the sum is exactly equivalent to a uniform load of 1 ksf on the footing. Note that I_σ = constant but load intensity varies going from A to C and from C to A. A table will be convenient (again refer to Fig. E5-5):

			For point A(A to C)		For point C(C to A)	
Strip No.	$M = B/z$	$N = L/z$	I_σ	$\Delta q = q_o I_o$		$\Delta q = q_o I_o$
1	1/10	8/10	0.0257 × 1/12 − 0.000	= 0.002 14	× 11/12 − 0	= 0.023 6
			↘ + 3/12		↘ 0.0257 × 9/12	
2	2/10	8/10	0.0504 × 3/12 − 0.006 425	= 0.006 18	× 9/12 − 0.019 28	= 0.018 5
			↘ + 5/12		↘ 0.0504 × 7/12	
3	3/10	8/10	0.0730 × 5/12 − 0.021 0	= 0.009 42	× 7/12 − 0.029 40	= 0.013 18
			↘ + 7/12		↘ 0.0730 × 5/12	
4	4/10	8/10	0.0931 × 7/12 − 0.042 58	= 0.011 73	× 5/12 − 0.030 42	= 0.008 37
			↘ + 9/12		↘ 0.0931 × 3/12	
5	5/10	8/10	0.1103 × 9/12 − 0.069 83	= 0.012 90	× 3/12 − 0.023 28	= 0.004 30
			↘ + 11/12		↘ 0.1103 × 1/12	
6	6.10	8/10	0.1245 × 11/12 − 0.101 11	= 0.013 20	× 1/12 − 0.009 19	= 0.001 20
				Total Δq = 0.055 56 ksf		Total Δq = 0.069 13 ksf

Summing, we have at A and C = 0.055 56 + 0.069 13 = 0.124 69 ksf. A uniform load of 1 ksf gives $\Delta q_a = \Delta q_c = 0.1247$ ksf based on Table 5-1 at $M = 0.6, N = 0.8$. Using Eq. (5-9), we have $R_D = 14.14$; $R_B^2 = 136$; $R_L^2 = 164$ and by substitution of values we obtain $\Delta q = 0.055\,36$ ksf for point A and 0.069 33 ksf for point C.

////

Example 5-6. Let us assume that we are to redo Example 5-5. We do not have access to the Newmark methodology or Eq. (5-9) but do have access to Eq. (5-4). From the data given in Example 5-5 we have $B = 6$ ft; $L = 8$ ft; and depth $z = 10$ ft. We are to use Fps units consistent with both the reference and Example 5-5.

Solution. Referring to Fig. 5-6*a*, we see the *center of the resultant* is at

$$x = \frac{2}{3}B = \frac{2}{3}\cdot 6 = 4.0\text{ ft} \qquad y = \frac{L}{2} = \frac{8}{2} = 4.0\text{ ft} \qquad z = 10\text{ ft}$$

$$R_A = \sqrt{x^2 + y^2 + z^2} = \sqrt{4^2 + 4^2 + 10^2} = 11.489\text{ ft (to corner } A)$$

$$R_C = \sqrt{2^2 + 4^2 + 10^2} = 10.954\text{ ft (to corner } C)$$

$$Q = BLq_o/2 = (6)(8)(1)/2 = \mathbf{24}\text{ kips}$$

From Eq. (5-4) we have

$$q_v = \frac{3Qz^3}{2\pi R^5}$$

Separating terms and computing $3Qz^3/2\pi$, we find

$$\frac{3 \cdot 24 \cdot 10^3}{2\pi} = 11\,459.129$$

$$q_{vA} = 11\,459.129/11.489^5 = 0.0572 \text{ ksf}$$
$$q_{vC} = 11\,459.129/10.954^5 = 0.0727 \text{ ksi}$$

The results from Example 5-5 and this example are next compared:

	Point *A*	Point *B*
Boussinesq [Eq. (5-4)]	0.0572	0.0727 ksf
Example 5-5	0.0566	0.0691
Difference	0.0016	0.0035

Refer to Table E5-6 for a complete comparison of pressure profiles. For the computational purist some of the differences shown in Table E5-6 are substantial, but may be adequate—even conservative—for design purposes in an engineering office—and certainly the point load equation [Eq. (5-4)] is the easiest of all methods to use.

TABLE E5-6

Comparison of stress values from the Boussinesq point load equation (Eq. 5-4) and Eq. (5-4) converted to a numerical format using program SMBWVP

Refer to example Fig. E5-5 for location of points *A* and *C*.

	Points for Boussinesq equation		Points for numerical method	
z, ft	*A*	*C*	*A*	*C*
0.0	0.0000	1.0000*	0.0000	1.0000*
2.0	0.0118	0.0325	0.0479	0.1972
4.0	0.0459	0.0943	0.0710	0.1527
6.0	0.0649	0.1055	0.0730	0.1168
8.0	0.0650	0.0907	0.0654	0.0895
10.0†	0.0572	0.0727	0.0555	0.0691
12.0	0.0482	0.0575	0.0463	0.0546
14.0	0.0401	0.0459	0.0384	0.0437
16.0	0.0333	0.0371	0.0321	0.0355
18.0	0.0279	0.0304	0.0270	0.0293

*Not from computations but known value.

†Depth used in Examples 5-5 and 5-6.

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A SIMPLE METHOD FOR ALL SPECIAL LOADING CASES. Example 5-6 illustrates that when the load pattern is difficult (for example, a base covered with an uneven pile of material producing a nonuniform load), the following procedure is adequate for design:

1. Locate the load resultant as best you can so critical footing locations such as corners, the center, and so forth can be located using x, y coordinates with respect to the load resultant.
2. For the case of depth $z = 0$, use the computed contact pressure as your best estimate. You must do this since $z = 0$ computes a value of $q_v = 0$ or undefined (∞) in Eq. (5-4).
3. For depth $z > 0$ compute the value R and use Eq. (5-4). For cases where $R < z$, Eq. (5-4) will not give very good values but may be about the best you can do. In Example 5-6 note that R is not much greater than z, but the answers compare quite well with the known values.
4. Consider using Table E5-6 as a guide to increase proportionately your Boussinesq pressures, as computed by Eq. (5-4), to approximate more closely the "exact" pressure values obtained by the numerical method. For example you actually have an $R = 2.11$ m (which corresponds exactly to the 4.0 ft depth on Table E5-6, so no interpolation is required), and you have a computed $q_{v,\text{comp}} = 9.13$ kPa at point A. The "corrected" (or at least more nearly correct) q_v can be computed as follows:

$$q_v = \frac{q_{v,\text{nm}}}{q_{v,b}} \times q_{v,\text{comp}}$$

where $q_{v,\text{nm}}$ = vertical pressure from numerical method (most correct)

$q_{v,b}$ = vertical pressure from Boussinesq Eq. (5-4)

so in our case above, we have

$$q_v = \frac{0.0710}{0.0459} \times 9.13 = 1.54 \times 9.13 = \mathbf{14.13}\ \text{kPa}$$

Pressures at other depth points would be similarly scaled. You might note that at the depth of 3.05 m (10 ft) the ratio is 0.0555/0.0572 = **0.970** at point A.

5-5 WESTERGAARD'S METHOD FOR COMPUTING SOIL PRESSURES

When the soil mass consists of layered strata of fine and coarse materials, as beneath a road pavement, or alternating layers of clay and sand, some authorities are of the opinion the Westergaard (1938) equations give a better estimate of the stress q_v.

The Westergaard equations, unlike those of Boussinesq, include Poisson's ratio μ, and the following is one of several forms given for a point load Q:

$$q_v = \frac{Q}{2\pi z^2} \frac{\sqrt{a}}{[a + (r/z)^2]^{3/2}} \tag{5-13}$$

where $a = (1 - 2\mu)/(2 - 2\mu)$ and other terms are the same as in the Boussinesq equation. We can rewrite this equation as

$$q_v = \frac{Q}{z^2} A_w \tag{5-13a}$$

as done for the Boussinesq equation. For $\mu = 0.30$ we obtain the following values:

r/z	0.000	0.100	0.200	0.300	0.400	0.500	0.750	1.000	1.500	2.000
A_w	0.557	0.529	0.458	0.369	0.286	0.217	0.109	0.058	0.021	0.010

Comparing the Boussinesq values A_b from Eq. (5-5), we see that generally the Westergaard stresses will be larger. This result depends somewhat on Poisson's ratio, however, since $\mu = 0$ and $r/z = 0.0$ gives $A_w = \mathbf{0.318}$ (versus $A_b = 0.477$); for $\mu = 0.25$ and $r/z = 0.0$ obtain $A_w = \mathbf{0.477}$.

Similarly as for the Boussinesq equation [Eq. (*a*) and using Fig. 5-2*b*] we can write

$$q = \frac{q_o\sqrt{a}}{2z^2}\int_0^A \left[a + \left(\frac{r}{z}\right)^2\right]^{-3/2} (2r)\,dr$$

After integration we have the direct solution for round footings analogous to Eq. (5-6):

$$q = q_o\left(1 - \sqrt{\frac{a}{(r/z)^2 + a}}\right) \tag{5-14}$$

From some rearranging and using the (+) root,

$$\frac{r}{z} = +\sqrt{\frac{a}{(1 - q/q_o)^2} - a}$$

If this equation is solved for selected values of Poisson's ratio and incremental quantities of q/q_0, as was done with the Boussinesq equation, values to plot a Westergaard influence chart may be computed. Since the Westergaard equation is not much used, construction of an influence chart (done exactly as for the Boussinesq method but for a given value of μ) is left as an exercise for the reader.

If use of the Westergaard equations is deemed preferable, this is an option programmed into SMBWVP on your diskette. For programming, the integration of stresses for a rectangle of $B \times L$ gives the following equation [used by Fadum (1948) for his stress charts] for the corner of a rectangular area (and programmed in SMNMWEST) as

$$q_v = \frac{q_o}{2\pi}\tan^{-1}\left(\frac{MN}{\sqrt{a}(M^2 + N^2 + a)^{1/2}}\right) \tag{5-15}$$

where M, N are previously defined with Eq. (5-8) and a has been defined with Eq. (5-13). The $\tan^{-1}$ term is in radians. This equation can be readily used to obtain a vertical stress profile as for the Boussinesq equation of Eq. (5-8) for rectangular and round (converted to equivalent square) footings. To check programming, use the following table of values:

μ	M	N	I_σ
0.45	1.0	1.0	0.1845
0.45	1.0	0.5	0.1529

At $z = 0$ we have a discontinuity where we arbitrarily set $\Delta q = q_o$ for any base-on-ground location.

5-6 IMMEDIATE SETTLEMENT COMPUTATIONS

The settlement of the corner of a rectangular base of dimensions $B' \times L'$ on the surface of an elastic half-space can be computed from an equation from the Theory of Elasticity [e.g., Timoshenko and Goodier (1951)] as follows:

$$\Delta H = q_o B' \frac{1-\mu^2}{E_s}\left(I_1 + \frac{1-2\mu}{1-\mu} I_2\right) I_F \qquad (5\text{-}16)$$

where q_o = intensity of contact pressure in units of E_s

B' = least lateral dimension of contributing base area in units of ΔH

I_i = influence factors, which depend on L'/B', thickness of stratum H, Poisson's ratio μ, and base embedment depth D

E_s, μ = elastic soil parameters—see Tables 2-7, 2-8, and 5-6

The influence factors (see Fig. 5-7 for identification of terms) I_1 and I_2 can be computed using equations given by Steinbrenner (1934) as follows:

$$I_1 = \frac{1}{\pi}\left[M \ln \frac{(1+\sqrt{M^2+1})\sqrt{M^2+N^2}}{M(1+\sqrt{M^2+N^2+1})} + \ln \frac{(M+\sqrt{M^2+1})\sqrt{1+N^2}}{M+\sqrt{M^2+N^2+1}}\right] \qquad (a)$$

$$I_2 = \frac{N}{2\pi}\tan^{-1}\left(\frac{M}{N\sqrt{M^2+N^2+1}}\right) \quad (\tan^{-1} \text{ in radians}) \qquad (b)$$

where $M = \dfrac{L'}{B'}$

Figure 5-7 Influence factor I_F for footing at a depth D. Use actual footing width and depth dimension for this D/B ratio. Use program FFACTOR for values to avoid interpolation.

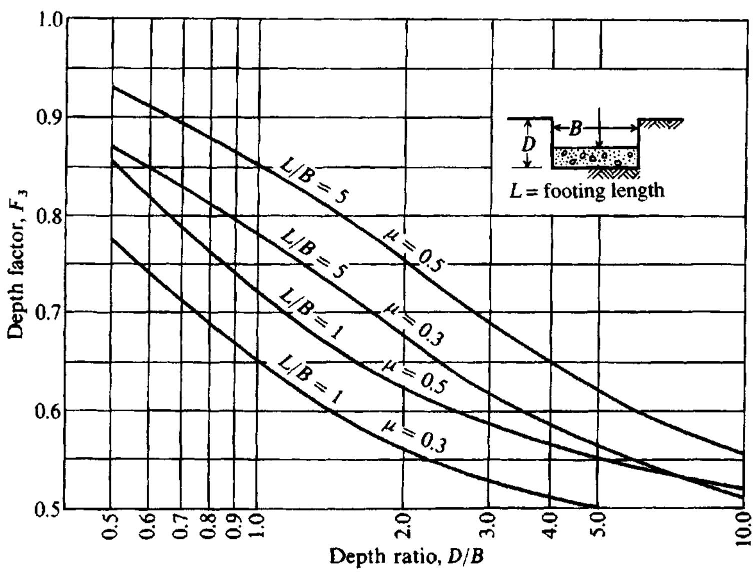

TABLE 5-2

Values of I_1 and I_2 to compute the Steinbrenner influence factor I_s for use in Eq. (5-16a) for several $N = H/B'$ and $M = L/B$ ratios

N	M = 1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.2	I_1 = 0.009	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007
	I_2 = 0.041	0.042	0.042	0.042	0.042	0.042	0.043	0.043	0.043	0.043	0.043
0.4	0.033	0.032	0.031	0.030	0.029	0.028	0.028	0.027	0.027	0.027	0.027
	0.066	0.068	0.069	0.070	0.070	0.071	0.071	0.072	0.072	0.073	0.073
0.6	0.066	0.064	0.063	0.061	0.060	0.059	0.058	0.057	0.056	0.056	0.055
	0.079	0.081	0.083	0.085	0.087	0.088	0.089	0.090	0.091	0.091	0.092
0.8	0.104	0.102	0.100	0.098	0.096	0.095	0.093	0.092	0.091	0.090	0.089
	0.083	0.087	0.090	0.093	0.095	0.097	0.098	0.100	0.101	0.102	0.103
1.0	0.142	0.140	0.138	0.136	0.134	0.132	0.130	0.129	0.127	0.126	0.125
	0.083	0.088	0.091	0.095	0.098	0.100	0.102	0.104	0.106	0.108	0.109
1.5	0.224	0.224	0.224	0.223	0.222	0.220	0.219	0.217	0.216	0.214	0.213
	0.075	0.080	0.084	0.089	0.093	0.096	0.099	0.102	0.105	0.108	0.110
2.0	0.285	0.288	0.290	0.292	0.292	0.292	0.292	0.292	0.291	0.290	0.289
	0.064	0.069	0.074	0.078	0.083	0.086	0.090	0.094	0.097	0.100	0.102
3.0	0.363	0.372	0.379	0.384	0.389	0.393	0.396	0.398	0.400	0.401	0.402
	0.048	0.052	0.056	0.060	0.064	0.068	0.071	0.075	0.078	0.081	0.084
4.0	0.408	0.421	0.431	0.440	0.448	0.455	0.460	0.465	0.469	0.473	0.476
	0.037	0.041	0.044	0.048	0.051	0.054	0.057	0.060	0.063	0.066	0.069
5.0	0.437	0.452	0.465	0.477	0.487	0.496	0.503	0.510	0.516	0.522	0.526
	0.031	0.034	0.036	0.039	0.042	0.045	0.048	0.050	0.053	0.055	0.058
6.0	0.457	0.474	0.489	0.502	0.514	0.524	0.534	0.542	0.550	0.557	0.563
	0.026	0.028	0.031	0.033	0.036	0.038	0.040	0.043	0.045	0.047	0.050
7.0	0.471	0.490	0.506	0.520	0.533	0.545	0.556	0.566	0.575	0.583	0.590
	0.022	0.024	0.027	0.029	0.031	0.033	0.035	0.037	0.039	0.041	0.043
8.0	0.482	0.502	0.519	0.534	0.549	0.561	0.573	0.584	0.594	0.602	0.611
	0.020	0.022	0.023	0.025	0.027	0.029	0.031	0.033	0.035	0.036	0.038
9.0	0.491	0.511	0.529	0.545	0.560	0.574	0.587	0.598	0.609	0.618	0.627
	0.017	0.019	0.021	0.023	0.024	0.026	0.028	0.029	0.031	0.033	0.034
10.0	0.498	0.519	0.537	0.554	0.570	0.584	0.597	0.610	0.621	0.631	0.641
	0.016	0.017	0.019	0.020	0.022	0.023	0.025	0.027	0.028	0.030	0.031
20.0	0.529	0.553	0.575	0.595	0.614	0.631	0.647	0.662	0.677	0.690	0.702
	0.008	0.009	0.010	0.010	0.011	0.012	0.013	0.013	0.014	0.015	0.016
500.0	**0.560**	0.587	0.612	0.635	0.656	0.677	0.696	0.714	0.731	0.748	0.763
	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001

TABLE 5-2

Values of I_1 and I_2 to compute the Steinbrenner influence factor I_s for use in Eq. (5-16*a*) for several $N = H/B'$ and $M = L/B$ ratios *(continued)*

N		M = 2.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.2	I_1 =	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
	I_2 =	0.043	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044
0.4		0.026	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
		0.074	0.075	0.075	0.075	0.076	0.076	0.076	0.076	0.076	0.076	0.076
0.6		0.053	0.051	0.050	0.050	0.050	0.049	0.049	0.049	0.049	0.049	0.049
		0.094	0.097	0.097	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098
0.8		0.086	0.082	0.081	0.080	0.080	0.080	0.079	0.079	0.079	0.079	0.079
		0.107	0.111	0.112	0.113	0.113	0.113	0.113	0.114	0.114	0.114	0.114
1.0		0.121	0.115	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
		0.114	0.120	0.122	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.125
1.5		0.207	0.197	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
		0.118	0.130	0.134	0.136	0.137	0.138	0.138	0.139	0.140	0.140	0.140
2.0		0.284	0.271	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
		0.114	0.131	0.136	0.139	0.141	0.143	0.144	0.145	0.147	0.147	0.148
3.0		0.402	0.392	0.386	0.382	0.378	0.376	0.374	0.373	0.368	0.367	0.367
		0.097	0.122	0.131	0.137	0.141	0.144	0.145	0.147	0.152	0.153	0.154
4.0		0.484	0.484	0.479	0.474	0.470	0.466	0.464	0.462	0.453	0.451	0.451
		0.082	0.110	0.121	0.129	0.135	0.139	0.142	0.145	0.154	0.155	0.156
5.0		0.553	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.519	0.519
		0.070	0.098	0.111	0.120	0.128	0.133	0.137	0.140	0.154	0.156	0.157
6.0		0.585	0.609	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
		0.060	0.087	0.101	0.111	0.120	0.126	0.131	0.135	0.153	0.157	0.157
7.0		0.618	0.653	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
		0.053	0.078	0.092	0.103	0.112	0.119	0.125	0.129	0.152	0.157	0.158
8.0		0.643	0.688	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
		0.047	0.071	0.084	0.095	0.104	0.112	0.118	0.124	0.151	0.156	0.158
9.0		0.663	0.716	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
		0.042	0.064	0.077	0.088	0.097	0.105	0.112	0.118	0.149	0.156	0.158
10.0		0.679	0.740	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
		0.038	0.059	0.071	0.082	0.091	0.099	0.106	0.112	0.147	0.156	0.158
20.0		0.756	0.856	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
		0.020	0.031	0.039	0.046	0.053	0.059	0.065	0.071	0.124	0.148	0.156
500.0		0.832	0.977	1.046	1.102	1.150	1.191	1.227	1.259	1.532	1.721	1.879
		0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.008	0.016	0.031

$$N = \frac{H}{B'}$$

$$B' = \frac{B}{2} \text{ for center; } = B \text{ for corner } I_i$$

$$L' = L/2 \text{ for center; } = L \text{ for corner } I_i$$

The influence factor I_F is from the Fox (1948*b*) equations, which suggest that the settlement is reduced when it is placed at some depth in the ground, depending on Poisson's ratio and L/B. Figure 5-7 can be used to approximate I_F. You can also use Table 5-2, which gives a select range of I_1 and I_2 values, to compute the composite Steinbrenner influence factor I_s as

$$I_s = I_1 + \frac{1 - 2\mu}{1 - \mu} I_2 \tag{c}$$

Program FFACTOR (option 6) can be used to obtain both I_F and I_s directly; you have only to input appropriate base dimensions (actual L, B for I_F and B', L' for I_s) and Poisson's ratio μ.

Equation (5-16) can be written more compactly as follows:

$$\Delta H = q_o B' \frac{1 - \mu^2}{E_s} m I_s I_F \tag{5-16a}$$

where I_s is defined in Eq. (*c*) and m = number of corners contributing to settlement ΔH. At the footing center $m = 4$; at a side $m = 2$, and at a corner $m = 1$. Not all the rectangles have to have the same L'/B' ratio, but for any footing, use a constant depth H.

This equation is strictly applicable to *flexible bases* on the half-space. The half-space may consist of either cohesionless materials of any water content or unsaturated cohesive soils. The soils may be either inorganic or organic; however, if organic, the amount of organic material should be very small, because both E_s and μ are markedly affected by high organic content. Also, in organic soils the foregoing equation has limited applicability since secondary compression or "creep" is usually the predominating settlement component.

In practice, most foundations are flexible. Even very thick ones deflect when loaded by the superstructure loads. Some theory indicates that if the base is rigid the settlement will be uniform (but may tilt), and the settlement factor I_s will be about 7 percent less than computed by Eq. (*c*). On this basis if your base is "rigid" you should reduce the I_s factor by about 7 percent (that is, $I_{sr} = 0.931 I_s$).

Equation (5-16*a*) is very widely used to compute immediate settlements. These estimates, however, have not agreed well with measured settlements. After analyzing a number of cases, the author concluded that the equation is adequate but the method of using it was incorrect. The equation should be used [see Bowles (1987)] as follows:

1. Make your best estimate of base contact pressure q_o.
2. For round bases, convert to an equivalent square.
3. Determine the point where the settlement is to be computed and divide the base (as in the Newmark stress method) so the point is at the corner or common corner of one or up to 4 contributing rectangles (see Fig. 5-7).

TABLE 5-3

Comparison of computed versus measured settlement for a number of cases provided by the references cited.

											Settlement, in.	
Reference	***H*, ft**	***B*, ft**	***L/B***	***D/B***	***N* or q_c**	**E_s, ksf**	**μ**	**Δp, ksf**	**I_s**	**I_f**	**Computed**	**Measured**
D'Appolonia et al. (1968)	4*B*	12.5	1.6	0.5	25*	1,200	0.33	3.4	0.589	0.75	0.33	0.3–0.4
Schmertmann (1970)												
Case 1	5*B*	8.5	8.8	0.78	40	310	0.4	3.74	0.805	0.87	1.45	1.53
Case 2	5*B*	9.8	4.2	1	120	620	0.3	3.34	0.774	0.75	0.67	0.8–0.9
Case 5	5*B*	62	1.0	0	65	350	0.45	1.56	0.50	1.0	2.64	2.48
Case 6	*B*	87	2.2	0.1	90	230	0.3	4.14	0.349	0.98	11.7	10.6
Case 8	5*B*	2	1.0	0.55	18	110	0.3	2.28	0.51	0.6	0.35	0.27
Tschebotarioff (1973)	0.8*B*	90	1.1	0.1	12*	270	0.3	7.2	0.152	0.95	3.9	3.9
Davisson and Salley (1972)	90	124	1	0	12–30*	390	0.3	3.14	0.255	1.0	5.6	5.3
Fischer et al. (1972)	1700	500	1	0.2	—	58 200	0.45	7.0	0.472	0.93	0.50	0.50
Webb and Melvill (1971)	150	177	1	0	—	1,100	0.3	4.5	0.161	1.0	1.27	1.50
Swiger (1974)	4*B*	32	1	0	—	3,900	0.3	2.75	0.493	1.0	0.24	0.24
Kantey (1965)	3.5*B*	20	1	0	50	260	0.3	4.0	0.483	1.0	3.25	3.20

Units: Used consistent with references.

**N* value, otherwise is q_c. Values not shown use other methods for E_s.

Source: Bowles (1987).

4. Note that the stratum depth actually causing settlement is not at $H/B \to \infty$, but is at either of the following:
   a. Depth $z = 5B$ where B = least total lateral dimension of base.
   b. Depth to where a hard stratum is encountered. Take "hard" as that where E_s in the hard layer is about $10E_s$ of the adjacent upper layer.
5. Compute the H/B' ratio. For a depth $H = z = 5B$ and for the center of the base we have $H/B' = 5B/0.5B = 10$. For a corner, using the same H, obtain $5B/B = 5$. This computation sets the depth $H = z$ = depth to use for all of the contributing rectangles. Do not use, say, $H = 5B = 15$ m for one rectangle and $H = 5B = 10$ m for two other contributing rectangles—use 15 m in this case for all.
6. Enter Table 5-2, obtain I_1 and I_2, with your best estimate for μ compute I_s, and obtain I_F from Fig. 5-7. Alternatively, use program FFACTOR to compute these factors.
7. Obtain the *weighted average* E_s in the depth $z = H$. The weighted average can be computed (where, for n layers, $H = \sum_i^n H_i$) as

$$E_{s,\text{av}} = \frac{H_1E_{s1} + H_2E_{s2} + \cdots + H_nE_{sn}}{H} \qquad (d)$$

Table 5-3 presents a number of cases reanalyzed by the author using the foregoing procedure. It can be seen that quite good settlement estimates can be made. Earlier estimates were poor owing to two major factors: One was to use a value of E_s just beneath the base and the other was to use a semi-infinite half-space so that $I_s = 0.56$ (but the I_2 contribution was usually neglected—i.e., $\mu = 0.5$). A curve-fitting scheme to obtain I_s used by Gazetas et al. (1985) appeared to have much promise for irregular-shaped bases; however, using the method for some cases in Table 5-3 produced such poor settlement predictions compared with the suggested method that these equations and computation details are not recommended for use. Sufficient computations are given in Bowles (1987) to allow the reader to reproduce E_s and ΔH in this table.

This method for immediate settlements was also used to compute estimated loads for a set of five spread footings [see Briaud and Gibbens (1994)] for purposes of comparison with reported measured values for a settlement of $\Delta H = 25$ mm. A substantial amount of data was taken using the test methods described in Chap. 3, including the SPT, CPT, PMT, DMT, and Iowa stepped blade. For this text the author elected to use only the CPT method with q_c obtained by enlarging the plots, estimating the "average" q_c by eye for each 3 m (10 ft) of depth, and computing a resulting value using

$$\frac{\sum H_i q_{c,i}}{\sum H_i}$$

It was reported that the sandy base soil was very lightly overconsolidated so the cone constant was taken as 5.5 for all footings except the 1 × 1 m one where the size was such that any soil disturbance would be in the zone of influence. Clearly one can play a numbers game on the coefficient, however, with 3.5 regularly used for normally consolidated soils and from 6 to 30 for overconsolidated soils (refer to Table 5-6), any value from 4 to 7 would appear to apply—5.5 is a reasonable average. With these data and using the program FFACTOR on your diskette for the I_F and I_s (which were all 0.505, because the bases were square and

TABLE 5-4
Comparison of computed versus measured spread footing loads for a 25-mm settlement after 30 min of load. Poisson's ratio $\mu = 0.35$ for all cases.

$B \times B$, m	Cone $q_{c,\text{average}}$, kPa	Cone k	$E_S = kq_{c,\text{average}}$, kPa	B', m	Fox I_F	Soil pressure Δq, kPa	Footing load, kN Computed	Footing load, kN Measured
3 × 3	5940	5.5	32 700	1.5	0.872	353	3177	4500
3 × 3	9580	5.5	52 690	1.5	0.892	555	4995	5200
2 × 2	7185	5.5	39 518	1.0	0.836	667	2668	3600
1.5 × 1.5	4790	5.5	26 345	0.75	0.788	629	1415	1500
1 × 1	6706	3.5	23 471	0.50	0.728	909	909	850

Load test data from Briaud and Gibbens (1994)

because we used an effective influence depth of $5B$) factors, Table 5-4 was developed. Any needed Fps values in the original reference were converted to SI.

Example 5-7. Estimate the settlement of the raft (or mat) foundation for the "Savings Bank Building" given by Kay and Cavagnaro (1983) using the author's procedure. Given data are as follows:

$q_o = 134$ kPa $\quad B \times L = 33.5 \times 39.5$ m $\qquad$ measured $\Delta H =$ about 18 mm

Soil is layered clays with one sand seam from ground surface to sandstone bedrock at -14 m; mat at -3 m.

E_s from 3 to 6 m $= 42.5$ MPa $\qquad E_s$ from 6 to 14 m $= 60$ MPa

E_s for sandstone ≥ 500 MPa

Solution. For clay, estimate $\mu = 0.35$ (reference used 0.2). Compute

$$E_{s(\text{average})} = \frac{3 \times 42.5 + 8 \times 60}{11} = 55 \text{ MPa}$$

From base to sandstone $H = 14 - 3 = 11$ m.

$$B' = \frac{33.5}{2} = 16.75 \text{ m (for center of mat)} \rightarrow \frac{H}{B'} = \frac{11}{16.75} = 0.66 \text{ (use 0.7)}$$

Interpolating in Table 5-2, we obtain $I_1 = 0.0815$; $I_2 = 0.086$:

$$I_s = 0.0815 + \frac{1 - 2(0.35)}{1 - 0.35}(0.0865) = 0.121$$

$$\frac{D}{B} = \frac{3}{33.5} = 0.09; \text{ use } I_F = 0.95$$

With four contributing corners $m = 4$ and Eq. (5-16a) gives

$$\Delta H = q_o B' \frac{1 - \mu^2}{E_s} 4 I_s I_F$$

$$\Delta H = 134(16.75)\frac{1 - 0.35^2}{55 \times 1000}(4 \times 0.121)(0.95)(1000) = \mathbf{16.5} \text{ mm}$$

(The factor 1000 converts MPa to kPa and m to mm.)

This estimate is rather good when compared to the measured value of 18 mm. If this were made for a semi-infinite elastic half-space (a common practice) we would obtain (using E_s just under the mat)

$$\Delta H = 134\left[16.75\left(\frac{0.878}{42\,500}\right)(4 \times 0.56 \times 0.95 \times 1000)\right] = 98.6 \text{ mm}$$

which is seriously in error. You should study the reference and these computations to appreciate the great difficulty in making settlement predictions and then later trying to verify them—even approximately.

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5-7 ROTATION OF BASES

It is sometimes necessary to estimate the rotation of a base. This is more of a problem with bases subjected to rocking moments producing vibrations (considered in more detail in Chap. 20), however, for static rotations as when a column applies an overturning moment it may be necessary to make some kind of estimate of the rotation.

A search of the literature produced five different solutions—none of which agreed well with any other—for flexible base rotation under moment. On this basis, and because theoretical solutions require full contact of base with the soil and with overturning often the full base area is not in contact, the best estimate would be made using a finite difference solution. The finite difference solution is recommended since the overturning moment can be modeled using statics to increase the node forces on the pressed side and decrease the node forces on the tension side. The average displacement profile across the base in the direction of the overturning effort can be used to obtain the angle of rotation. This computer program is B-19 in the package of useful programs for Foundation Design noted on your diskette.

Alternatively, the footing rotation can be expressed (see Fig. 5-8) as

$$\tan\theta = \frac{1-\mu^2}{E_s}\frac{M}{B^2L}I_\theta \qquad (5\text{-}17)$$

where M = overturning moment *resisted by base dimension B*. Influence values I_θ that may be used for a rigid base were given by Taylor (1967) as in Table 5-5. Values of I_θ for a flexible base are given by Tettinek and Matl (1953, see also Frolich in this reference p. 362). These flexible values are intermediate to those of several other authorities. The rotation spring of

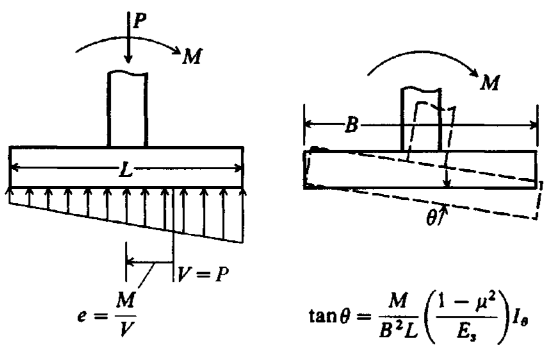

Figure 5-8 Rotation of a footing on an elastic base.

TABLE 5-5
Influence factors I_θ to compute rotation of a footing

L/B	Flexible	Rigid†	
0.1	1.045	1.59	
0.2	1.60	2.42	
0.50	2.51	3.54	
0.75	2.91	3.94	
1.00 (circle)	3.15 (3.00)*	4.17 (5.53)*	For rigid:
1.50	3.43	4.44	$I_\theta = 16/[\pi(1 + 0.22B/L)]$
2.00	3.57	4.59	
3.00	3.70	4.74	
5.00	3.77	4.87	
10.00	3.81	4.98	
100.00	3.82	$5.06 = 16/\pi$	

*For circle B = diameter.

†There are several "rigid" values; these are from equations given by Taylor (1967, Fig. 9, p. 227). They compare reasonably well with those given by Poulos and Davis (1974, p. 169, Table 7.3).

Table 20-2 may also be used to compute base rotation. Most practical footings are intermediate between "rigid" and "flexible" and require engineering judgment for the computed value of footing rotation θ.

Using a computer program such as B19 or the influence factors of Table 5-5 gives a nearly linear displacement profile across the footing length L. This result is approximately correct and will produce the *constant* pressure distribution of Fig. 4-4*a* since the soil will behave similarly to the compression block zone used in concrete beam design using the USD method. In that design the concrete strains are assumed linear but the compression stress block is rectangular. One could actually produce this case using program B19 (FADMATFD) if the nonlinear switch were activated and the correct (or nearly correct) value of maximum linear soil displacement XMAX were used. Enough concrete beam testing has been done to determine that the maximum linear strain is approximately 0.003. Finding an XMAX that would produce an analogous rectangular pressure profile under the pressed part of a footing undergoing rotation would involve trial and error. Making several runs of program B19 with a different XMAX for each trial would eventually produce a reasonable rectangular pressure profile, but this is seldom of more than academic interest. The only XMAX of interest in this type of problem is one that gives

$$q = \text{XMAX} \cdot k_s \leq q_a$$

When this is found the resulting average displacement profile can be used to estimate base and/or superstructure tilt.

Example 5-8.

Given. A rectangular footing with a column moment of 90 kN · m and P = 500 kN. Footing is $3 \times 2 \times 0.5$ m thick. The soil parameters are E_s = 10 000 kPa, μ = 0.30. The concrete column is 0.42×0.42 m and has a length of 2.8 m, and $E_c = 27.6 \times 10^6$ kPa. Estimate the footing rotation

Next Page

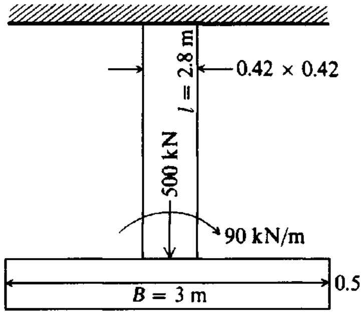

Figure E5-8

and find the footing moment after rotation assuming the upper end of the column is fixed as shown in Fig. E5-8.

Solution.

$$\frac{L}{B} = \frac{2}{3} = 0.67$$

$$I_\theta = 2.8 \text{ (interpolated from Table 5-5, column "Flexible")}$$

$$\tan\theta = \frac{1-\mu^2}{E_s}\frac{M}{B^2L}I_\theta$$

$$\tan\theta = \frac{1-0.3^2}{10\,000}\frac{90}{3^2 2}(2.8)$$

$$\tan\theta = 0.001\,274 \text{ rad}$$

$$\theta = \mathbf{0.073^\circ}$$

From any text on mechanics of materials the relationship between beam rotation and moment (when the far end is fixed, the induced $M' = M/2$) is

$$\theta = \frac{ML}{4EI}$$

from which the moment to cause a column rotation of θ is

$$M = \frac{4EI}{L}\theta$$

The column moment of inertia is

$$I = \frac{bh^3}{12} = \frac{0.42^3}{12} = 2.593 \times 10^{-3} \text{ m}^4$$

Substitution of I, E_c, L, and θ gives the released column moment of

$$M = 4\frac{(27.6 \times 10^6)(2.593 \times 10^{-3})(1.274 \times 10^{-3})}{2.8} = 130 \text{ kN} \cdot \text{m}$$

Since the rotation is equivalent to applying a moment of 130 kN · m opposite to the given M of 90 kN · m, the footing moment is reduced to zero **and the base $\theta \le$ 0.073°**. There is also a change in the "far-end" column moment that is not considered here.

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