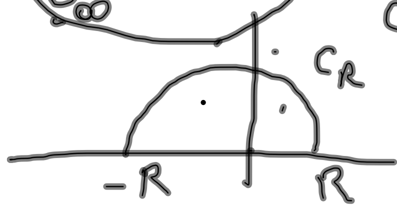
$$\int_0^{2\pi} \frac{\text{sen}\, 2x}{1+\cos 4x}\, dx$$

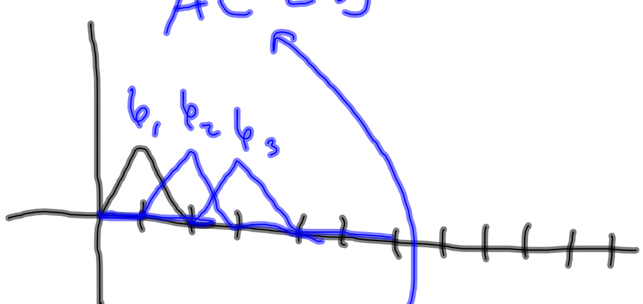
$\text{sen}\, x =$

$z = Re^{i\theta}$

$$\int_{-\infty}^{\infty} \frac{x+1}{x^3+x}\, dx \sim \int_{C_R} \frac{z+1}{z^3+z} \sim \frac{1}{R} \longrightarrow 0$$



$AC = b$



$u \approx \sum c_i \varphi_i$

$$\boxed{u_{tt} - u_{xx} = 0}\ , \quad t>0, \quad 0<x<\ell$$

$u(t,0) = u(t,\ell) = 0, \ t>0$ (Dirichlet)

$\rightarrow u(0,x) = x(\ell - x), \quad 0<x<\ell$

$\rightarrow u_t(0,x) = 1, \quad t>0$

$u(t,x) = T(t)X(x)$ ($u \equiv 0$ no es sol.)

así

$$T''X - TX'' = 0 \quad / \div TX$$

entonces

$$\frac{T''}{T} - \frac{X''}{X} = 0 \Rightarrow \frac{T''}{T} = \frac{X''}{X} = \lambda = cte$$

e.d.,

$$\boxed{\begin{array}{l} T'' - \lambda T = 0 \\ X'' - \lambda X = 0 \end{array}}$$

$u(t,0)=0 \Rightarrow T(t)\,X(0)=0 \quad \forall t>0.$

$\therefore X(0)=0$

$u(t,l)=0 \Rightarrow T(t)\,X(l)=0 \quad , \forall t>0$

$X(l)=0$

$$\therefore \boxed{\begin{array}{l} X''-\lambda X=0 \\ X(0)=X(l)=0 \end{array}} \quad (P_\lambda)$$

$\underline{\lambda=0}$

$X''=0 \Rightarrow X(x)=ax+b$

$X(0)=b=0 \Rightarrow b=0$

$X(l)=0 \Rightarrow al=0$

$\therefore a=b=0$

$\therefore X\equiv 0$

no sirve

$\lambda > 0$, $\lambda = k^2$, $k > 0$.

$$X'' - k^2 X = 0$$

$$X(x) = a e^{kx} + b e^{-kx}$$

así

$$X(0) = 0 \Rightarrow a + b = 0 \Rightarrow a = -b$$

$$X(\ell) = 0 \Rightarrow a e^{k\ell} - a e^{-k\ell} = 0$$

$$a\left(e^{k\ell} - e^{-k\ell}\right) = 0$$

$$\therefore\ a = b = 0$$

$$X \equiv 0$$

NO SIRVE

$\lambda < 0$, $\lambda = -k^2$

$$X'' + k^2 X = 0$$

$$X(x) = a\cos(kx) + b\,\mathrm{sen}(kx)$$

$$X(0) = a = 0$$

$$X(\ell) = b\,\mathrm{sen}(k\ell)$$

buscamos $k > 0$ tq:

$$\mathrm{sen}(k\ell) = 0 \Rightarrow k\ell = n\pi$$

$$k = \frac{n\pi}{\ell}$$

$$\boxed{\lambda_n = -\left(\frac{n\pi}{\ell}\right)^2 , \; n \geq 1 \qquad X_n = b_n \operatorname{sen}\left(\frac{n\pi x}{\ell}\right)}$$

así

$$T_n'' - \lambda_n T_n = 0 \Rightarrow T'' + \left(\frac{n\pi}{\ell}\right)^2 T = 0$$

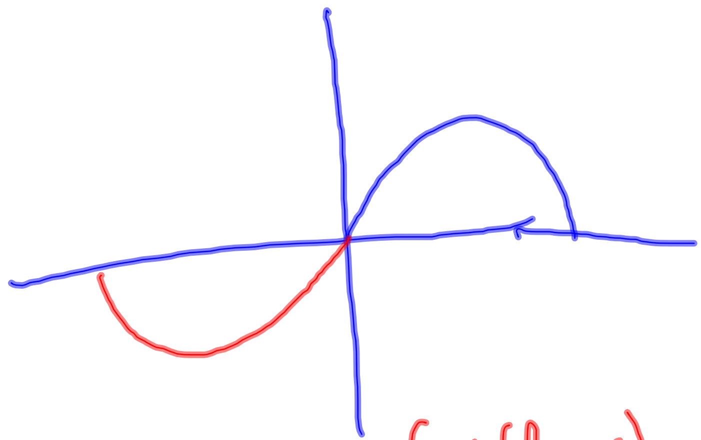
$$T_n(t) = \alpha_n \cos\left(\frac{n\pi t}{\ell}\right) + \beta_n \operatorname{sen}\left(\frac{n\pi t}{\ell}\right)$$

$$u_n(t,x) = T_n(t) X_n(x) \quad , \forall n \geq 1$$

$$= \operatorname{sen}\left(\frac{n\pi x}{\ell}\right)\left[c_n^1 \cos\left(\frac{n\pi t}{\ell}\right) + c_n^2 \operatorname{sen}\left(\frac{n\pi t}{\ell}\right)\right]$$

$$\therefore \; u(t,x) = \sum_{n \geq 1} u_n$$

$$u(0,x) = x(1-x) = \sum_{n \geq 1} c_n^1 \operatorname{sen}\left(\frac{n\pi x}{\ell}\right)$$



$$f_p(x) = \begin{cases} x(\ell - x), & 0 < x < \ell \\ x(\ell + x), & -\ell < x < 0 \end{cases}$$

$f(x)$

$$-(-x(\ell-(-x)) = x(\ell+x)$$

$$f_p(x) = \begin{cases} f(x), & 0 < x < \ell \\ f(-x), & -\ell < x < 0 \end{cases}$$

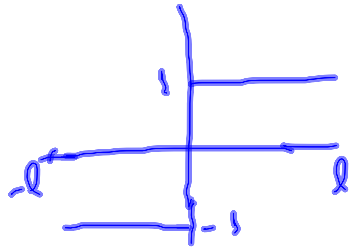
$$f_i(x) = \begin{cases} f(x), & 0 < x < \ell \\ -f(-x), & -\ell < x < 0 \end{cases}$$

$$C_n^1 = \frac{1}{\ell}\int_{-\ell}^{\ell} f_i(x)\,\text{sen}\left(\frac{n\pi x}{\ell}\right)dx$$

$$\boxed{C_n^1 = \frac{2}{\ell}\int_{0}^{\ell} x(l-x)\,\text{sen}\left(\frac{n\pi x}{\ell}\right)dx}$$

$$u_t(t,x) = \text{sen}\left(\frac{n\pi x}{\ell}\right)\left[\sum_{n\geq 1} -C_n^1\,\frac{n\pi}{\ell}\,\text{sen}\left(\frac{n\pi t}{\ell}\right) + C_n^2\,\frac{n\pi}{\ell}\cos\left(\frac{n\pi t}{\ell}\right)\right]$$

$$u_t(0,x) = 1 = \sum_{n\geq 1} \text{sen}\left(\frac{n\pi x}{\ell}\right) C_n^2\,\frac{n\pi}{\ell} \qquad 0<x<\ell$$



$c_n^2 = ?$

$$c_n^2 \frac{n\pi}{l} = \frac{1}{l}\int_{-l}^{l} f(x)\,\operatorname{sen}\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{2}{l}\int_{0}^{l} \operatorname{sen}\left(\frac{n\pi x}{l}\right) dx$$

$u_{tt} - u_{xx} + u = 0 \quad , \quad 0 < x < l$

$u(t,0) = u_x(t,l) = 0$

$u(0,x) = x^2$

$u_t(0,x) = 1 + x$

$T''X - TX'' + TX = 0$

$$\frac{T''}{T} = \frac{X'' + X}{\cancel{X}} = \lambda$$

$X'' = \lambda X - X$

$X'' - (\lambda - 1)X = 0$

$u_t - u_{xxx} = 0, \quad 0 < x < \pi, \; t > 0$

$u(t,0) = u(t,\pi)$

$u_x(t,0) = u_x(t,\pi) \qquad t > 0$

$u_{xx}(t,0) = u_{xx}(t,\pi)$

$u(0,x) = x, \quad 0 < x < \pi$