TABLE 3

**Definitions and Procedures, Analysis of Beams on Elastic Foundation**

Definitions:

$K_{v_1}$ = Modulus of subgrade reaction for a 1 sq ft bearing plate.

$K_b$ = Modulus of subgrade reaction for beam of width b, $K_b = (K_{v_1})/b$

y = Deflection of beam at a point.

p = Pressure intensity on the subgrade at a point, $p = y(K_b)$

b = Width of beam at contact surface

I = Moment of inertia of beam

E = Modulus of elasticity of beam material

$\ell$ = Beam length

$\lambda$ = Characteristics of the system of beam and supporting soil =

$$\lambda = \sqrt[4]{\frac{K_b b}{4\,EI}}$$

Procedure for Analysis:

1. Determine E and establish $K_{v_1}$ from Figure 6 in DM-7.01, Chapter 5 or from plate bearing tests.

2. Determine depth of beam from shear requirements at critical section and width from allowable bearing pressure. Compute characteristic $\lambda$ of beam and supporting soil.

3. Classify beams in accordance with relative stiffness into the following three groups. Analysis procedure differs with each group.

   Group 1 - Short beams: $\lambda\ell < \pi/4$. Beam is considered rigid. Assume linear distribution of foundation contract pressure as for a rigid footing. Compute shear and moment in beam by simple statics.

TABLE 3 (continued)

Definitions and Procedures, Analysis of Beams on Elastic Foundation

Group 2 - Beams of medium length: $\pi/4 < \lambda\ell < \pi$. End conditions influence all sections of the beam. Compute moments and shears throughout the beam length by the infinite beam formulas, top panel of Figure 10. Determine in this way the shear and moments at the two ends of the beam. By superposing on the loaded beam two pairs of concentrated forces and moments at the ends of the beam, solutions for the infinite beam are modified to conform to the actual end conditions. For example, if Q = 0 and M = 0 at the ends of a free-ended beam, apply redundant shear and moment at the ends equal and opposite to that detemined from the infinite beam formulas. See reference cited in text for formulas for moments and shears in end loaded beam of finite length.

Group 3 - Long beams: $\lambda\ell > \pi$ . End condition at distant end has negligible influence on moment and shear in the interior of the beam. Consider beam as extending an infinite distance away from loaded end. Compute moment and shear caused by interior loads by formulas for infinite beam, top panel of Figure 10. Compute moment and shear for loads applied near the beam ends by formulas for semi-infinite beam. bottom panel of Figure 10. Superpose moment and shear obtained from the two load systems.

4. Obtain functions $A_{\lambda x}$ , $B_{\lambda x}$ , $C_{\lambda x}$ , $D_{\lambda x}$ , for use in formulas of Figure 10 from Figure 11.

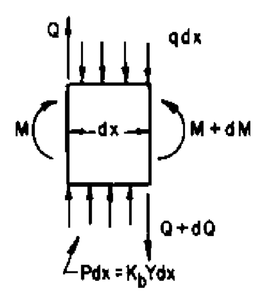
Sign Convention:

Consider infinitely small element of beam between two vertical cross sections at a distance $d_x$ apart.

+Q = Upward acting shear force to left of section.

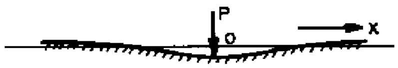
+M = Clockwise movement acting from the left to the section.

+y = Downward delection.



## INFINITE BEAM

### CONCENTRATED LOAD




DEFLECTION: $y = \frac{P\lambda}{2K} A_{\lambda x}$

MOMENT: $M = \frac{P}{4\lambda} C_{\lambda x}$

SHEAR: $Q = -\frac{P}{2} D_{\lambda x}$

### APPLIED MOMENT

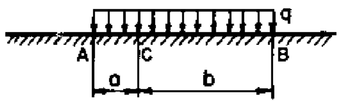

DEFLECTION: $y = \frac{M_o \lambda^2}{K} B_{\lambda x}$

MOMENT: $M = \frac{M_o}{2} D_{\lambda x}$

SHEAR: $Q = -\frac{M_o \lambda}{2} A_{\lambda x}$

### UNIFORMLY DISTRIBUTED LOAD

POINT C IS UNDER LOAD

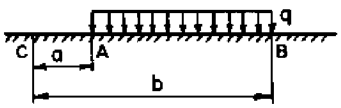



DEFLECTION: $y_C = \frac{q}{2K}(2 - D_{\lambda a} - D_{\lambda b})$

MOMENT: $M_C = \frac{q}{4\lambda^2}(B_{\lambda a} + B_{\lambda b})$

SHEAR: $Q_C = \frac{q}{4\lambda}(C_{\lambda a} - C_{\lambda b})$

POINT C IS LEFT OF LOAD

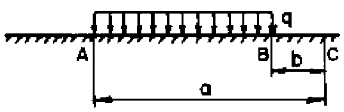



DEFLECTION: $y_C = \frac{q}{2K}(D_{\lambda a} - D_{\lambda b})$

MOMENT: $M_C = -\frac{q}{4\lambda^2}(B_{\lambda a} - B_{\lambda b})$

SHEAR: $Q_C = \frac{q}{4\lambda}(C_{\lambda a} - C_{\lambda b})$

POINT C IS RIGHT OF LOAD




DEFLECTION: $y_C = -\frac{q}{2K}(D_{\lambda a} - D_{\lambda b})$

MOMENT: $M_C = \frac{q}{4\lambda^2}(B_{\lambda a} - B_{\lambda b})$

SHEAR: $Q_C = \frac{q}{4\lambda}(C_{\lambda a} - C_{\lambda b})$

## SEMI-INFINITE BEAM

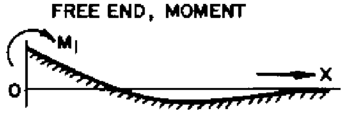
### FREE END, CONCENTRATED LOAD


DEFLECTION: $y = \frac{2P_1\lambda}{K} D_{\lambda x}$

MOMENT: $M = -\frac{P_1}{\lambda} B_{\lambda x}$

SHEAR: $Q = -P_1 C_{\lambda x}$

### FREE END, MOMENT

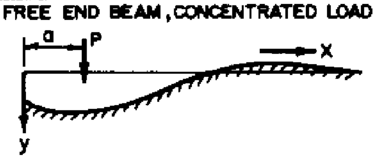



DEFLECTION: $y = -\frac{2M_1\lambda^2}{K} C_{\lambda x}$

MOMENT: $M = M_1 A_{\lambda x}$

SHEAR: $Q = -2M_1 \lambda B_{\lambda x}$

### FREE END BEAM, CONCENTRATED LOAD NEAR END




DEFLECTION: $y = \frac{P\lambda}{2K}\left[(C_{\lambda a} + 2D_{\lambda a})A_{\lambda x} - 2(C_{\lambda a} + D_{\lambda a})B_{\lambda x} + A_{\lambda(a+x)}\right]$

IF NOTATION $(C_{\lambda a} + 2D_{\lambda a}) = \alpha$ AND $(C_{\lambda a} + D_{\lambda a}) = \beta$ IS USED

MOMENT: $M = \frac{P}{4\lambda}\left\{\alpha C_{\lambda x} - 2\beta D_{\lambda x} + C_{\lambda(a-x)}\right\}$

SHEAR: $Q = -\frac{P}{2}\left\{\alpha D_{\lambda x} - \beta A_{\lambda x} \pm D_{\lambda(a-x)}\right\}$

FIGURE 10
Computation of Shear, Moment, and Deflection, Beams on Elastic Foundation

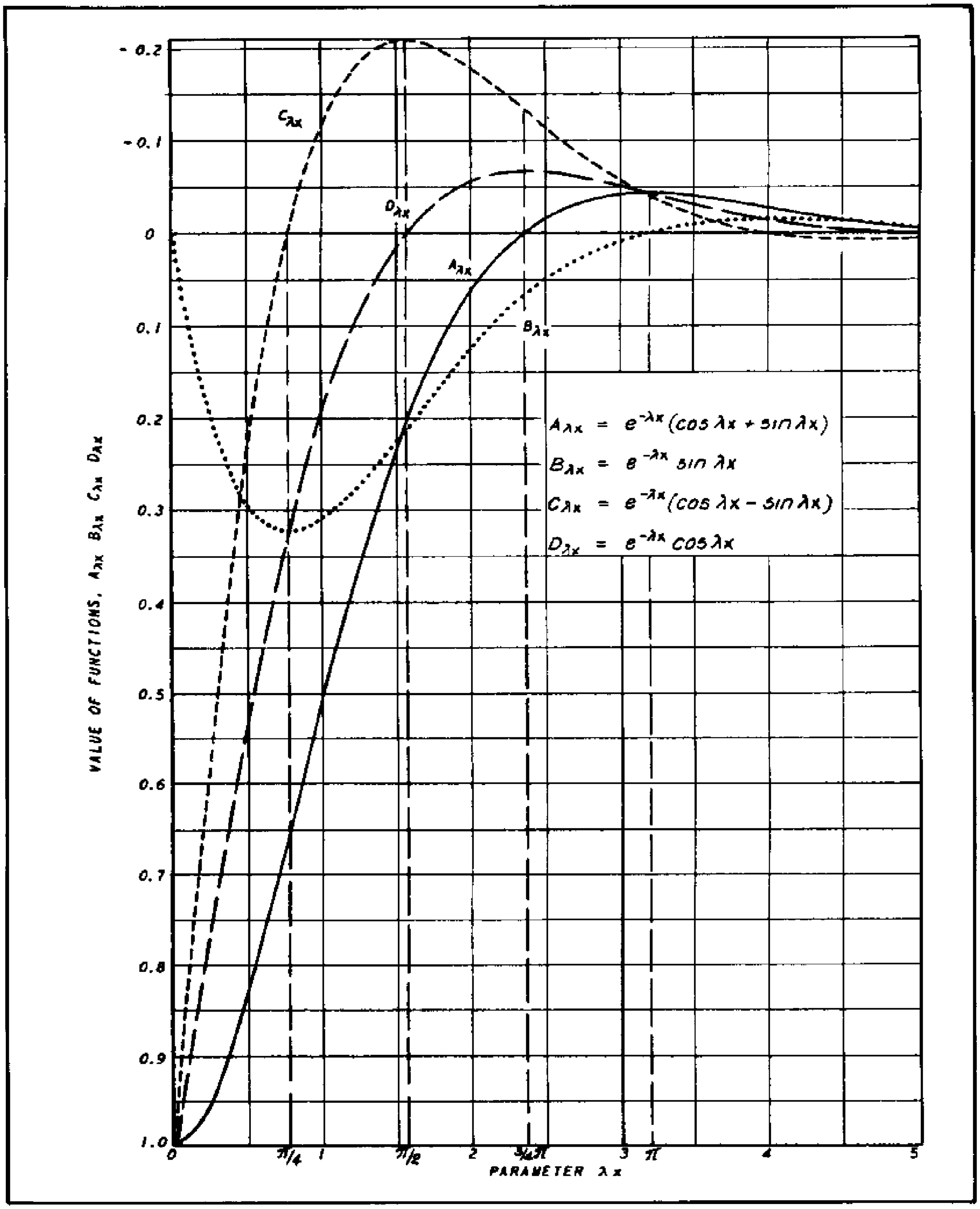



FIGURE 11

Functions for Shear, Moment, and Deflection, Beams on Elastic Foundations