$\boldsymbol{Auxiliar\ 04\ MA1002}$
$\boldsymbol{Sección\ 03\ 2010}$

Definición

(1) F *continua en* I, *derivable en* $int(I)$ *es primitiva de* $f \iff \forall x \in int(I),\ F'(x) = f(x)$

(2) $\int f = \{F, F \text{ es primitiva de } f\} \iff \{F + c,\ F' = f\} \iff \int f(x)dx = F(x) + c$

Observaciones:

$$\int f(x)'dx = f(x) + c$$

$$\frac{d}{dx}\int f(x)dx = f(x)$$

$$\int [f(x) + \alpha g(x)]dx = \int f(x)dx + \alpha \int g(x)dx \quad \forall \alpha \in$$

Teorema Cambio de Variable:

$$Si\ u = g(x) \implies \int f(u)du = \int f(g(x))g'(x)dx$$

Integración por partes

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

Estimados:
La pauta de algunos problemas del auxiliar 5. Ese día les comenté la propuesta de realizar un auxiliar extra la semana del Control 2. Resolveremos ejercicios de todos los tipos, algunos pendientes que "conviene" mirar más en detalle ;D y otros propuestos para repasar/aclarar toda la meteria..

Buen fin de semana LARGO
=)

Problema 0 > AQUI
Problema 1 > CLASE
Problema 2 > CLASE
Problema 3> CLASE
Problema 4 > AQUI
Problema 5> AQUI
Problema 6> AUX extra
Problema 7 > AUX extra
Problema 8> Aux extra
Problrma 9> AQUI

sbh

$$I = \int \frac{1}{a^2 + x^2}\, dx$$

$\boxed{\text{C.V}}$ $x = au \Rightarrow dx = a\,du$

$$\Rightarrow I = \int \frac{1}{a^2 + a^2u^2} \cdot a\,du$$

$$= \frac{1}{a} \int \frac{1}{1 + u^2}\, du$$

$$= \frac{1}{a} \arctan(u) + C$$

$$= \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$I = \int \frac{x}{a^2 + x^2}\, dx$$

[C.V] $u = x^2 \Rightarrow du = 2x\,dx$

[i.e. $x\,dx = \frac{du}{2}$]

$$I = \int \frac{\left[\frac{du}{2}\right]}{a^2 + u}$$

$$= \frac{1}{2} \int \frac{du}{a^2 + u} = \frac{1}{2} \ln\left(|a^2 + x^2|\right) + C$$

[equivalente $I = \frac{1}{2} \int \frac{2x}{a^2 + x^2}\, dx = \frac{1}{2} \ln\left(|a^2 + x^2|\right) + C$]

$\frac{f'}{f}$

$$I = \int \frac{x^2\,dx}{1+x^2} = \int \frac{1 + x^2 - 1}{1+x^2}\,dx$$

$$= \int \cancelto{1}{\frac{1+x^2}{1+x^2}}\,dx - \int \frac{1}{1+x^2}\,dx$$

$$= x - \arctan(x) + C$$

$$I = \int \frac{x}{\sqrt{1+x}}\,dx$$

$$= \int \frac{1+x}{\sqrt{1+x}} - \frac{1}{\sqrt{1+x}}\,dx$$

$$= \int (1+x)^{1/2}\,dx - \int (1+x)^{-1/2}\,dx$$

$$= \frac{2}{3}(1+x)^{3/2} - 2\sqrt{1+x} + \mathbb{C}$$

BONUS $\frac{11}{0}$

$$I = \int \frac{x^2}{\sqrt{1+x}}\, dx$$

[C.V] $u = 1 + x$ i.e $x = u - 1$

$\Rightarrow du = dx$

$$\Rightarrow I = \int \frac{u^2 - 2u + 1}{\sqrt{u}}\, du$$

$$= \int u^{3/2}\, du - 2\int u^{1/2}\, du - \int u^{-1/2}\, du$$

$$= \frac{2}{5}(1+x)^{5/2} - \frac{4}{3}(1+x)^{2/3} + 2\sqrt{1+x} + C$$

$$I = \int \frac{\operatorname{sen}(x)\cos(x)}{\sqrt{1+\operatorname{sen}(x)}}\,dx$$

$[C.V]$ $u = \operatorname{sen}(x) \Rightarrow du = \cos(x)\,dx$

$$\Rightarrow I = \int \frac{u \cdot du}{\sqrt{1+u}}$$

$[C.V]$ $z = 1+u \Rightarrow dz = du$

$$\Rightarrow I = \int \frac{z-1}{\sqrt{z}}\,dz = \int z^{1/2}\,dx - \int z^{-1/2}\,dx$$

$$= \frac{2}{3}\left(1+\operatorname{sen}(x)\right)^{3/2} - 2\sqrt{1+\operatorname{sen}(x)} + C$$

$$I = \int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}\,dx$$

[C.V] $u = \sqrt{x} \Rightarrow du = \frac{dx}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$, $\sqrt{x} = u$

$$\Rightarrow I = \int \frac{u \cdot (2u\,du)}{\sqrt{1+u}} = 2\int \frac{u^2}{\sqrt{1+u}}\,du$$

[C.V] $z = 1+u \Rightarrow dx = du$

$$\Rightarrow I = \int \frac{z^2 - 2z + 1}{\sqrt{z}}\,dz$$

$$= \frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{8}{3}(1+\sqrt{x})^{3/2} + 4\sqrt{1+\sqrt{x}} + C$$

$$I = \int \cos(\ln(x))\,dx \qquad [C.V] \quad y = \ln(x) \Rightarrow dy = \frac{1}{x}\,dx \Rightarrow x = e^y$$

$$\Rightarrow I = \int \cos(y)\, e^y\, dy \quad (*)$$

$[I.P.P]$ $\quad u = \cos(y) \Rightarrow du = -\operatorname{sen}(y)\,dy$

$\quad dv = e^y\,dy \Rightarrow v = e^y$

$$\Rightarrow I = \cos(y)\, e^y dy + \int e^y \operatorname{sen}(y)\,dy$$

$[I.P.P]$ $\quad u = \operatorname{sen}(y) \Rightarrow du = \cos(y)\,dy$

$\quad dv = e^y\,dy \Rightarrow v = e^y$

$$\Rightarrow I = \cos(y)\, e^y dy + \operatorname{sen}(y)\, e^y - \underbrace{\int \cos(y)\, e^y\, dy}_{I}$$

$$\Rightarrow 2I = e^y \left[\cos(y) + \operatorname{sen}(y)\right]$$

$$\Rightarrow I = \frac{x}{2}\left[\cos(\ln(x) + \operatorname{sen}(\ln(x))\right]$$

$$I = \int \operatorname{sen}^2(x)\,dx$$

[I.P.P] $u = \operatorname{sen}(x) \Rightarrow du = \cos(x)\,dx$

$dv = \operatorname{sen}(x)\,dx \Rightarrow v = -\cos(x)$

$$\Rightarrow I = -\operatorname{sen}(x)\cos(x) + \int \cos^2(x)\,dx$$

$$= -\operatorname{sen}(x)\cos(x) + \int dx - \underbrace{\int \operatorname{sen}^2(x)\,dx}_{I}$$

$$\Rightarrow 2I = x - \operatorname{sen}(x)\cos(x)$$

$$\Rightarrow I = \frac{x - \operatorname{sen}(x)\cos(x)}{2} + C$$

$$I = \int \frac{4x^3 - 3x^2 + 3}{(x-1)^2(x^2+1)}\,dx$$

(1) SE DESCOMPONE:

$$\frac{4 - 3x^2 + 3}{(x-1)^2(x^2+1)} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+1)}$$

$$\Rightarrow 4 - 3x^2 + 3 = A(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)^2$$

$$\Rightarrow 4 - 3x^2 + 3 = A(x^2+1) + B(x^3 - x^2 + x - 1) + (Cx+D)(x^2 - 2x + 1)$$

de $x^3 \Rightarrow$ $4 = B + C$

de $x^2 \Rightarrow$ $A - B - 2C + D = -3$

de $x \Rightarrow$ $B + C - 2D = 0$

Cte $\Rightarrow$ $A - B + D = 3$

Luego $3-2C=-3 \Rightarrow \boxed{C=3}$

de (x^3) $B=4-C=1$

de (x) $2D=B+C=4 \Rightarrow \boxed{D=2}$

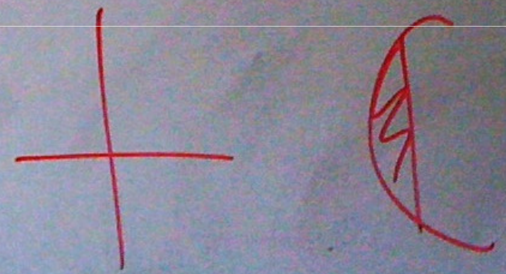
de cte $A=B-D+3=1-2+3$

$\Rightarrow \boxed{A=2}$

Entonces:

$$I=\int \frac{2}{(x-1)^2}\,dx+\int \frac{dx}{x-1}+\int \frac{3x+2}{x^2+1}\,dx$$

$$I = \frac{-2}{x-1} + \ln\left(|x-1|\right) + \frac{3}{2}\ln\left(x^2+1\right) + 2\arctan(x)$$



P_3. $\int e^{-x} \ln(1+e^x)\,dx$

[I.P.P] $\quad u = \ln(1+e^x)\,dx$

$dv = e^{-x}dx \Rightarrow v - e^{-x}$

$$\Rightarrow I = uv - \int v\,du = -e^{-x}\ln(1+e^x) + \int \frac{dx}{1+e^x}$$

[C.V] $\quad z = e^x \Rightarrow dz = e^x dx \Rightarrow dx = \frac{dz}{z}$

$$\Rightarrow I = -e^{-x}\ln(1+e^x) + \int \frac{1}{1+z}\cdot\frac{dz}{z}$$

$$\Rightarrow I = -e^{-x}\ln(1+e^x) + \int\left[\frac{1}{z} - \frac{1}{1+z}\right]dz$$

$$= -e^{-x}\ln(1+e^x) + \ln(e^x) - \ln(1+e^x) + C$$

$$= \ln\left(\frac{e^x}{1+e^x}\right) - e^{-x}\ln(1+e^x) + C$$

P4. $f : \mathbb{R} \to \mathbb{R}_+ \;/\; \int f(x)\,dx = f(x)$

(i) EN EFECTO $\int f(x)\,dx = f(x) \;/\; \frac{d}{dx}(\;)$

$$\Rightarrow \frac{d}{dx}\int f(x)\,dx = \frac{d}{dx} f(x)$$

$$\Rightarrow f(x) = f'(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 1 \quad / \int (\;)$$

$$\Rightarrow \int \frac{f'(x)}{f(x)}\,dx = \int 1\,dx$$

$$\therefore \int \frac{f'(x)}{f(x)}\,dx = X + C$$

P 4.

(ii) $\int \frac{f'(x)}{f(x)} = x + c$

$\Rightarrow \ln(f(x)) = x + c \quad / e^{(\cdot)}$

$\Rightarrow f(x) = e^{x+c}$

P_5. $\displaystyle\int \frac{x}{(1+x^2)(1+x)}\,dx$

(*) SE UTILIZAN FRACCIONES PARCIALES !! ☺

$$\Rightarrow \frac{x}{(1+x^2)(1+x)}dx = \frac{Ax+B}{(1+x^2)} + \frac{C}{(1+x)}$$

$$\Rightarrow (Ax+B)(1+x) + C(1+x^2)$$
$$= (A+C)x^2 + (A+B)x + B+C$$

$$\Rightarrow \left.\begin{array}{l} A+C=0 \\ B+C=0 \\ A+B=1 \end{array}\right] \Rightarrow A = B = -C = \frac{1}{2}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{x+1}{1+x^2}dx + \frac{1}{2}\int \frac{1}{1+x}dx$$

$$= \frac{1}{2}\left(\arctan(x) + \ln\left(\sqrt{1+x^2}\right) - \ln(|1+x|)\right)$$