PAUTA P2 C1 - FI2002-4

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(a) Sabemos que el potencial está dado por:

(1p) $$\left\{ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{1} \frac{\lambda\, dr'}{\|\vec{r}-\vec{r}\,'\|} \right. \quad \left(\begin{array}{l} \text{Se usa la convención de que} \\ V(\infty)=0 \end{array} \right)$$

(1p) $\Rightarrow$ donde: $\vec{r} = z\hat{k}$

$\vec{r}\,' = \rho\hat{\rho} \rightarrow$ pero $\rho = R \Rightarrow \vec{r}\,' = R\hat{\rho}$

$dr' = \rho\, d\theta \Rightarrow dr' = R\, d\theta$

$\vec{r}-\vec{r}\,' = z\hat{k} - \rho\hat{\rho}$

$\Rightarrow \|\vec{r}-\vec{r}\,'\| = \sqrt{z^2+\rho^2} = \sqrt{z^2+R^2}$ ($\rho = R$)

$\Rightarrow$ En la fórmula general:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\lambda R\, d\theta}{\sqrt{z^2+R^2}}$$

$$\Rightarrow V(z) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda R \cdot 2\pi}{\sqrt{z^2+R^2}}$$

(1p) $$\Rightarrow \boxed{V(z) = \frac{\lambda R}{2\varepsilon_0\sqrt{z^2+R^2}} = \frac{Q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}}$$

(b) $\cdot\ V_1(z) = \dfrac{Q_1}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$ } potencial anillo (1) = carga Q_1

$\cdot\ V_2(z) = \dfrac{Q_2}{4\pi\varepsilon_0\sqrt{(z-L)^2+R^2}}$ } potencial anillo (2) = carga Q_2

$\vec{r}\,' = R\hat{\rho} + L\hat{k}$, $\vec{r} = z\hat{k}$ } $\vec{r}-\vec{r}\,' = (z-L)\hat{k} - R\hat{\rho}$, $\|\vec{r}-\vec{r}\,'\| = \sqrt{(z-L)^2+R^2}$

②

- LA ENERGÍA QUE "SIENTE" q DEBIDO A Q_1 Y Q_2 ACERCÁNDOSE AL ORIGEN DE Q_1 (CENTRO DE ARGOLLA DE CARGA Q_1) ES:

$$U_1 = \frac{1}{2}\left(q V_1(0) + q V_2(0)\right) \Big\} \text{0.5 p}$$

- LA ENERGÍA QUE "SIENTE" q DEBIDO A Q_1 Y Q_2 ACERCÁNDOSE AL ORIGEN DE Q_2 (CENTRO DE ARGOLLA DE CARGA Q_2) ES:

$$U_2 = \frac{1}{2}\left(q V_1(L) + q V_2(L)\right) \Big\} \text{0.5 p}$$

↳ PERO: $V_1(0) = \dfrac{Q_1}{4\pi\varepsilon_0 R}$; $V_2(0) = \dfrac{Q_2}{4\pi\varepsilon_0 \sqrt{L^2+R^2}}$

$V_1(L) = \dfrac{Q_1}{4\pi\varepsilon_0 \sqrt{L^2+R^2}}$; $V_2(L) = \dfrac{Q_2}{4\pi\varepsilon_0 R}$

⇒ QUEDA QUE:

$$\boxed{\begin{aligned} 2U_1 &= \frac{qQ_1}{4\pi\varepsilon_0 R} + \frac{qQ_2}{4\pi\varepsilon_0\sqrt{L^2+R^2}} \\ 2U_2 &= \frac{qQ_1}{4\pi\varepsilon_0\sqrt{L^2+R^2}} + \frac{qQ_2}{4\pi\varepsilon_0 R} \end{aligned}} \Bigg\} \text{1p}$$

⇒ DESPEJANDO QUEDA QUE:

$$\boxed{\begin{aligned} Q_1 &= \frac{8\pi\varepsilon_0 R}{qL^2}\left(U_1(L^2+R^2) - U_2 R\sqrt{L^2+R^2}\right) \\ Q_2 &= \frac{8\pi\varepsilon_0 R}{qL^2}\left(U_2(L^2+R^2) - U_1 R\sqrt{L^2+R^2}\right) \end{aligned}} \Bigg\} \text{1p}$$

PAUTA P3 C1 FI2002-4

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(a) • $r<a \Rightarrow \boxed{\vec{E}=0}$ (esfera conductora) } 1pt

• $a<r<b \Rightarrow$ Gauss

↳ Asumimos simetría esférica $\Rightarrow \vec{D}=D(r)\hat{r}$

$\Rightarrow d\vec{s} = r^2 \operatorname{sen}\varphi \, d\varphi \, d\theta \, \hat{r}$

$\Rightarrow ds = r^2 \operatorname{sen}\varphi \, d\varphi \, d\theta$

$$\Rightarrow \oint_S \vec{D}\cdot d\vec{s} = Q_{T\,libre} \quad ; \quad Q_T = \int_0^{2\pi}\int_0^{\pi} \sigma_c r^2 \operatorname{sen}\varphi \, d\varphi \, d\theta \Big|_{r=a}$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi} D(r) r^2 \operatorname{sen}\varphi \, d\varphi \, d\theta = \sigma_c a^2 4\pi$$

$$\Rightarrow D(r) r^2 \cancel{4\pi} = \sigma_c a^2 \cancel{4\pi}$$

$$\Rightarrow D(r) = \frac{\sigma_c a^2}{r^2} \quad \sigma_c = \frac{Q}{4\pi a^2}$$

$$\Rightarrow \vec{D}(r) = \frac{Q}{4\pi a^2}\cdot\frac{a^2}{r^2}\hat{r}$$

$$\Rightarrow \vec{D}(r) = \frac{Q}{4\pi r^2}\hat{r}$$

$$\Rightarrow \boxed{\vec{E}(r) = \frac{Q}{4\pi \varepsilon r^2}\hat{r}} \quad \text{1pt}$$

• $r>b \Rightarrow$ lo mismo $\Rightarrow \boxed{\vec{E}(r) = \frac{Q}{4\pi \varepsilon_0} r^2}$ } 1pt

pero $\varepsilon = \varepsilon_0$ pues estamos en el vacío

(b) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$$\Rightarrow \vec{P} = \vec{D} - \varepsilon_0 \vec{E}$$

$$\Rightarrow \vec{P} = \left(\frac{Q}{4\pi r^2} - \frac{\varepsilon_0 Q}{4\pi \varepsilon r^2}\right)\hat{r}$$

$$\left.\Rightarrow \boxed{\vec{P} = \frac{Q}{4\pi r^2}\left(1 - \frac{\varepsilon_0}{\varepsilon}\right)\hat{r}}\right\} \text{0.5pt}$$

Luego: $\sigma_p = \vec{P}\cdot\hat{n}$

$$\bullet\ \sigma_p\Big|_{r=a} = \frac{Q}{4\pi a^2}\left(1 - \frac{\varepsilon_0}{\varepsilon}\right)\hat{r}\cdot(-\hat{r}) = \frac{Q}{4\pi a^2}\left(\frac{\varepsilon_0}{\varepsilon} - 1\right) \Big\} \text{0.5pt}$$

$$\bullet\ \sigma_p\Big|_{r=b} = \frac{Q}{4\pi b^2}\left(1 - \frac{\varepsilon_0}{\varepsilon}\right)\hat{r}\cdot\hat{r} = \frac{Q}{4\pi b^2}\left(1 - \frac{\varepsilon_0}{\varepsilon}\right) \Big\} \text{0.5pt}$$

↳ NOTE QUE: $\rho_p = -\nabla\cdot\vec{P} = 0$ // (NO HAY DENSIDAD VOLUMÉTRICA DE POLARIZACIÓN)

(c) $\bullet\ r < a \Rightarrow \vec{E}(\vec{r}) = 0 \Rightarrow \left(\oint \vec{E}\cdot d\vec{s} = \frac{Q_L}{\varepsilon_0} \Rightarrow \vec{E} = 0\right)$ • IGUAL AL CALCULADO • EN a } 0.5pt

$$\bullet\ r > b \Rightarrow \oint_S \vec{E}\cdot d\vec{s} = \frac{Q_L}{\varepsilon_0} \Rightarrow E(r)\cdot r^2 4\pi = \frac{\sigma_c \cdot a^2 \cdot 4\pi}{\varepsilon_0}$$

$$\Rightarrow \boxed{\vec{E}(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}}$$ ↙ IGUAL AL CALCULADO EN (a) } 0.5pt

$$\bullet\ a < r < b \Rightarrow \oint_S \vec{E}\cdot ds = \frac{Q_L}{\varepsilon_0}; \quad Q_L = Q_c + Q_{pa} \rightarrow Q_{pa} = 4\pi a^2 \cdot \sigma_{pa}$$

↳ $Q_c = 4\pi a^2 \sigma_c$ $\wedge$

$$\Rightarrow Q_{pa} = \cancel{4\pi a^2}\cdot\frac{Q}{\cancel{4\pi a^2}}\left(\frac{\varepsilon_0}{\varepsilon} - 1\right) \wedge Q_c = 4\pi a^2 \cdot \frac{Q}{4\pi a^2}$$

$$Q_{pa} = \frac{Q\varepsilon_0}{\varepsilon} - Q \wedge Q_c = Q$$

$$\Rightarrow \oint \vec{E}\cdot d\vec{s} = \frac{Q_L}{\varepsilon_0} \Rightarrow E(r)\, r^2 \cdot 4\pi = \frac{1}{\varepsilon_0}\left(\frac{Q\varepsilon_0}{\varepsilon} - Q + Q\right) = \frac{Q}{\varepsilon}$$ } 0.5pt

$$\Rightarrow \boxed{\vec{E}(r) = \frac{Q}{4\pi\varepsilon r^2}\hat{r}}$$ IGUAL AL DE LA PARTE (a)