

# Karnaugh Maps (K-Maps)

- Boolean expressions can be minimized by combining terms
- K-maps minimize equations graphically
- $PA + P\bar{A} = P$

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

| $Y$<br>$\diagdown$<br>$AB$ |   | $C$ |    |    |    |
|----------------------------|---|-----|----|----|----|
|                            |   | 00  | 01 | 11 | 10 |
| $C$                        | 0 | 1   | 0  | 0  | 0  |
|                            | 1 | 1   | 0  | 0  | 0  |

| Y<br>C \ AB |   | AB                      |                   |             |                   |
|-------------|---|-------------------------|-------------------|-------------|-------------------|
|             |   | 00                      | 01                | 11          | 10                |
| C           | 0 | $\bar{A}\bar{B}\bar{C}$ | $\bar{A}B\bar{C}$ | $AB\bar{C}$ | $A\bar{B}\bar{C}$ |
|             | 1 | $\bar{A}\bar{B}C$       | $\bar{A}BC$       | $ABC$       | $A\bar{B}C$       |

# K-map

- Circle 1's in adjacent squares
- In the Boolean expression, include only the literals whose true and complement form are *not* in the circle

| <i>A</i> | <i>B</i> | <i>C</i> | <i>Y</i> |
|----------|----------|----------|----------|
| 0        | 0        | 0        | 1        |
| 0        | 0        | 1        | 1        |
| 0        | 1        | 0        | 0        |
| 0        | 1        | 1        | 0        |
| 1        | 0        | 0        | 0        |
| 1        | 0        | 1        | 0        |
| 1        | 1        | 0        | 0        |
| 1        | 1        | 1        | 0        |

  

|          |   |           |    |    |    |
|----------|---|-----------|----|----|----|
|          |   | <i>AB</i> |    |    |    |
|          |   | 00        | 01 | 11 | 10 |
| <i>C</i> | 0 | 1         | 0  | 0  | 0  |
|          | 1 | 1         | 0  | 0  | 0  |

$$Y = \overline{A}\overline{B}$$

# 3-input K-map

| Y<br>C |                         | AB                |                   |             |    |
|--------|-------------------------|-------------------|-------------------|-------------|----|
|        |                         | 00                | 01                | 11          | 10 |
| 0      | $\bar{A}\bar{B}\bar{C}$ | $\bar{A}B\bar{C}$ | $A\bar{B}\bar{C}$ | $ABC$       |    |
| 1      | $\bar{A}\bar{B}C$       | $\bar{A}BC$       | $ABC$             | $A\bar{B}C$ |    |

Truth Table

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

K-Map

| Y<br>C |  | AB |    |    |    |
|--------|--|----|----|----|----|
|        |  | 00 | 01 | 11 | 10 |
| 0      |  |    |    |    |    |
| 1      |  |    |    |    |    |

# 3-input K-map

| Y<br>C |   | AB                |             |             |             |
|--------|---|-------------------|-------------|-------------|-------------|
|        |   | 00                | 01          | 11          | 10          |
| 0      | 0 | $ABC$             | $\bar{A}BC$ | $AB\bar{C}$ | $A\bar{B}C$ |
|        | 1 | $\bar{A}\bar{B}C$ | $\bar{A}BC$ | $ABC$       | $A\bar{B}C$ |

Truth Table

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

K-Map

| Y<br>C |   | AB |    |    |    |
|--------|---|----|----|----|----|
|        |   | 00 | 01 | 11 | 10 |
| 0      | 0 | 0  | 1  | 1  | 0  |
|        | 1 | 0  | 1  | 0  | 0  |

$$Y = \bar{A}B + B\bar{C}$$

# K-map Definitions

- Complement: variable with a bar over it  
 $\bar{A}, \bar{B}, \bar{C}$
- Literal: variable or its complement  
 $A, \bar{A}, B, \bar{B}, C, \bar{C}$
- Implicant: product of literals  
 $ABC, \bar{A}C, BC$
- **Prime implicant:** implicant corresponding to the largest circle in a K-map

# K-map Rules

- Every 1 in a K-map must be circled at least once
- Each circle must span a power of 2 (i.e. 1, 2, 4) squares in each direction
- Each circle must be as large as possible
- A circle may wrap around the edges of the K-map
- A “don't care” (X) is circled only if it helps minimize the equation

# 4-input K-map

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        | 1        |
| 0        | 0        | 0        | 1        | 0        |
| 0        | 0        | 1        | 0        | 1        |
| 0        | 0        | 1        | 1        | 1        |
| 0        | 1        | 0        | 0        | 0        |
| 0        | 1        | 0        | 1        | 1        |
| 0        | 1        | 1        | 0        | 1        |
| 0        | 1        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        | 1        |
| 1        | 0        | 0        | 1        | 1        |
| 1        | 0        | 1        | 0        | 1        |
| 1        | 0        | 1        | 1        | 0        |
| 1        | 1        | 0        | 0        | 0        |
| 1        | 1        | 0        | 1        | 0        |
| 1        | 1        | 1        | 0        | 0        |
| 1        | 1        | 1        | 1        | 0        |

| <i>Y</i>  |    | <i>AB</i> |    |    |    |
|-----------|----|-----------|----|----|----|
| <i>CD</i> |    | 00        | 01 | 11 | 10 |
|           | 00 |           |    |    |    |
| 01        |    |           |    |    |    |
| 11        |    |           |    |    |    |
| 10        |    |           |    |    |    |

# 4-input K-map

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        | 1        |
| 0        | 0        | 0        | 1        | 0        |
| 0        | 0        | 1        | 0        | 1        |
| 0        | 0        | 1        | 1        | 1        |
| 0        | 1        | 0        | 0        | 0        |
| 0        | 1        | 0        | 1        | 1        |
| 0        | 1        | 1        | 0        | 1        |
| 0        | 1        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        | 1        |
| 1        | 0        | 0        | 1        | 1        |
| 1        | 0        | 1        | 0        | 1        |
| 1        | 0        | 1        | 1        | 0        |
| 1        | 1        | 0        | 0        | 0        |
| 1        | 1        | 0        | 1        | 0        |
| 1        | 1        | 1        | 0        | 0        |
| 1        | 1        | 1        | 1        | 0        |

| <i>Y</i><br><i>CD</i> \ <i>AB</i> |  |    |    |    |    |
|-----------------------------------|--|----|----|----|----|
|                                   |  | 00 | 01 | 11 | 10 |
| 00                                |  | 1  | 0  | 0  | 1  |
| 01                                |  | 0  | 1  | 0  | 1  |
| 11                                |  | 1  | 1  | 0  | 0  |
| 10                                |  | 1  | 1  | 0  | 1  |

# 4-input K-map

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        | 1        |
| 0        | 0        | 0        | 1        | 0        |
| 0        | 0        | 1        | 0        | 1        |
| 0        | 0        | 1        | 1        | 1        |
| 0        | 1        | 0        | 0        | 0        |
| 0        | 1        | 0        | 1        | 1        |
| 0        | 1        | 1        | 0        | 1        |
| 0        | 1        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        | 1        |
| 1        | 0        | 0        | 1        | 1        |
| 1        | 0        | 1        | 0        | 1        |
| 1        | 0        | 1        | 1        | 0        |
| 1        | 1        | 0        | 0        | 0        |
| 1        | 1        | 0        | 1        | 0        |
| 1        | 1        | 1        | 0        | 0        |
| 1        | 1        | 1        | 1        | 0        |

| <i>Y</i>  |    | <i>AB</i> |    |    |    |
|-----------|----|-----------|----|----|----|
| <i>CD</i> |    | 00        | 01 | 11 | 10 |
|           | 00 | 1         | 0  | 0  | 1  |
| 01        | 01 | 0         | 1  | 0  | 1  |
| 11        | 11 | 1         | 1  | 0  | 0  |
| 10        | 10 | 1         | 1  | 0  | 1  |

$$Y = \bar{A}\bar{C} + \bar{A}BD + A\bar{B}\bar{C} + \bar{B}\bar{D}$$

# K-maps with Don't Cares

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        | 1        |
| 0        | 0        | 0        | 1        | 0        |
| 0        | 0        | 1        | 0        | 1        |
| 0        | 0        | 1        | 1        | 1        |
| 0        | 1        | 0        | 0        | 0        |
| 0        | 1        | 0        | 1        | X        |
| 0        | 1        | 1        | 0        | 1        |
| 0        | 1        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        | 1        |
| 1        | 0        | 0        | 1        | 1        |
| 1        | 0        | 1        | 0        | X        |
| 1        | 0        | 1        | 1        | X        |
| 1        | 1        | 0        | 0        | X        |
| 1        | 1        | 0        | 1        | X        |
| 1        | 1        | 1        | 0        | X        |
| 1        | 1        | 1        | 1        | X        |

| <i>Y</i> \ <i>AB</i> |    | 00 | 01 | 11 | 10 |
|----------------------|----|----|----|----|----|
| <i>CD</i>            | 00 |    |    |    |    |
|                      | 01 |    |    |    |    |
|                      | 11 |    |    |    |    |
|                      | 10 |    |    |    |    |

# K-maps with Don't Cares

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        | 1        |
| 0        | 0        | 0        | 1        | 0        |
| 0        | 0        | 1        | 0        | 1        |
| 0        | 0        | 1        | 1        | 1        |
| 0        | 1        | 0        | 0        | 0        |
| 0        | 1        | 0        | 1        | X        |
| 0        | 1        | 1        | 0        | 1        |
| 0        | 1        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        | 1        |
| 1        | 0        | 0        | 1        | 1        |
| 1        | 0        | 1        | 0        | X        |
| 1        | 0        | 1        | 1        | X        |
| 1        | 1        | 0        | 0        | X        |
| 1        | 1        | 0        | 1        | X        |
| 1        | 1        | 1        | 0        | X        |
| 1        | 1        | 1        | 1        | X        |

| <i>Y</i><br><i>CD</i> \ <i>AB</i> |  | 00 | 01 | 11 | 10 |
|-----------------------------------|--|----|----|----|----|
|                                   |  | 00 | 01 | 11 | 10 |
| 00                                |  | 1  | 0  | X  | 1  |
| 01                                |  | 0  | X  | X  | 1  |
| 11                                |  | 1  | 1  | X  | X  |
| 10                                |  | 1  | 1  | X  | X  |

# K-maps with Don't Cares

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        | 1        |
| 0        | 0        | 0        | 1        | 0        |
| 0        | 0        | 1        | 0        | 1        |
| 0        | 0        | 1        | 1        | 1        |
| 0        | 1        | 0        | 0        | 0        |
| 0        | 1        | 0        | 1        | X        |
| 0        | 1        | 1        | 0        | 1        |
| 0        | 1        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        | 1        |
| 1        | 0        | 0        | 1        | 1        |
| 1        | 0        | 1        | 0        | X        |
| 1        | 0        | 1        | 1        | X        |
| 1        | 1        | 0        | 0        | X        |
| 1        | 1        | 0        | 1        | X        |
| 1        | 1        | 1        | 0        | X        |
| 1        | 1        | 1        | 1        | X        |

|          |           | <i>AB</i> |    |    |    |
|----------|-----------|-----------|----|----|----|
| <i>Y</i> | <i>CD</i> | 00        | 01 | 11 | 10 |
|          |           | 00        | 01 | 11 | 10 |
|          | 00        | 1         | 0  | X  | 1  |
|          | 01        | 0         | X  | X  | 1  |
|          | 11        | 1         | 1  | X  | X  |
|          | 10        | 1         | 1  | X  | X  |

$$Y = A + \bar{B}\bar{D} + C$$