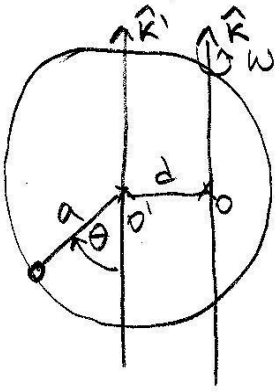
Pauta Ej. 7

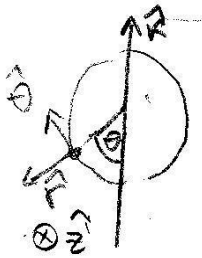
a) Encuentre la ecuación que describe el movimiento del anillo c/respecto al aro



Cilíndricas: $S \sim (\rho, \phi, z)$ ↓ Cilíndricas: $S' \sim (r, \theta, z')$

$$\vec{R} = d\hat{\rho} \quad , \quad \vec{\Omega} = \omega\hat{k}$$
$$\Rightarrow \ddot{\vec{R}} = -d\omega^2\hat{\rho} \qquad \dot{\vec{\Omega}} = 0$$

$$\vec{r}' = a\hat{r} \Rightarrow \vec{v} = a\dot{\theta}\hat{\theta}$$
$$\Rightarrow \vec{a} = -a\dot{\theta}^2\hat{r} + a\ddot{\theta}\hat{\theta}$$



$$\hat{k} = \operatorname{sen}\theta\hat{\theta} - \cos\theta\hat{r}$$
$$\Rightarrow \vec{\Omega} = \omega(\operatorname{sen}\theta\hat{\theta} - \cos\theta\hat{r})$$

- $\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = \vec{\Omega}\times\left[(\omega\operatorname{sen}\theta\hat{\theta} - \omega\cos\theta\hat{r})\times a\hat{r}\right]$
$= \vec{\Omega}\times\left[-a\omega\operatorname{sen}\theta\,\hat{z}'\right] = \left(\omega\operatorname{sen}\theta\hat{\theta} - \omega\cos\theta\hat{r}\right)\times\left(-a\omega\operatorname{sen}\theta\,\hat{z}'\right)$
$= -a\omega^2\operatorname{sen}^2\theta\,\hat{r} - a\omega^2\operatorname{sen}\theta\cos\theta\,\hat{\theta}$
- $2\vec{\Omega}\times\vec{v}' = (2\omega\operatorname{sen}\theta\hat{\theta} - 2\omega\cos\theta\hat{r})\times a\dot{\theta}\hat{\theta} = -2a\dot{\theta}\omega\cos\theta\,\hat{z}'$
- $\dot{\vec{\Omega}}\times\vec{r}' = 0$

$\vec{F} = N_r\hat{r} + N_{z'}\hat{z}'$ como $\hat{\rho} = \cos\theta\hat{\theta} + \operatorname{sen}\theta\hat{r}$

Ecuación vectorial de movimiento del SRNI:

$$-ma\dot{\theta}^2\hat{r} + ma\ddot{\theta}\hat{\theta} = N_r\hat{r} + N_{z'}\hat{z}' + md\omega^2\cos\theta\hat{\theta} + md\omega^2\operatorname{sen}\theta\hat{r}$$
$$+ ma\omega^2\operatorname{sen}^2\theta\hat{r} + ma\omega^2\operatorname{sen}\theta\cos\theta\hat{\theta} + 2ma\omega\dot{\theta}\cos\theta\hat{z}'$$

b) Determine las componentes de la normal en función de θ y $\dot{\theta}$

de a)

$\hat{r}$) $-ma\dot{\theta}^2 = N_r + md w^2 \operatorname{sen}\theta + maw^2 \operatorname{sen}^2\theta$

$\hat{\theta}$) $ma\ddot{\theta} = md w^2 \cos\theta + maw^2 \operatorname{sen}\theta\cos\theta$

$\hat{z}$) $0 = N_z + 2ma w\dot{\theta}\cos\theta$

de $\hat{r}$)

$$\boxed{N_r = -ma\dot{\theta}^2 - mw^2(d\operatorname{sen}\theta + a\operatorname{sen}^2\theta)}$$

de $\hat{z}$)

$$\boxed{N_z = -2ma w\dot{\theta}\cos\theta}$$

c) Encuentre pos. de equilibrio θ_e

de $\hat{\theta}$)

$$\ddot{\theta} = \frac{d}{a}w^2\cos\theta + w^2\operatorname{sen}\theta\cos\theta = f(\theta)$$

- $f(\theta)=0 \Rightarrow w^2\cos\theta\left(\frac{d}{a} + \operatorname{sen}\theta\right) = 0$

por geometría $a > d \Rightarrow \frac{d}{a} < 1$

① $w^2\cos\theta = 0 \Rightarrow \boxed{\theta_1 = \frac{\pi}{2}}$, $\boxed{\theta_2 = \frac{3\pi}{2}}$

② $\frac{d}{a} + \operatorname{sen}\theta = 0 \Rightarrow \operatorname{sen}(-\theta) = \frac{d}{a}$

$$\Rightarrow \boxed{\theta_3 = -\operatorname{arcsen}\left(\frac{d}{a}\right)}$$