

# 10

## Bearing Capacity of Shallow Foundations on Soil

### 10 Bearing Capacity of Shallow Foundations on Soil

#### 10.1 Introduction

One possible ultimate limit state of a shallow foundation involves the case where the applied loads exceed the resistance of the ground beneath the foundation. The geotechnical resistance at this ultimate limit state is termed the ultimate bearing capacity of the ground that supports the foundation. The ultimate bearing capacity depends on the strength of the ground, ground conditions (e.g., thickness and presence of weak layers, depth to bedrock), and the nature of applied loading (e.g., vertical, horizontal and inclined forces; moments). Methods to estimate the ultimate bearing capacity of shallow foundations on fine- and coarse-grained soils are presented in this Chapter. Other possible ultimate limit states for shallow foundations may include sliding, overturning and general slope stability and their influence on foundation design need to be assessed for each individual project. The serviceability limit state of the foundation is considered separately from the ultimate limit state, as presented in Chapter 11. Shallow foundations are those constructed on or embedded near the ground surface such that the distance from the ground surface to the underside of the foundation is not greater than the width (or least plan dimension) of the foundation.

#### 10.2 Conventional Bearing Capacity

##### 10.2.1 Bearing Capacity Equation

The ultimate bearing capacity (i.e. the geotechnical bearing resistance at the ultimate limit state) of a shallow foundation on uniform soil as shown in Figure 10.1 with shear strength parameters  $c$  and  $\phi$  may be calculated from:

$$q_u = c N_c S_c + q_s N_q S_q + \frac{1}{2} \gamma B N_\gamma S_\gamma \quad (10.1)$$

where:

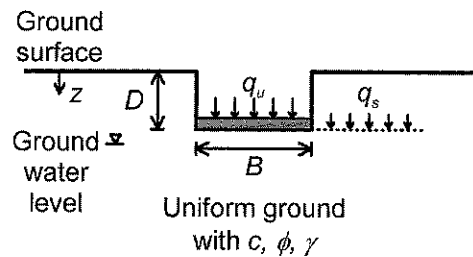
- $q_u$  = ultimate bearing capacity (denoted as  $R_n$  in limit states design—see Section 8.4),
- $N_c, N_q, N_\gamma$  = dimensionless bearing capacity factors (see 10.2.3),
- $S_c, S_q, S_\gamma$  = dimensionless modification factors for foundation shape, inclination, depth and tilt and ground slope (see 10.2.4),
- $q_s$  = vertical stress acting at the elevation of the base of foundation (see 10.2.2),
- $B$  = width of foundation or least plan dimension of the foundation,
- $c$  = soil cohesion (see 10.2.2),
- $\gamma$  = soil unit weight (see 10.2.6).

Unless otherwise noted, any consistent set of units may be used for the parameters in Equation 10.1.

Equation 10.1 expresses the ultimate bearing capacity of a foundation experiencing general shear failure as the sum of: the shear resistance of a weightless material with cohesive strength parameter  $c$  ( $N_c$  term), the shear resistance

of a frictional but weightless material with angle of friction  $\phi'$  on addition of a surcharge  $q_s$  at the foundation level ( $N_q$  term), and the shear resistance of a frictional material with angle of friction  $\phi'$  and weight  $\gamma$  but no surcharge ( $N_\gamma$  term).

Shear strength parameters  $c$  and  $\phi'$  are normally selected within depth  $B$  beneath the base of the foundation.



**FIGURE 10.1** Definition of geometry and parameters for ultimate bearing capacity of a shallow foundation

### 10.2.2. Undrained and Drained Conditions

The values of  $c$  and  $\phi'$  for use in the general bearing capacity equation (Equation 10.1) depend on the type of soil and whether short-term (undrained) or long-term (drained) conditions are being examined. The short-term stability of a foundation involving fine-grained soils can be calculated by taking  $c$  equal to the undrained shear strength,  $s_u$ , and  $\phi = 0$ . The long-term stability of a foundation can be obtained with  $c$  equal to the effective cohesion intercept,  $c'$ , and  $\phi'$  equal to the effective angle of internal friction of the soil,  $\phi'$ . In most cases, short-term stability controls design, especially for soft to very stiff clays.

The surcharge  $q_s$  for use in the general bearing capacity also depends on whether undrained or drained conditions are being considered. For undrained conditions  $q_s$  is the total vertical stress acting adjacent to the base of the foundation; whereas, for drained conditions it is equal to the vertical effective stress and consequently will be influenced by the position of the groundwater level (see Section 10.2.6).

### 10.2.3 Bearing Capacity Factors

Bearing capacity factors have been derived based on modified plasticity solutions for uniform ground conditions. Bearing capacity factors  $N_c$  and  $N_q$  have been reported by Meyerhof (1963), Hansen (1970) and Vesic (1975) to be equal to:

$$N_c = (N_q - 1) \cot \phi \quad (10.2)$$

$$N_q = e^{\pi \tan \phi} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \quad (10.3)$$

Several formulations of the bearing capacity factor  $N_\gamma$  are available (Terzaghi, 1943; Meyerhof, 1963; Hansen, 1970; Vesic, 1975) but tend to overestimate  $N_\gamma$  when compared with the more rigorous plasticity solution of Davis and Booker (1971). An approximate value of  $N_\gamma$  suitable for  $\phi' > 10^\circ$  obtained from Davis and Booker (1971) is:

$$N_\gamma \cong 0.0663 e^{0.1623 \phi} \quad (10.4)$$

for a smooth interface between the foundation and the ground, while for a rough interface it equals:

$$N_\gamma \cong 0.0663 e^{0.1623 \phi} \quad (10.5)$$

where  $\phi$  is in degrees.

For the case of undrained stability ( $c = s_u$ ,  $\phi' = 0$ ) the bearing capacity factors become:

$$N_c = (2 + \pi) \quad (10.6)$$

$$N_q = 1 \text{ and} \quad (10.7)$$

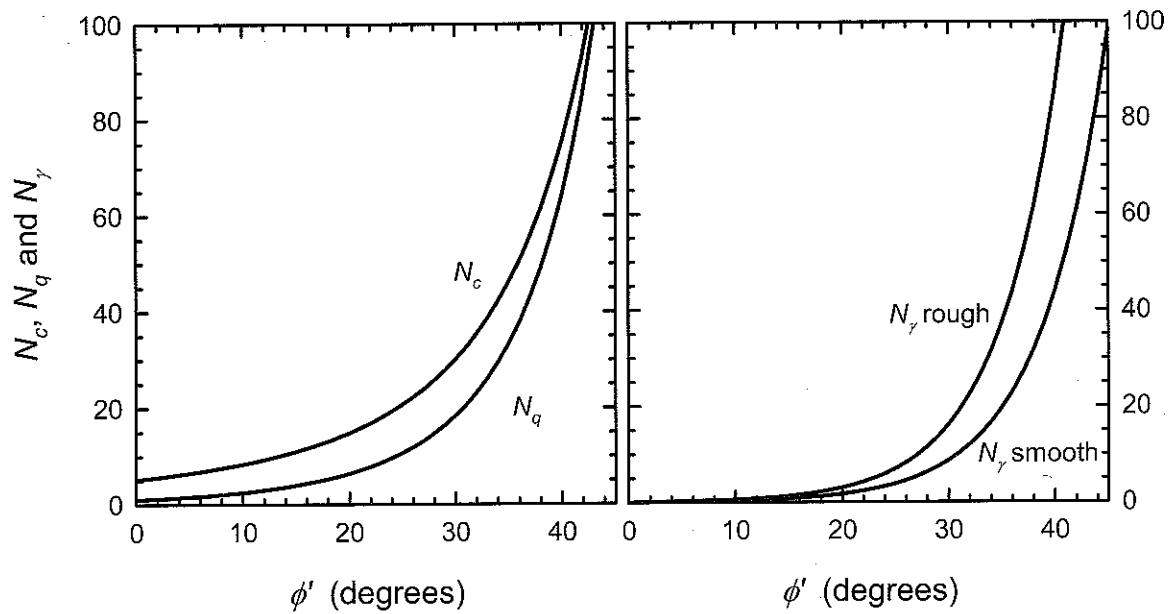
$$N_\gamma = 0 \quad (10.8)$$

Bearing capacity factors  $N_c$ ,  $N_q$ , and  $N_\gamma$  for uniform ground conditions are presented in Table 10.1 and plotted in Figure 10.2.

**TABLE 10.1** Bearing capacity factors  $N_c$  and  $N_q$  from Meyerhof (1963) and  $N_\gamma$  from Davis and Booker (1971)

$\phi^\circ$	$N_c$	$N_q$	$N_\gamma$ rough	$N_\gamma$ smooth
0	5.1	1	0	0
10	8.3	2.5	0.6	0.3
15	11	3.9	1.3	0.8
20	15	6.4	3.0	1.7
21	16	7.1	3.6	2.0
22	17	7.8	4.2	2.4
23	18	8.7	5.0	2.8
24	19	9.6	5.9	3.3
25	21	11	7.0	3.8
26	22	12	8.2	4.5
27	24	13	9.7	5.3
28	26	15	11	6.2
29	28	16	14	7.3
30	30	18	16	8.6
31	33	21	19	10
32	35	23	22	12
33	39	26	27	14
34	42	29	31	17
35	46	33	37	19
36	51	38	44	23
37	56	43	52	27
38	61	49	61	32
39	68	56	73	37
40	75	64	86	44

Small (2001) and Poulos et al. (2001) present useful summaries of bearing capacity factors for soils with an increase in strength with depth, finite depth, fissured clays, layered soils, and foundations near slopes.



**FIGURE 10.2** Bearing capacity factors  $N_c$  and  $N_q$  from Meyerhof (1963) and  $N_\gamma$  from Davis and Booker (1971)

#### 10.2.4 Modification Factors

The bearing capacity factors were derived for the case of strip footing on a level base subjected to loading perpendicular to the foundation. Deviations from these conditions can be accounted for, where appropriate, by factors to modify the bearing capacity factors for the effects of foundation shape ( $S_{cs}$ ,  $S_{qs}$  and  $S_\gamma$ ), load inclination ( $S_{ci}$ ,  $S_{qi}$  and  $S_{\gamma i}$ ), foundation depth ( $S_{cd}$ ,  $S_{qd}$  and  $S_{\gamma d}$ ), surface slope ( $S_{c\beta}$ ,  $S_{q\beta}$  and  $S_{\gamma\beta}$ ) and foundation tilt ( $S_{c\delta}$ ,  $S_{q\delta}$  and  $S_{\gamma\delta}$ ) via:

$$S_c = S_{cs} S_{ci} S_{cd} S_{c\beta} S_{c\delta} \quad (10.9)$$

$$S_q = S_{qs} S_{qi} S_{qd} S_{q\beta} S_{q\delta} \quad (10.10)$$

$$S_\gamma = S_{\gamma s} S_{\gamma i} S_{\gamma d} S_{\gamma\beta} S_{\gamma\delta} \quad (10.11)$$

where expressions for the various modification factors are given in Table 10.2 based on Vesic (1975).

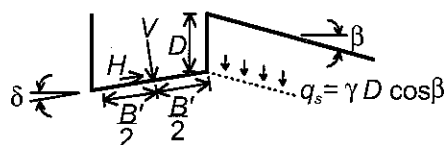
**TABLE 10.2** Modification Factors for General Bearing Capacity Equation (based on Vesic, 1975)

Factor	$S_c$	$S_q$	$S_\gamma$
Foundation shape, $s$	$S_{cs} = 1 + \frac{B' N_q}{L' N_c}$	$S_{qs} = 1 + \frac{B'}{L'} \tan \phi$	$S_{\gamma s} = 1 - 0.4 \frac{B'}{L'}$
Inclined loading, $i^{[1]}$	$\phi = 0, \quad S_{ci} = 1 - \frac{mH}{B'L'cN_c}$ $\phi > 0, \quad S_{ci} = S_{qi} - \frac{1 - S_{qi}}{N_c \tan \phi}$	$S_{qi} = \left(1 - \frac{H}{V + B'L'c \cot \phi}\right)^m$	$S_{\gamma i} = \left(1 - \frac{H}{V + B'L'c \cot \phi}\right)^{m+1}$
Foundation depth, $d^{[2]}$	$\phi = 0, \quad S_{cd} = 1 + 0.4k$ $\phi > 0, \quad S_{cd} = S_{qd} - \frac{1 - S_{qd}}{N_c \tan \phi}$	$S_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 k$	$S_{\gamma d} = 1$

**TABLE 10.2** *Modification Factors for General Bearing Capacity Equation (based on Vesic, 1975) (continued)*

Surface slope, $\beta^{[3]}$	$\phi = 0, \quad S_{c\beta} = 1 - \frac{2\beta}{\pi + 2}$ $\phi > 0, \quad S_{c\beta} = S_{q\beta} - \frac{1 - S_{q\beta}}{N_c \tan \phi}$	$S_{q\beta} = (1 - \tan \beta)^2$	$S_{\gamma\beta} = (1 - \tan \beta)^2 \quad [4]$
Base inclination, $\delta^{[5]}$	$\phi = 0, \quad S_{c\delta} = 1 - \frac{2\delta}{\pi + 2}$ $\phi > 0, \quad S_{c\delta} = S_{q\delta} - \frac{1 - S_{q\delta}}{N_c \tan \phi}$	$S_{q\delta} = (1 - \delta \tan \phi)^2$	$S_{\gamma\delta} = (1 - \delta \tan \phi)^2$

- [1]  $V$  = vertical force;  $H$  = horizontal force;  $m$  depends on direction of inclined loading  $\theta$  relative to long side of the foundation: If force inclined in  $B$  direction ( $\theta=90^\circ$ )  $m = m_B = (2+B/L)/(1+B/L)$ , if inclined in  $L$  direction ( $\theta=0^\circ$ )  $m = m_L = (2+L/B)/(1+L/B)$ , and if inclined at angle  $\theta$  to  $L$  direction  $m = m_\theta = m_L \cos^2 \theta + m_B \sin^2 \theta$ .
- [2]  $k = D/B$  if  $D/B \leq 1$ ;  $k = \tan^{-1}(D/B)$  if  $D/B > 1$ .
- [3]  $\beta$  = inclination below horizontal of the ground surface away from the edge of the foundation (see Figure 10.4); for  $\beta < \pi/4$ ;  $\beta$  in radians.
- [4] For sloping ground case where  $\phi = 0$   $N_\gamma = -2\sin\beta$  must be used in bearing capacity equation.
- [5]  $\delta$  = inclination from the horizontal of the underside of the foundation (see Figure 10.4); for  $\delta < \pi/4$ ;  $\delta$  in radians.



**FIGURE 10.4** *Definition of parameters for shallow foundation with ground slope  $\beta$  and base tilt  $\delta$*

### 10.2.5 Eccentric Forces and Moments

If the foundation is subjected to vertical forces that act eccentric to the centroid of the foundation, the size of the foundation used in the bearing capacity equation should be reduced:

$$B' = B - 2e_B \quad (10.12)$$

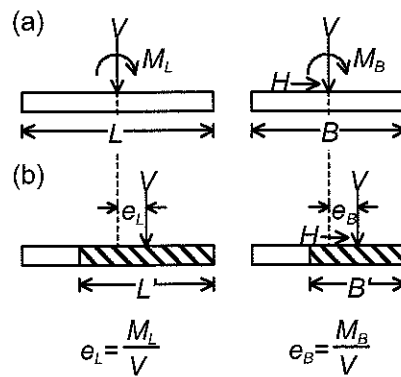
$$L' = L - 2e_L \quad (10.13)$$

where

- $B, L$  = actual foundation dimensions,  
 $B', L'$  = reduced dimensions for use in bearing capacity equation, and  
 $e_B, e_L$  = eccentricities of force in directions  $B$  and  $L$  from the centroid.

This is an approximate but reasonable approach to account for eccentricities provided that the resultant loading acts within the middle third of the foundation (i.e.  $e < B/6$ ). Values of  $B'$  and  $L'$  are to be used in all bearing capacity calculations. The term  $k$  for depth modification factors  $S_{cd}$  and  $S_{qd}$ , and the term  $m$  for load inclination factors  $S_{ci}$ ,  $S_{qi}$  and  $S_{\gamma i}$  as shown in Table 10.2 remain in terms of  $L$  and  $B$ .

Foundations that are subject to moments  $M_B$  and  $M_L$  in the  $B$  and  $L$  directions and vertical load  $V$  acting through the centroid can be treated as an equivalent loading system with vertical load  $V$  acting at eccentricities  $e_B$  and  $e_L$  as shown in Figure 10.3.



**FIGURE 10.3** Shallow foundation subjected to moments and vertical force

### 10.2.6 Influence of Groundwater

The position of the groundwater level will influence the selection of  $\gamma$  and  $q_s$  for use in the general bearing capacity equation when considering drained conditions as summarized in Table 10.3.

**TABLE 10.3** Unit Weight and Surcharge for Drained Conditions in the General Bearing Capacity Equation depending on Depth from Surface to the Groundwater Level  $z$  (as defined in Figure 10.1). The foundation is located at depth  $D$  beneath the ground surface

Depth from surface of groundwater level	Unit weight $\gamma$ for $N_\gamma$ term	Surcharge term $q_s$
$z = 0$	$\gamma_{sub}$	$\gamma_{sub} D$
$z = D$	$\gamma_{sub}$	$\gamma_{bulk} D$
$D < z < D+B$	$\gamma_{sub} + \frac{z-D}{B}(\gamma_{bulk} - \gamma_{sub})$	$\gamma_{bulk} D$
$z > D+B$	$\gamma_{bulk}$	$\gamma_{bulk} D$

The bulk unit weight  $\gamma_{bulk}$  should be selected based on the minimum water content of the soil above the water table. Effective stresses can be introduced into the  $N_\gamma$  term by using the submerged unit weight  $\gamma_{sub}$ , which is equal to:

$$\gamma_{sub} = \gamma_{sat} - \gamma_w \quad (10.14)$$

where

$\gamma_{sat}$  is the saturated unit weight and  $\gamma_w$  is the unit weight of water.

In all cases in Table 10.3,  $q_s$  is the vertical effective stress adjacent to the foundation at its base.

## 10.3 Bearing Capacity Directly from In-Situ Testing

### 10.3.1 Standard Penetration Test (SPT)

There is no direct relationship between standard penetration test (SPT) resistance  $N$  and the ultimate bearing

capacity. Shear strength parameters for use in the general bearing capacity equation can be estimated from empirical correlations with SPT-N (e.g., Hatanaka and Uchida, 1996; Terzaghi et al., 1996). Empirical design charts relating the design bearing pressure for foundations on sand to SPT-N are available; however, since these are also based on limiting settlement of the foundation they are presented in Section 11.8.1. Such empirical correlations need to be treated with caution and adjusted as appropriate by experience.

### 10.3.2 Cone Penetration Test (CPT)

Shear strength parameters for use in the general bearing capacity equation can be estimated from empirical correlations with cone penetration test (CPT) results (e.g., Lunne et al., 1997). Empirical methods are also available to estimate the ultimate bearing capacity directly from CPT tip resistance  $q_c$ .

For coarse-grained soils:

$$q_u = K_\phi \bar{q}_c \quad (10.15)$$

where

- $K_\phi$  = empirical factor relating ultimate bearing capacity and average CPT tip resistance for coarse-grained soils, and
- $\bar{q}_c$  = average tip resistance over a depth  $B$  beneath the foundation.

Values of  $K_\phi$  depend on soil density and foundation shape and range between 0.16 to 0.3 (Lunne et al., 1997). A value of  $K_\phi = 0.16$  can be used for most cases, recognizing that limiting settlement will generally control foundation design.

For fine grained soils and undrained conditions:

$$q_u = K_{su} \bar{q}_c + \gamma D \quad (10.16)$$

where

- $K_{su}$  = empirical factor relating ultimate bearing capacity and average CPT tip resistance for fine-grained soils, and

all other parameters are as previously defined. Factor  $K_{su}$  ranges from 0.3 to 0.6 depending on foundation shape and embedment, and soil stress history and sensitivity. A value of  $K_{su} = 0.3$  can be conservatively used for most cases.

These empirical correlations need to be treated with caution and adjusted where appropriate based on experience.

### 10.3.3 Pressuremeter and Dilatometer Tests

In-situ tests such as the pressuremeter test (PMT) and flat dilatometer test (DMT) can be used to obtain shear strength parameters for use in the general bearing capacity equation (e.g., Lunne et al., 1989; Marchetti et al., 2001).

### 10.3.4 Plate Load Test

A plate load test, if loaded to failure, can be used to assess the ultimate bearing capacity. In this test a reduced-scale foundation is subjected to load and the deflection is recorded. The plate load test involves the actual ground material beneath the foundation and can be useful to obtain soil parameters and to verify the method of analysis. The general bearing capacity equation can be used to interpret results if ground conditions are homogeneous with depth. Scale effects are important as the results will depend on the size of the reduced-scale foundation relative to the underlying sequence of soil strata. Appropriate engineering judgment must be exercised prior to any extrapolation to larger foundations. An additional disadvantage is the costs required to conduct the tests. As a result, plate load tests may only be appropriate for medium to higher risk projects. The plate load test is also useful in the evaluation of ground

stiffness (e.g., see Sections 7.7.1 and 11.7)

#### 10.4 Factored Geotechnical Bearing Resistance at Ultimate Limit States

Geotechnical resistance at the ultimate limit state is reduced (multiplied by the appropriate geotechnical resistance factor (see Tables 8.1 and 8.2 in Chapter 8) to provide the factored geotechnical bearing resistance for foundation design.

##### 10.4.1 Net Ultimate Bearing Pressure

The ultimate bearing capacity  $q_u$  is the total stress that can be applied at foundation level. If an excavation is made for the foundation, stresses in excess of the original overburden stress at the foundation level contribute to bearing failure. The net bearing capacity is defined as:

$$q_{netu} = q_u - q_{ob} \quad (10.17)$$

where

- $q_{netu}$  = net bearing capacity,
- $q_u$  = ultimate bearing capacity, and
- $q_{ob}$  = total overburden stress removed at foundation level.

There is no possibility of bearing failure if the applied load at the foundation level is equal to that of the excavated soil. This is the basis for the design of what is termed full-compensated (or floating) foundations.

##### 10.4.2 Allowable Bearing Capacity

In a working stress design (WSD) approach (see Chapter 8) all uncertainty is accounted for in one parameter called the global factor of safety against ultimate bearing capacity  $FS$ . The allowable bearing capacity,  $q_{all}$ , that can be applied at the foundation level is:

$$q_{all} = \frac{q_{netu}}{FS} + q_{ob} \quad (10.18)$$

The value of  $FS$  against ultimate bearing capacity of a shallow foundation is normally taken equal to 3 (see Section 8.8 in Chapter 8).

For shallow foundations on the ground surface or neglecting the effect of the excavated ground, the allowable bearing pressure becomes:

$$q_{all} = \frac{q_u}{FS} \quad (10.19)$$

##### 10.4.3 Factored Geotechnical Bearing Resistance

Using the load and resistance factor design (LRFD) approach (see Chapter 8), uncertainty in loads acting on the foundation and the resistance of the foundation are treated separately. Loads acting on the foundation are increased using appropriate factors for live and dead loads, while the geotechnical resistance is decreased using a geotechnical resistance factor  $\Phi$ .

For the bearing resistance of shallow foundations the geotechnical resistance factor  $\Phi$  may be taken to be 0.5 (see Tables 8.1 and 8.2 in Chapter 8).