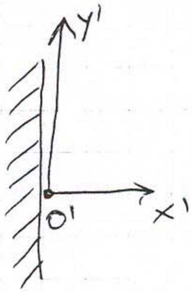
(P1) PAUTA P1 - C3 FI 21A - 2004/2
SEC.01

1/2

$$\vec{\Omega}_0 = \Omega_0 \hat{k}$$

(1)



$$\vec{r}\,' = y'\hat{j}'$$
$$\vec{v} = \dot{y}\,\hat{j}$$
$$\vec{a} = \ddot{y}\,\hat{j}$$

Fuerzas reales: $\vec{N} = N\hat{\imath}$

Fuerzas inerciales:

$$-m\vec{a}_0 = -m\left(D\Omega_0^2\,\hat{\imath}\right)$$

$$-2m\vec{\Omega}_0 \times \vec{v} = -2m\,\Omega_0\hat{k} \times \dot{y}\,\hat{j} = 2m\Omega_0\dot{y}\,\hat{\imath}$$

$$-m\vec{\Omega}_0 \times (\vec{\Omega}_0 \times \vec{r}) = -m\vec{\Omega}_0 \times \left(\Omega_0\hat{k} \times y\,\hat{j}\right)$$
$$= -m\vec{\Omega}_0 \times \Omega_0 y\,(-\hat{\imath}) = m\Omega_0^2\, y\,\hat{j}$$

$$-m\,\vec{\alpha}_0 = 0$$

Ec. de momento

$$X: \qquad 0 = N - mD\Omega_0^2 + 2m\Omega_0\dot{y} \qquad (1)$$

$$Y: \qquad m\ddot{y} = m\,\Omega_0^2\, y \qquad (2)$$

$$(2) \Rightarrow \ddot{y} = \Omega_0^2\, y$$

sea $\dot{y} = a y$

$$\ddot{y} = a\dot{y} = a^2 y$$

$$a = \Omega_0$$

$$\therefore \boxed{\dot{y} = \Omega_0 y}$$

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b) $(1) \Rightarrow \quad N = mD\Omega_0^2 - 2m\Omega_0\dot{y}$

pero $\dot{y} = \Omega_0 y$

$\Rightarrow \quad N = mD\Omega_0^2 - 2m\Omega_0^2 y$

$N = 0 \quad \Rightarrow \quad \boxed{y_* = \frac{D}{2}}$

c) Matemáticamente: $y_* = \frac{D}{2}$ es la única solución $\Rightarrow$ no existe despegue para $y < 0$.

Físicamente: si la partícula se mueve según $-\hat{j}$ tanto la fuerza centrífuga como la fuerza de Coriolis mantienen a la partícula pegada al muro. No se produce separación.

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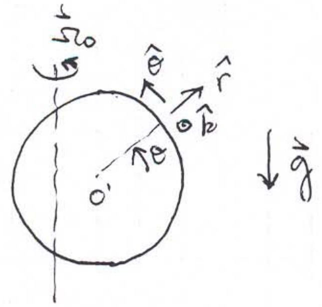
(2)



$\hat{K}$, $\hat{I}$

$$\hat{K} = \operatorname{sen}\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$

$$\hat{I} = \cos\theta\,\hat{r} - \operatorname{sen}\theta\,\hat{\theta}$$

$$m\vec{a}\,' = \vec{F}^{Real} + \vec{F}^{inercial}$$

$$\vec{a}\,' = -R\dot{\theta}^2\,\hat{r} + R\ddot{\theta}\,\hat{\theta}$$

$$\vec{F}^{Reales} = -gm\,(\underbrace{\operatorname{sen}\theta\,\hat{r} + \cos\theta\,\hat{\theta}}_{\hat{K}}) + N_r\,\hat{r} + N_z\,\hat{k}$$

F. inerciales:

$$-m\vec{a}_0 = -m\,\Omega_0^2\frac{R}{2}\,(\overbrace{-\cos\theta\,\hat{r} + \operatorname{sen}\theta\,\hat{\theta}}^{-\hat{I}})$$

$$= m\Omega_0^2\frac{R}{2}\,(\cos\theta\,\hat{r} - \operatorname{sen}\theta\,\hat{\theta})$$

$$-2m\vec{\Omega}_0\times\vec{v}\,' = -2m\Omega_0(\operatorname{sen}\theta\,\hat{r} + \cos\theta\,\hat{r})\times R\dot{\theta}\,\hat{\theta}$$

$$= -2m\Omega_0\operatorname{sen}\theta\,R\dot{\theta}\,\hat{k}$$

$$-m\vec{\Omega}_0\times(\vec{\Omega}_0\times\vec{r}) = -m\vec{\Omega}_0\times\left(\Omega_0(\operatorname{sen}\theta\,\hat{r} + \cos\theta\,\hat{\theta})\times R\,\hat{r}\right)$$

$$= -m\Omega_0(\operatorname{sen}\theta\,\hat{r} + \cos\theta\,\hat{\theta})\times\Omega_0 R\cos\theta\,(-\hat{k})$$

$$= m\Omega_0^2 R\cos\theta\,(\operatorname{sen}\theta\,(-\hat{\theta}) + \cos\theta\,\hat{r})$$

Ec de mto.

$$\hat{r}:\quad -mR\dot{\theta}^2 = -mg\operatorname{sen}\theta + N_r + m\Omega_0^2\frac{R}{2}\cos\theta + m\Omega_0^2 R\cos^2\theta \quad (1)$$

$$\hat{\theta}:\quad mR\ddot{\theta} = -mg\cos\theta - m\Omega_0^2\frac{R}{2}\operatorname{sen}\theta - m\Omega_0^2 R\cos\theta\operatorname{sen}\theta \quad (2)$$

$$\hat{k}:\quad 0 = N_z - 2m\Omega_0 R\dot{\theta}\operatorname{sen}\theta \quad (3)$$

(P2) 2/2

(2) ⇒

$$\boxed{\ddot{\theta} = -\frac{g}{R}\cos\theta - \frac{\Omega_0^2}{2}\operatorname{sen}\theta - \Omega_0^2\operatorname{sen}\theta\cos\theta} \quad (4)$$

integrando entre

$$\theta = \frac{\pi}{2} \quad , \dot{\theta} = 0$$

$$\frac{1}{2}\dot{\theta}^2 = -\frac{g}{R}\operatorname{sen}\theta\Big|_{\pi/2}^{\theta} + \frac{\Omega_0^2}{2}\cos\theta\Big|_{\pi/2}^{\theta} - \frac{\Omega_0^2}{2}\operatorname{sen}^2\theta\Big|_{\pi/2}^{\theta}$$

$$\frac{1}{2}\dot{\theta}^2 = -\frac{g}{R}(\operatorname{sen}\theta - 1) + \frac{\Omega_0^2}{2}(\cos\theta) - \frac{\Omega_0^2}{2}(\operatorname{sen}^2\theta - 1)$$

$$\frac{1}{2}\dot{\theta}^2 = \frac{g}{R}(1 - \operatorname{sen}\theta) + \frac{\Omega_0^2}{2}\cos\theta\left(1 + \cos\theta\right)$$

$$\boxed{\dot{\theta} = \sqrt{2(\ \%\ )}} \qquad (5)$$

b) Fuerzas del aro.

de (5)

(1) ⇒

$$N_r = mg\operatorname{sen}\theta - m\Omega_0^2\frac{R}{2}\cos\theta - m\Omega_0^2 R\cos^2\theta - mR\dot{\theta}^2$$

de (5)

$$N_z = 2m\Omega_0 R\dot{\theta}\operatorname{sen}\theta$$

c) $\dot{\theta} = 0 = \frac{g}{R}(1 - \operatorname{sen}\theta_*) + \frac{\Omega_0^2}{2}\cos\theta_*(1 + \cos\theta_*)$

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SET 01

(3)

a) Tiempo que tarda la nave en su recorrido

$$\tau = \frac{T}{2} = \frac{1}{2}\left(\frac{2\pi}{\sqrt{c}} a^{3/2} \right)$$

a : semi eje mayor :

$$a = \frac{R_T + R_M}{2} = R_T \frac{1+f}{2}$$

$\therefore$ $$\tau = \frac{R_T^{3/2}}{c^{1/2}} \pi \left(\frac{1+f}{2} \right)^{3/2}$$

Necesitamos calcular la velocidad angular de Marte.

$$h_M = R_M^2 \omega_M$$

$$\omega_M = \frac{h_M}{R_M^2} = \frac{h_M}{R_T^2 f^2}$$

pero para órbita circular

$$\frac{h_M^2}{c} = R_M \Rightarrow h_M = \sqrt{c} \sqrt{R_T} f^{1/2}$$

$$\Rightarrow \omega_M = \frac{\sqrt{c}}{R_T^{3/2}} \frac{f^{1/2}}{f^2} = \frac{\sqrt{c}}{R_T^{3/2}} \frac{1}{f^{3/2}}$$

$\therefore$ $$\pi - \alpha_0 = \omega_M \cdot \tau = \frac{1}{f^{3/2}} \cdot \pi \left(\frac{1+f}{2} \right)^{3/2} = \pi \left(\frac{1+f}{2f} \right)^{3/2}$$

$$\Rightarrow \alpha_0 = \pi - \%$$

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b) Energías de órbitas circulares

$$\mathcal{E}_T = -\frac{C}{2R_T}$$

$$\mathcal{E}_M = -\frac{C}{2R_M} = -\frac{C}{2fR_T}$$

falta la energía de la órbita de la nave.

Sabemos que

$$\sqrt{1+\frac{2\mathcal{E}h^2}{C^2}} = e \quad (*)$$

Necesitamos calcular e y h.

Pero $r_{MIN} = R_T$

$r_{MAX} = R_M = fR_T$

$$fR_T = R_T\frac{(1+e)}{(1-e)} \Rightarrow \boxed{e = \frac{f-1}{f+1}}$$

Sabemos que

$$R_T(1+e) = \frac{h^2}{C} \rightarrow h^2 = CR_T(1+e)$$

$$\boxed{h^2 = CR_T\frac{2f}{1+f}}$$

Tenemos todo para usar (*) y despejar $\mathcal{E}$.

P3 3/3

de (*) :

$$\mathcal{E} = \frac{C^2}{2h^2}\left(e^2 - 1\right)$$

$$= \frac{C^2}{2\left(C R_T \frac{2f}{1+f}\right)}\left(\frac{(f-1)^2}{(f+1)^2} - 1\right)$$

$$= \frac{C}{4 \cdot R_T f} \frac{(f-1)^2 - (f+1)^2}{(f+1)}$$

$$\boxed{\mathcal{E} = -\frac{C}{R_T}\frac{1}{(f+1)}}$$ energía de órbita elíptico

$$\Delta\mathcal{E}_1 = -\frac{C}{R_T}\frac{1}{(1+f)} - \left(-\frac{C}{2R_T}\right) = \frac{f-1}{2(f+1)}\frac{C}{R_T}$$

$$\Delta\mathcal{E}_2 = -\frac{C}{2fR_T} - \left(-\frac{C}{R_T(1+f)}\right) = \frac{f-1}{2f(1+f)}\frac{C}{R_T}$$