Auxiliar de Repaso

Pregunta 1

Fashionables is a franchisee of The Limited, the well-known retailer of fashionable clothing. Prior to the winter season, The Limited offers Fashionables the choice of five different colors of a particular sweater design. The sweaters are knit overseas by hand, and because of the leadtimes involved, Fashionables will need to order its assortment in advance of the selling season. As per the contracting terms offered by The Limited, Fashionables will also not be able to cancel, modify or reorder sweaters during the selling season. Demand for each color during the season is Normally distributed with a mean of 500 and a standard deviation of 200. Further, you may assume that the demands for each sweater are independent of those for a different color.

The Limited offers the sweaters to Fashionables at the wholesale price of $40 per sweater, and Fashionables plans to sell each sweater at the retail price of $70 per unit. The Limited delivers orders placed by Fashionables in truckloads at a cost of $2,000 per truckload. The transportation cost of $2,000 is borne by Fashionables. Assume unless otherwise specified that all the sweaters ordered by Fashionables will fit into one truckload. Also assume that all other associated costs, such as unpacking and handling, are negligible.

The Limited does not accept any returns of unsold inventory. However, Fashionables can sell all of the unsold sweaters at the end of the season at the fire-sale prices of $20 each.

a) How many units of each sweater-type should Fashionables order to maximize its expected profit?
b) If Fashionables wishes to ensure a 97.5% in-stock probability, what should its order quantity be for each type of sweater?
c) If Fashionables wishes to ensure a 97.5% fill rate, what should its order quantity be for each type of sweater?

For the next three parts (d – e) assume Fashionables' orders 725 of each sweater.

d) What is Fashionables' expected profit?
e) What is Fashionables' expected fill rate for each sweater?
f) What is the stockout probability for each sweater?
g) Now suppose that The Limited announces that the unit of truckload capacity is 2500 total units of sweaters. If Fashionables orders more than 2500 units in total (actually, from 2501 to 5,000 units in total), it will have to pay for two truckloads. What now is Fashionable's optimal order quantity for each sweater?

Respuesta:

Q 9.4

a) Use Exhibit 9.6. The underage cost is $C_u = 70 - 40 = 30$ and the overage cost is $C_o = 40 - 20 = 20$. The critical ratio is $C_u / (C_o + C_u) = 30/50 = 0.6$. From the *Standard Normal Distribution Function Table*, $\Phi(0.25) = 0.5987$ and $\Phi(0.26) = 0.6026$, so we choose $z = 0.26$. Convert that *z*-statistic into an order quantity $Q = \mu + z \times \sigma = 500 + 0.26 \times 200 = 552$. Note that the cost of a truckload has no impact on the profit maximizing order quantity.

b) Use Exhibit 9.11. We need to find the *z* in the *Standard Normal Distribution Function Table* such that $\Phi(z) = 0.9750$ because $\Phi(z)$ is the in-stock probability. We see that $\Phi(1.96) = 0.9750$, so we choose $z = 1.96$. Convert to $Q = \mu + z \times \sigma = 500 + 1.96 \times 200 = 892$.

c) Use Exhibit 9.10. We first find our target lost sales with the equation $L(z) = (\mu / \sigma) \times (1 - \mathit{Fill\ rate}) = 0.0625$. We see in the *Standard Normal Loss Function Table* that $L(1.14) = 0.0634$ and $L(1.15) = 0.0621$, so we use the round up rule to choose the higher *z*, *z* = 1.15. Now convert to *Q*: $Q = 500 + 1.15 \times 200 = 730$.

d) If 725 units are ordered, then the corresponding *z*-statistic is $z = (Q - \mu)/\sigma = (725 - 500)/200 = 1.13$. We need to evaluate lost sales, expected sales and expected left over inventory before we can evaluate the expected profit. Expected lost sales with the Standard Normal is obtained from the *Standard Normal Loss Function Table*, $L(1.13) = 0.0646$. Expected lost sales is $\sigma \times L(z) = 200 \times 0.0646 = 12.9$. Expected sales is 500 – 12.9 = 487.1. Expected left over inventory is 725 – 487.1 = 237.9. Expected profit is

$$\begin{aligned} \mathit{Expected\ profit} &= (70 - 40) \times 487.1 - (40 - 20) \times 237.9 \\ &= 9855 \end{aligned}$$

So the expected profit per sweater is 9,855. The total expected profit is five times that amount, minus 2000 times the number of truckloads required.

e) The fill rate is $1 - \mathit{Expected\ lost\ sales} / \mu = 1 - 12.9/500 = 97.4\%$.

f) The stockout probability is the probability demand exceeds the order quantity 725, which is $1 - \Phi(1.13) = 12.9\%$.

g) If we order the expected profit maximizing order quantity for each sweater, then that equals $5 \times 552 = 2{,}760$ sweaters. With an order quantity of 552 sweaters expected lost sales is $56.5 = 200 \times L(0.26) = 200 \times 0.2824$, expected sales is 500 – 56.5 = 443.5 and expected left over inventory is 552 – 443.5 = 108.5. Expected profit per sweater is

$$\begin{aligned} \mathit{Expected\ profit} &= (70 - 40) \times 443.5 - (40 - 20) \times 108.5 \\ &= 11{,}135 \end{aligned}$$

Because two truckloads are required, the total profit is then $5\times 11{,}136-2\times 2000=51{,}675$. If we order only 500 units per sweater type, then we can evaluate the expected profit per sweater to be 11,010. Total profit is then $5\times 11{,}010-2000=53{,}050$. Therefore, we are better off just ordering one truckload with 500 sweaters of each type.

Pregunta 2

Teddy Bower is an outdoor clothing and accessories chain that purchases a line of parkas at \$10 each from its Asian supplier, TeddySports. Unfortunately, at the time of order placement, demand is still uncertain: Teddy Bower forecasts that its demand is Normally distributed with mean 2100 and standard deviation 1200. Teddy Bower sells these parkas at \$22 each. Unsold parkas have little salvage value; Teddy Bower simply gives them away to a charity.

a) What is the probability this parka turns out to be a "dog", defined as a product that sells less than half of the forecast?
b) How many parkas should Teddy Bower buy from TeddySports to maximize expected profit?
c) If Teddy Bower wishes to ensure a 98.5% fill rate, how many parkas should Teddy Bower's order?
d) If Teddy Bower wishes to ensure a 98.5% in-stock probability, how many parkas should Teddy Bower's order?

For the next three questions (e-g), assume Teddy Bower orders 3000 parkas.

e) Evaluate Teddy Bower's expected profit.
f) Evaluate Teddy Bower's fill rate.
g) Evaluate Teddy Bower's stock-out probability

Respuesta:

a) The parka sells less than half of the forecast if demand is 2100/2 = 1050 or fewer units. Normalize the quantity 1050: $z=(1050-2100)/1200=-0.88$. From the *Standard Normal Distribution Function Table*, $\Phi(-0.88)=0.1894$, which implies there is a 18.9% probability that the parka will be a dog.
b) To determine the profit maximizing order quantity, begin with the underage cost, $C_u=22-10=12$, and the overage cost, $C_o=10-0=10$. The critical ratio is $12/(10+12)=0.5455$. We see from the *Standard Normal Distribution Function Table* that $\Phi(0.11)=0.5438$ and $\Phi(0.12)=0.5478$, so we choose $z=0.12$. Convert that *z*-statistic back into an order quantity, $Q=\mu+z\times\sigma=2100+0.12\times 1200=2{,}244$.
c) Use Exhibit 9.10. To hit the target fill rate of 98.5% we need to evaluate the target lost sales:

$$L(z) = (\mu / \sigma) \times (1 - \textit{Fill rate}) = (2100/1200) \times (1\text{-}0.985) = 0.0263$$

From the *Standard Normal Loss Function Table* we see that $L(1.54) = 0.0267$ and $L(1.55) = 0.0261$, so we choose z = 1.55. Convert to Q: $Q = 2100 + 1.55 \times 1200 = 3960$.

d) Use Exhibit 9.11. To hit the target in-stock probability of 98.5%, we need to find the z-statistic such that $\Phi(z) = 0.9850$. We see from the *Standard Normal Distribution Function Table* that $\Phi(2.17) = 0.9850$, so we choose $z = 2.17$. Convert to Q: $Q = 2100 + 2.17 \times 1200 = 4704$.

e) If 3000 parkas are ordered then the corresponding z-statistic is $(3000 - 2100)/1200 = 0.75$. Now look up expected lost sales with the Standard Normal distribution in the *Standard Normal Loss Function Table*: $L(0.75) = 0.1312$. Convert that lost sales into the expected lost sales with the actual demand distribution: $\sigma \times L(z) = 1200 \times 0.1312 = 157.4$. Expected sales = expected demand – expected lost sales = 2100 – 157.4 = 1942.6. Expected left over inventory = 3000 – 1942.6 = 1057.4. Finally,

$$\textit{Expected profit} = (22 - 10) \times 1942.6 - (10 - 0) \times 1057.4$$
$$= 12{,}737$$

f) The fill rate equals expected sales divided by expected demand = 1942.6 / 2100 = 92.5%.

g) The stock out probability is $1 - \Phi(z) = 1 - \Phi(0.75) = 22.7\%$

Pregunta 3

Considere una red logística compuesta tan solo por centros de distribución y locales de venta. Identifique al menos tres objetivos conflictivos que pueden amenazarla estabilidad de la relación entre estos agentes.

Sol: A modo de ejemplo, algunos objetivos conflictivos son:

- Tiempos de Entrega: Los locales de venta requieren de flexibilidad en la entrega, en cambio los centros de distribucion prefieren calendarios r?gidos para programar bien sus operaciones.
- Costos de Transporte: Los centros de distribucion desean cargar los mas posible sus camiones debido a la existencia de costos fijos, en cambio, los locales de ventas requieren de entregas en pequeñas cantidades para no generar costos de inventario de los productos.
- Inventario: Los locales de venta quieren administrar el menor inventario posible en tienda, por lo que realizan pedidos pequeños con mucha frecuencia. Los centros de distribucion por su parte desean pedidos grandes y de varios productos a la vez.
- Calidad de Servicio: Los locales de ventas requieren de la entrega de los productos a tiempo para satisfacer la demanda direta de sus clientes, para esto los centros de distribucion deben generar mayores niveles de inventarios, mejorando el servicio pero aumentando sus costos.

¿Cuáles son las ventajas y desventajas de tener un sistema de bodegas con una "Gran" bodega central versus tener varias bodegas "pequeñas" individuales?

Sol: Ventajas: Una gran bodega central permite tener mejor control del inventario total que se tiene. Uno evita problemas de coordinación de inventario y/o costos posicionamiento de stock que ocurririan con muchas bodegas individuales. Ademas de que evita repetir costos fijos como luz, equipos, seguridad y se ganan economias de escala.

Desventajas: Con una bodega habran mayores costos de transporte de bodegas a consumidores finales. Ademas como el nivel de inventario sera mayor habria mayor costo de inventario en la gran bodega.

Muestre que una formulación del VRP con ventanas de tiempo y tiempo de viaje $t_{ij}>0$ no necesita incluir restricciones de eliminación de subtours explícitamente

Sol: Vamos a mostrar que simplemente teniendo variables de llegada de tiempo a los nodos, necesarias para ver si satisfacen ventanas de tiempo, nos permite eliminar subtours. Con lo que incluir las restricciones de eliminación de subtours explicitamente no es necesario.

Supongamos que si hay un subtour, que visita los puntos 1, 2, 3 y luego vuelve a 1. Esto quiere decir que los tiempos de llegada a estos puntos, w_i deben satisfacer lo siguiente:

$$\begin{aligned} w_1 + t_{12} &\leq w_2 \\ w_2 + t_{23} &\leq w_3 \\ w_3 + t_{31} &\leq w_1 \end{aligned}$$

sumando estas tres desigualdades tenemos que $0 < t_{12} + t_{23} + t_{31} \leq 0$ lo que es imposible.

¿Cuál es la relación entre costos fijos y hacer revenue management?

Sol: En general el revenue management es exitoso en situaciones en que los costos fijos son elevados y no es posible producir más (o cambiar la forma de producir) para adecuarse a los cambios en la demanda. Por ejemplo en una linea aerea, es relativamente caro poner aviones adicionales para aumentar el número de asientos para satisfacer una demanda mayor que la estimada. En estas situaciones, uno puede trabajar con los precios para ajustar la demanda a la capacidad existente producto de los costos fijos.

Pregunta 4

Suponga que dispone de m vehiculos homogeneos para satisfacer una demanda en n clientes distribuidos en una ciudad, tal que el tiempo de viaje entre el cliente i y j esta dada por c_{ij} y cada cliente tiene una demanda igual a 1.

1. (0,8 pts) Escriba este problema como un problema de ruteo de vehiculos que busca minimizar el tiempo de viaje total, suponiendo además que todos los vehiculos salen y terminan en una ubicación especial, nodo 0, y que deben completar esta ruta en 10 horas.

2. Muestre como se pude adaptar este modelo para considerar:

   a) (0,4 pts) Costo por demanda insatisfecha.
   b) (0,4 pts) Que la capacidad de cada vehiculo j es K_j.
   c) (0,4 pts) Que el cliente i tiene una demanda d_i que puede ser satisfecha por mas de un camión.

3. En esta parte usaremos una heuristica para resolver el problema planteado en 1.

   a) (0,5 pts) Describa una heurística de vecino mas cercano para encontrar una solución para el problema de la parte 1.
   b) (0,5 pts) Utilizela para encontrar una solución para el problema dado por la siguiente matriz (simetrica) de tiempos de viaje con $m = 2$ y donde la primera linea indica el depot, nodo 0.

$$\begin{array}{cccccc}
0 & 3 & 6 & 4 & 5 & 10 \\
 & 0 & 4 & 2 & 4 & 6 \\
 & & 0 & 7 & 3 & 4 \\
 & & & 0 & 4 & 4 \\
 & & & & 0 & 8
\end{array}$$

4. (Bonus) Muestre la siguiente equivalencia entre restricciones para eliminar subtours en el TSP

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \quad \Leftrightarrow \quad \sum_{i \in S, j \notin S} x_{ij} \geq 1$$