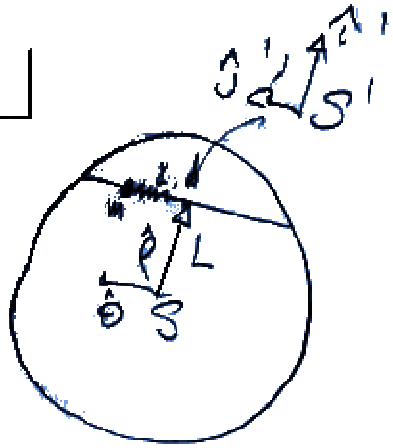
P1

Pauta C3

a)



$$\vec{r}_0 = L\hat{\rho} \qquad \vec{r}\,' = y\hat{\jmath}\,' \qquad \vec{\Omega} = \Omega\hat{k}$$
$$\vec{v}_0 = L\Omega\hat{\theta} \qquad \vec{v}\,' = \dot{y}\hat{\jmath}\,'$$
$$\vec{a}_0 = -L\Omega^2\hat{\rho} \qquad \vec{a}\,' = \ddot{y}\hat{\jmath}\,'$$

$$\hat{\rho} = \hat{\imath}\,' \qquad \hat{\theta} = \hat{\jmath}\,' \qquad \hat{k} = \hat{k}\,'$$

$$\vec{F} = -N_\rho\hat{\rho} - k(y-d)\hat{\jmath}\,'$$

$$\vec{F}_t = 0 \quad (\dot{\vec{\Omega}} = 0)$$

$$\vec{F}_{Cor} = -2m\vec{\Omega}\times\vec{v}\,' = -2m\Omega\dot{y}(\hat{k}\times\hat{\jmath}\,') = 2m\Omega\dot{y}\hat{\imath}\,'$$

$$\vec{F}_{cent} = -m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}\,') = -m\Omega^2 y(\hat{k}\times(\hat{k}\times\hat{\jmath}))$$
$$= -m\Omega^2 y(\hat{k}\times(-\hat{\imath}))$$
$$= m\Omega^2 y\hat{\jmath}\,'$$

b) • $m\vec{a}\,' = \vec{F} - m\vec{a}_0 + \vec{F}_I$

$$\Rightarrow m\ddot{y}\hat{\jmath}\,' = -N_\rho\hat{\imath}\,' - k(y-d)\hat{\jmath}\,' + mL\Omega^2\hat{\imath}\,' + 2m\Omega\dot{y}\hat{\imath}\,' + m\Omega^2 y\hat{\jmath}\,'$$

$\hat{\imath}\,'$) $\quad N_\rho = mL\Omega^2 + 2m\Omega\dot{y} \qquad (1)$

$\hat{\jmath}\,'$) $\quad m\ddot{y} = -k(y-d) + m\Omega^2 y$

$$\Rightarrow \ddot{y} + \left(\frac{k}{m} - \Omega^2\right)y = \frac{k}{m}d \qquad (2)$$

Si $\frac{k}{m} - \Omega^2 > 0$. $\Rightarrow$ mov es armónico simple.

Si $\frac{k}{m} - \Omega^2 = 0 \Rightarrow$ mov es unif. acelerado.

Si $\frac{k}{m} - \Omega^2 = 0 \Rightarrow$ mov es exponencial $(y = Ae^{\lambda t} + Be^{-\lambda t}$; con $\lambda \in \mathbb{R}$

c) Eq. $\Rightarrow \ddot{y}_{eq}=0$ $\Rightarrow$ $\boxed{y_{eq} = \dfrac{\frac{k}{m} d}{\frac{k}{m} - \Omega^2}}$ $\left[\text{Solo si } \frac{k}{m} - \Omega^2 > 0\right]$

d) Definimos $\omega_0^2 \equiv \frac{k}{m} - \Omega^2$

$$\ddot{y} + \omega_0^2 y = \frac{k}{m} d \quad (3)$$

$$y = y_H + y_p$$

Claramente $y_H = A\cos(\omega_0 t + \delta)$

$y_p = a$ cte en (3) $\Rightarrow a = \dfrac{\frac{k}{m} d}{\omega_0^2} = y_{eq}$

$\Rightarrow y = A\cos(\omega_0 t + \delta) + y_{eq}$

$\dot{y} = -A\omega_0 \operatorname{sen}(\omega_0 t + \delta)$

$\dot{y}(0) = 0 \Rightarrow \operatorname{sen}(\delta) = 0 \Rightarrow \boxed{\delta = 0}$

$y(0) = \mathcal{E} + y_{eq} \Rightarrow A\cos(0) + y_{eq} = \mathcal{E} + y_{eq} \Rightarrow \boxed{A = \mathcal{E}}$

$\Rightarrow$ $\boxed{y = \mathcal{E}\cos(\omega_0 t) + y_{eq}}$ //

e) Necesitamos $\dot{y}$ $\Rightarrow$ $\dot{y} = -\mathcal{E}\,\omega_0 \operatorname{sen}(\omega_0 t)$

en (1)

$\Rightarrow$ $\boxed{N_p = mL\Omega^2 - 2m\Omega\mathcal{E}\,\omega_0 \operatorname{sen}(\omega_0 t)}$ //

P2 | órbita parabólica punto $r = R$:

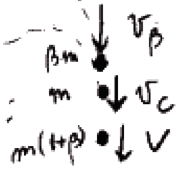
$$E = \frac{1}{2}(\beta m)\, v_\beta^2 - \frac{G(\beta m)M}{R} = 0$$

↑ parábola

$$\Rightarrow \quad v_\beta = \sqrt{\frac{2GM}{R}}$$

órbita circular: Fuerzas:

$$\frac{GmM}{R^2} = \frac{m v_c^2}{R} \Rightarrow v_c = \sqrt{\frac{GM}{R}}$$



choque: conserv. momentum lineal:

$$(\beta m)\, v_\beta + m v_c = m(1+\beta) V$$

$$\Rightarrow V = \frac{1}{1+\beta}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{2GM}{R}} + \sqrt{\frac{GM}{R}}\right) = \frac{2}{1+\frac{1}{\sqrt{2}}}\sqrt{\frac{GM}{R}}$$

energía después del choque

$$E(0+) = \frac{1}{2} m(1+\beta) V^2 - \frac{Gm(1+\beta)M}{R}$$

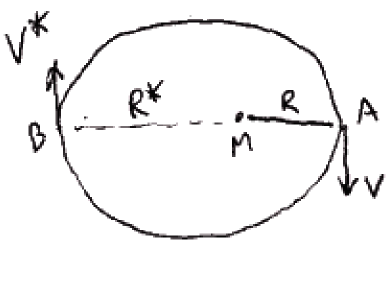
$$= \frac{1}{2} m(1+\beta)\frac{4}{(1+\beta)^2}\frac{GM}{R} - \frac{GmM}{R}(1+\beta)$$

$$= \frac{GmM}{R}\left(\frac{2}{1+\beta} - (1+\beta)\right)$$

$\beta = \frac{1}{\sqrt{2}}$

$$= \frac{GmM}{R}\left(\frac{1-2\beta-\beta^2}{1+\beta}\right) = \frac{GmM}{R}\left(\frac{1-\sqrt{2}-\frac{1}{2}}{1+\frac{1}{\sqrt{2}}}\right) = \frac{GmM}{R}\left(\frac{\frac{1}{2}-\sqrt{2}}{1+\frac{1}{\sqrt{2}}}\right) < 0$$

$E(0^+) < 0 \Rightarrow$ órbita después es elíptica.
sea R^* y V^* el radio y la veloc. en el otro punto de retorno de esta elipse (punto B)



por cons. momento angular: $RV = R^* V^*$

$$\Rightarrow V^* = \left(\frac{R}{R^*} V\right)$$

energía en punto B $= \frac{1}{2} m(1+\beta) V^{*2} - \frac{Gm(1+\beta)M}{R^*}$

= energía en punto A justo después del choque

$$E_B \qquad = \qquad E_A(0+)$$

$$\frac{1}{2} m(1+\beta) V^{*2} - \frac{G m(1+\beta) M}{R^*} = \frac{1}{2} m(1+\beta) V^2 - \frac{G m(1+\beta) M}{R}$$

$$\frac{1}{2} V^{*2} - \frac{GM}{R^*} = \frac{1}{2} V^2 - \frac{GM}{R}$$

$$\frac{1}{2}\left(\frac{R}{R^*}\right)^2 V^2 - \frac{GM}{R^*} = \frac{1}{2} V^2 - \frac{GM}{R}$$

pero $V^2 = \frac{4}{(1+\beta)^2} \frac{GM}{R}$

$$\Rightarrow \frac{2}{(1+\beta)^2}\left(\frac{R}{R^*}\right)^2 \frac{GM}{R} - \frac{GM}{R^*} = \frac{2}{(1+\beta)^2} \frac{GM}{R} - \frac{GM}{R} \quad \Big/ \times \frac{R}{GM}$$

$$\underbrace{\frac{2}{(1+\beta)^2}}_{\gamma}\left(\frac{R}{R^*}\right)^2 - \frac{R}{R^*} - \left(\frac{2}{(1+\beta)^2} - 1\right) = 0$$

$$\gamma x^2 - x - (\gamma - 1) = 0 \longrightarrow \text{una sol. es } x = 1$$

la otra sol.:

$$x = \frac{1 \pm \sqrt{1 + 4\gamma(\gamma - 1)}}{2\gamma} = \frac{1 \pm \sqrt{4\gamma^2 - 4\gamma + 1}}{2\gamma} = \frac{1 \pm (2\gamma - 1)}{2\gamma}$$

$$x_- = \frac{2 - 2\gamma}{2\gamma} = \frac{1}{\gamma} - 1 = \frac{(1+\beta)^2}{2} - 1$$

$$(1+\beta)^2 = 1 + 2\beta + \beta^2 = 1 + \sqrt{2} + \frac{1}{2} = \frac{3}{2} + \sqrt{2}$$

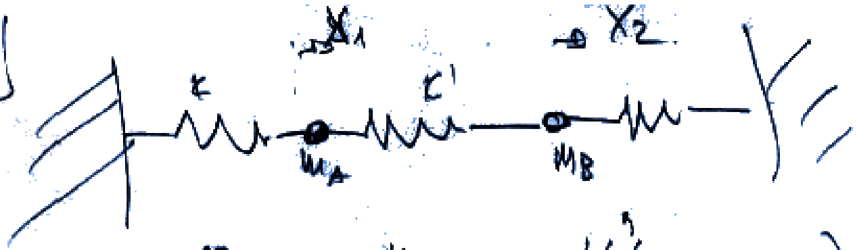
$$x_- = \frac{3}{4} + \frac{\sqrt{2}}{2} - 1 = -\frac{1}{4} + \frac{1}{\sqrt{2}} = \frac{2\sqrt{2} - 1}{4} = \frac{R}{R^*}$$

con esto:

$$\boxed{R^* = \frac{4}{2\sqrt{2} - 1} R}$$

$$\boxed{R^* = 2{,}19\ R}$$

P3



$$m_A \ddot{x}_1 = -k x_1 + k'(x_2 - x_1)$$
$$m_B \ddot{x}_2 = -k'(x_2 - x_1) - k x_2$$

$$\boxed{\begin{aligned} \ddot{x}_1 &= \frac{-(k+k')}{m_A} x_1 + \frac{k'}{m_A} x_2 \\ \ddot{x}_2 &= \frac{k'}{m_B} x_1 - \frac{(k+k')}{m_B} x_2 \end{aligned}}$$

a) $\underline{m_A = \infty} \Rightarrow \ddot{x}_1 = 0$

$$\Rightarrow \ddot{x}_2 = -\frac{(k+k')}{m_B} x_2 \Rightarrow \omega_p^2 = \boxed{\frac{(k+k')}{m_B}}$$

b) $\underline{k = 0} \Rightarrow$

$$\begin{aligned} \ddot{x}_1 &= -\frac{k'}{m_A} x_1 + \frac{k'}{m_A} x_2 = -\omega^2 x_1 \\ \ddot{x}_2 &= \frac{k'}{m_B} x_1 - \frac{k'}{m_B} x_2 = -\omega^2 x_2 \end{aligned}$$

$$\Rightarrow A = \begin{bmatrix} \omega^2 - \frac{k'}{m_A} & \frac{k'}{m_A} \\ \frac{k'}{m_B} & \omega^2 - \frac{k'}{m_B} \end{bmatrix}$$

$$\det(A) = 0 \Rightarrow (\omega^2)^2 - k'\left(\frac{1}{m_A} + \frac{1}{m_B}\right)\omega^2 + \cancel{\frac{k'^2}{m_A m_B}} = \cancel{\frac{k'^2}{m_A m_B}}$$

$$\Rightarrow \omega^2 = 0 \quad o \quad \omega^2 = k'\left(\frac{1}{m_A} + \frac{1}{m_B}\right) = \boxed{\frac{k'}{\mu}}$$

c) $k' = 0$ $\Rightarrow$ Chocante

$\omega_1^2 = \dfrac{k}{m_A}$

$\omega_2^2 = \dfrac{k}{m_B}$

Las partículas están desacopladas

d)
$$A = \begin{bmatrix} \omega^2 - \frac{(k+k')}{m_A} & \frac{k'}{m_A} \\ \frac{k'}{m_B} & \omega^2 - \frac{(k+k')}{m_B} \end{bmatrix}$$

$$\det(A) = 0 \Rightarrow (\omega^2)^2 - (k+k')\left[\frac{1}{m_A} + \frac{1}{m_B}\right]\omega^2 + \frac{(k+k')^2}{m_A m_B} = \frac{k'^2}{m_A m_B}$$

$$(\omega^2)^2 - (k+k')\left[\frac{1}{m_A} + \frac{1}{m_B}\right]\omega^2 + \frac{k^2 + 2kk'}{m_A m_B} = 0$$

$$\frac{1}{2}\left[-b \pm \sqrt{b^2 - 4ac}\right] = \frac{1}{2}\left[\underbrace{\frac{(k+k')}{\mu}}_{\lambda_1} \pm \underbrace{\sqrt{\frac{(k+k')^2}{\mu^2} - \left(4\frac{k^2 + 8kk'}{m_A \cdot m_B}\right)}}_{\lambda_2}\right]$$

$$\omega_1^2 = \frac{\lambda_1 + \lambda_2}{2} \qquad \omega_2^2 = \frac{\lambda_1 - \lambda_2}{2}$$

e) MN $\Rightarrow$ $X_1 = \dfrac{\frac{k'}{m_A}}{\omega_i^2 - \frac{(k+k')}{m_A}} X_2$

$$\Rightarrow \text{Modo } i = \begin{pmatrix} \frac{k'}{m_A} \Big/ \left(\omega_i^2 - \frac{(k+k')}{m_A}\right) \\ 1 \end{pmatrix}$$

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