MA2002-2 Cálculo Avanzado y Aplicaciones
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Pauta Auxiliar 1 (lo que quedó pendiente)

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P3. *Problema 1.2 del apunte*

(a) Hecha

(b)

$$\begin{aligned} rot[\overrightarrow{F}(t\vec{r}) \times t\vec{r}] &= rot[trg(tr)\hat{\varphi}] \\ &= \frac{1}{r^2 \sin\varphi}(r\sin\varphi \frac{\partial}{\partial r} tr^2 g(tr))\hat{\theta} \\ &= \frac{1}{r}\frac{\partial}{\partial r} tr^2 g(tr))\hat{\theta} \\ &= \frac{1}{r}(2trg(tr) + tr^2 \frac{\partial}{\partial r} g(tr) \cdot t)\hat{\theta} \\ &= (2tg(tr) + t^2 r \frac{\partial}{\partial r} g(tr))\hat{\theta} \\ &= (2t\overrightarrow{F}(t\vec{r}) + t^2 r \frac{\partial}{\partial r} g(tr)\hat{\theta}) \end{aligned}$$

Por otro lado.

$$\frac{d}{dt}\overrightarrow{F}(t\vec{r}) = \frac{d}{dt} g(tr)\hat{\theta} = [\frac{\partial}{\partial r} g(tr)\frac{\partial}{\partial t}(tr) + \frac{\partial}{\partial \theta} g(tr)\frac{\partial}{\partial t}(tr) + \frac{\partial}{\partial \varphi} g(tr)\frac{\partial}{\partial t}(tr)]\hat{\theta} = \frac{\partial}{\partial r} g(tr) \cdot r\hat{\theta}$$

Entonces

$$rot[\overrightarrow{F}(t\vec{r}) \times t\vec{r}] = (2t\overrightarrow{F}(t\vec{r}) + t^2 \frac{d}{dt}\overrightarrow{F}(t\vec{r}))$$

Que era lo que buscábamos.

(c) Sea $\overrightarrow{G}(\vec{r}) = \int_0^1 [\overrightarrow{F}(t\vec{r}) \times t\vec{r}]dt$
PDQ: $rot\overrightarrow{G} = \overrightarrow{F}$ en B.
En efecto:

$$\begin{aligned} rot\overrightarrow{G} &= rot \int_0^1 [\overrightarrow{F}(t\vec{r}) \times t\vec{r}]dt \\ &= \int_0^1 rot[\overrightarrow{F}(t\vec{r}) \times t\vec{r}]dt, \quad (a) \\ &= \int_0^1 2t[\overrightarrow{F}(t\vec{r})]dt + \underbrace{\int_0^1 t^2 \frac{d}{dt}\overrightarrow{F}(t\vec{r})dt}_{I}, \quad (b) \text{ pues } div\overrightarrow{F} = 0 \text{ en B} \end{aligned}$$

Además

$$\begin{aligned} I &= \int_0^1 t^2 \frac{d}{dt} \overrightarrow{F}(t\vec{r})dt \\ &= t^2 \overrightarrow{F}(t\vec{r}) \mid_0^1 -2\int_0^1 t\overrightarrow{F}(t\vec{r}) \\ &= \overrightarrow{F}(\vec{r}) - 2\int_0^1 t\overrightarrow{F}(t\vec{r}) \end{aligned}$$

Entonces

$$rot\overrightarrow{G} = 2\int_0^1 t\overrightarrow{F}(t\vec{r}) + \overrightarrow{F}(\vec{r}) - 2\int_0^1 t\overrightarrow{F}(t\vec{r}) = \overrightarrow{F}(\vec{r})$$

Que es lo que queríamos.