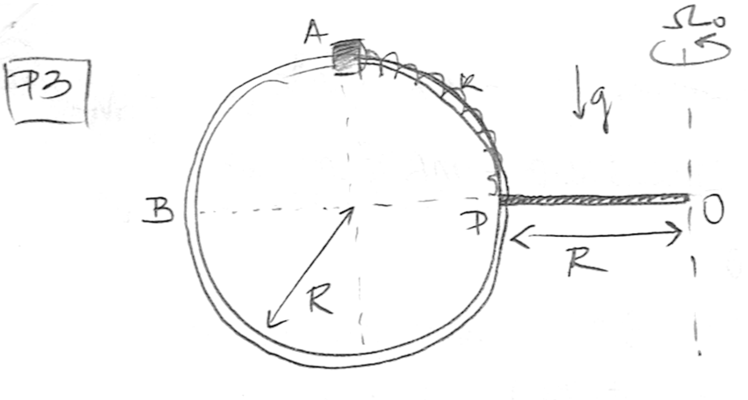
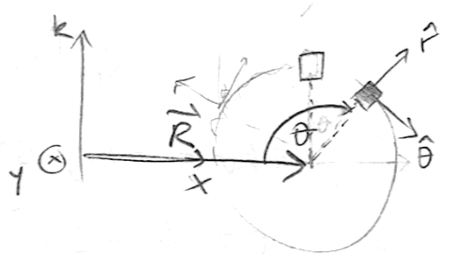
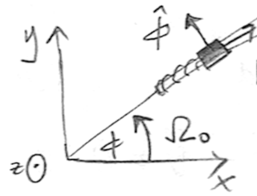
$\boxed{P3}$







$\vec r\,' = R\hat r$; $\vec v\,' = R\dot\theta\hat\theta$; $\vec a\,' = -R\dot\theta^2\hat r + R\ddot\theta\hat\theta$

$\ddot{\vec R} = -2R\Omega_0^2\hat\rho$ $\qquad \vec\Omega_0 = \Omega_0\hat k = \Omega_0(\text{sen}\theta\,\hat r + \cos\theta\,\hat\theta)$

$\ddot{\vec R} = -2R\Omega_0^2(\text{sen}\theta\,\hat\theta - \cos\theta\,\hat r)$

$\text{sen}\theta\,\hat\theta - \cos\theta\,\hat r$

$$\vec F = \underbrace{N_r\hat r + N_k\hat k'}_{\text{Normal}} - \underbrace{mg(\text{sen}\theta\,\hat r + \cos\theta\,\hat\theta)}_{\text{peso}} - \underbrace{kR\theta\hat\theta}_{\text{resorte}}$$

* $\vec\Omega_0\times(\vec\Omega_0\times\vec r\,') = (\underbrace{\Omega_0\text{sen}\theta\,\hat r + \Omega_0\cos\theta\,\hat\theta}_{\Omega_0\hat k})\times\Big[\underbrace{(\Omega_0\text{sen}\theta\,\hat r + \Omega_0\cos\theta\,\hat\theta)\times R\hat r}_{-R\Omega_0\cos\theta\,\hat k'}\Big]$ $= -R\Omega_0\cos\theta\,\hat\phi$

$= -R\Omega_0^2\cos\theta(-\hat\rho) = R\Omega_0^2\cos\theta(\text{sen}\theta\,\hat\theta - \cos\theta\,\hat r)$

* $2\vec\Omega_0\times\vec v\,' = 2(\Omega_0\text{sen}\theta\,\hat r + \cancel{\Omega_0\cos\theta\,\hat\theta})\times R\dot\theta\hat\theta = 2R\Omega_0\dot\theta\,\text{sen}\theta\,\hat k'$

* $\dot{\vec\Omega}\times\vec r\,' = 0$

Luego:

$-mR\dot\theta^2\hat r + mR\ddot\theta\hat\theta = N_r\hat r + N_k\hat k' - mg\,\text{sen}\theta\,\hat r - mg\cos\theta\,\hat\theta - kR\theta\hat\theta$

$+2mR\Omega_0^2\text{sen}\theta\,\hat\theta - 2mR\Omega_0^2\cos\theta\,\hat r - mR\Omega_0^2\cos\theta\,\text{sen}\theta\,\hat\theta + mR\Omega_0^2\cos^2\theta\,\hat r$

$-2mR\Omega_0\dot\theta\,\text{sen}\theta\,\hat k'$

$\hat{r})\ -mR\dot{\theta}^2 = N_r - mg\,\text{sen}\theta - 2mR\Omega_0^2\cos\theta + mR\Omega_0^2\cos^2\theta$

$\hat{\theta})\ mR\ddot{\theta} = -mg\cos\theta - kR\theta + 2mR\Omega_0^2\,\text{sen}\theta - mR\Omega_0^2\cos\theta\,\text{sen}\theta$

$\hat{k}')\ 0 = N_k - 2mR\Omega_0\dot{\theta}\,\text{sen}\theta$

(a) Para que esté en reposo en A ($\theta=\pi/2$), debemos imponer que $\ddot{\theta}=\dot{\theta}=0$, con $\theta=\pi/2$.

$\Rightarrow 0 = -mg\cos(\pi/2) - \frac{kR\pi}{2} + 2mR\Omega_0^2\,\text{sen}(\pi/2) - mR\Omega_0^2\cos(\pi/2)\,\text{sen}(\pi/2)$

$\Rightarrow 2mR\Omega_0^2 = \frac{kR\pi}{2} \Rightarrow \boxed{\Omega_0^2 = \frac{k\pi}{4m}}$

(b)

$\ddot{\theta} = -\frac{g}{R}\cos\theta - \frac{k}{m}\theta + 2\Omega_0^2\,\text{sen}\theta - \Omega_0^2\cos\theta\,\text{sen}\theta$

$\dot{\theta}\,d\dot{\theta} = \left[-\frac{g}{R}\cos\theta - \frac{k\theta}{m} + 2\Omega_0^2\,\text{sen}\theta - \Omega_0^2\cos\theta\,\text{sen}\theta\right]d\theta \quad / \int_{\dot{\theta}}^{0}, \int_{\pi/2}^{\pi}$

$\frac{\dot{\theta}^2}{2}\Big|_{\dot{\theta}_A}^{0} = -\frac{g}{R}\,\text{sen}\theta\Big|_{\pi/2}^{\pi} - \frac{k}{m}\frac{\theta^2}{2}\Big|_{\pi/2}^{\pi} - 2\Omega_0^2\cos\theta\Big|_{\pi/2}^{\pi} + \Omega_0^2\frac{\cos^2\theta}{2}\Big|_{\pi/2}^{\pi}$

$= \frac{g}{R} - \frac{k}{m}\frac{\pi^2}{8}3 + 2\Omega_0^2 + \frac{\Omega_0^2}{2}$

$\Rightarrow \dot{\theta}_A^2 = \frac{3}{4}\frac{k\pi^2}{m} - \frac{2g}{R} - 5\Omega_0^2$

$\frac{3}{24}\frac{k\pi^2}{m} - \frac{2g}{5R} \geq \Omega_0^2$

$\frac{3}{4}\frac{k\pi^2}{m}$.

$\frac{3}{4}\frac{k\pi^2}{m} - \frac{5k\pi}{4m} > 0$

(c)

$$N_r = mg\,\text{sen}\,\theta + 2mR\Omega_0^2\cos\theta + mR\Omega_0^2\cos^2\theta - mR\dot{\theta}^2$$

$$N_k' = 2mR\Omega_0\,\dot{\theta}\,\text{sen}\,\theta$$

A: $\theta = \pi/2 \Rightarrow N_r(\theta=\pi/2) = mg - mR\left[\frac{3k\pi^2}{4m} - \frac{2g}{R} - 5\Omega_0^2\right]$

$$\boxed{N_r(\pi/2) = 3mg + 5mR\Omega_0^2 - \frac{3kR\pi^2}{4}}$$

$$\boxed{N_k = 2mR\Omega_0\left[\frac{3k\pi^2}{4m} - \frac{2g}{R} - 5\Omega_0^2\right]^{1/2}}$$

B: $\theta = \pi$, $\dot{\theta} = 0$ (condición de (b)):

$$\boxed{N_r(\pi) = 3mR\Omega_0^2}$$

$$\boxed{N_k = 0}$$

en A: $\boxed{\vec{N}_A = \left(3mg + 5mR\Omega_0^2 - \frac{3kR\pi^2}{4}\right)\hat{r} + 2mR\Omega_0^2\left[\frac{3k\pi^2}{4m} - \frac{2g}{R} - 5\Omega_0^2\right]^{1/2}\hat{k}}$

en B: $\boxed{\vec{N}_B = 3mR\Omega_0^2\,\hat{r}}$