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Would rational voters acquire costly information?

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Abstract

We analyze an election in which voters are uncertain about which of two alternatives is better for them. Voters can acquire some costly information about the alternatives. In agreement with Downs's rational ignorance hypothesis, individual investment in political information declines to zero as the number of voters increases. However, if the marginal cost of information is near zero for nearly irrelevant information, there is a sequence of equilibria such that the election outcome is likely to correspond to the interests of the majority for arbitrarily large numbers of voters. Thus, "rationally ignorant" voters are consistent with a well-informed electorate.

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1. Introduction

One of the most influential contributions of Anthony Downs's *An Economic Theory of Democracy* to the economic modelling of politics is the concept of "rational ignorance." Given that each individual voter has a negligible probability of affecting the outcome in a large election, voters will not have an incentive to acquire political information before voting. In a situation in which discovering their interests or "true views" takes time and effort from individual citizens, the result may be a failure of democracy to produce a result consistent with the interests of the majority. In Downs's words,

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> If all others express their true views, he [the voter] gets the benefit of a well-informed electorate no matter how well-informed he is; if they are badly informed, he cannot produce those benefits himself. Therefore, as in all cases of individual benefits, the individual is motivated to shirk his share of the costs: he refuses to get enough information to discover his true views. Since all men do this, the election does not reflect the true consent of the governed. [8, p. 246]

We can actually draw a distinction between two versions of the rational ignorance hypothesis. The "weak version" is that individual voters, realizing that each vote has a negligible probability of affecting the outcome of the election, invest very little or no effort in acquiring political information. The "strong version" is that the election outcome itself will not be more likely to reflect the interests of the majority than, say, a fair coin toss. In this paper, we develop a formal model that is consistent with the weak version of the rational ignorance hypothesis, but contradicts the strong version.

A good deal of the literature on the influence activities of interest groups assumes that a decisive fraction of the electorate is uninformed because individual voters have little incentive to get political information (see [5] for an explicit discussion). Becker [6] argues that efficiency may be restored in the voting market because of the activity of influence groups. Coate and Morris [7] point out that the re-election motive may induce incumbent politicians to behave efficiently unless voters are uncertain about politicians' types. (In their view, and Becker's, efficiency does not mean that transfers from the majority to interest groups do not occur; it only means that those transfers are carried out with minimum dead weight costs.) Closer to our point, Wittman [20] calls into question the idea that the costs of information fall on the voter instead of on political entrepreneurs.

We provide a different rationale for elections to reflect the interests of the majority. In our model, there are no interest groups or active politicians. Voters do not have access to free information. Instead, they may acquire some information, at a cost. Crucially, acquiring poor information is cheap. We show that, as the number of voters increases, voters acquire less and less information. However, there is an equilibrium sequence such that along the sequence the outcome of the election is very likely to correspond to the interests of a majority of voters. Thus, the electorate may be quite well-informed even if individual voters are (at least asymptotically) rationally ignorant.

We study an election in which voters have common preferences, but they do not know which of two alternatives is better for them. Voters may acquire a costly signal about the alternatives. The signal is correct with probability $\frac{1}{2}+x$, where x is chosen by the voter. We refer to x as the quality of the signal. The cost of acquiring the signal is given by some convex function $C(x)$. Our first three theorems describe information acquisition and aggregation under the assumption that voters would be indifferent between the alternatives if they were to decide solely on the basis of their prior beliefs.

Theorem 1 shows that, if $C'(0) = 0$, then there is an equilibrium in which the quality of information acquired by voters is positive for an arbitrarily large electorate. In agreement with the weak version of the rational ignorance hypothesis, individual investment in information approaches zero as the size of the electorate increases.

Theorem 2 provides an estimate of the limit probability of choosing the best alternative along the sequence of equilibria in which voters acquire some information. If $C''(0) < \infty$, this probability is strictly larger than $\frac{1}{2}$. Moreover, this probability goes to one as $C''(0)$

approaches zero, or as the importance attached by voters to the election grows unboundedly. If $C''(0) = 0$, the limit probability of choosing the best alternative is actually one. Successful information aggregation is possible because the information acquired by each voter goes to zero but it does so slowly enough to allow the effect of large numbers to kick in.

It is reasonable to believe that voters are involuntarily exposed to a flow of political information in the course of everyday activities—a point already acknowledged by Downs [8, p. 245], who relies on the unwillingness of voters to assimilate even freely available information in order to support the rational ignorance hypothesis. If the function C simply reflects the cost of "paying a little attention," the conditions for at least partially successful information aggregation, that is $C'(0) = 0$ and $C''(0) < \infty$, do not appear unduly restrictive.

Theorem 3 establishes that the aggregate cost of information acquisition declines to zero as the number of voters increases if $C'(0) = C''(0) = 0$. If $C'(0) = 0$ and $C''(0) > 0$, then the aggregate cost converges to a positive constant. Combining Theorems 2 and 3, we obtain that the equilibrium with information acquisition is asymptotically efficient if $C'(0) = C''(0) = 0$. Moreover, universal or near universal participation in elections is desirable in that case. However, if $C'(0) = 0$ and $C''(0) > 0$, the equilibrium with information acquisition is not asymptotically efficient, and the optimal size of the electorate may be small in relation to the size of the society.

Political information in our model is a public good. As in other instances of privately provided public goods, there is an incentive to free ride on other voters, and in fact voters underinvest in political information in relation to a symmetric optimal profile. In the traditional problem of private provision of public goods in large economies [2], the marginal cost of contributing is constant, and the contributions of others reduce the marginal utility of additional units of the public good up to the point where it does not compensate most agents to contribute. In our model, the marginal cost is small for small contributions. The marginal benefit of contributing is positive because the probability of being pivotal is nonzero, though it decreases with the number of voters. Opposite to what happens in the traditional problem, approximate efficiency can be obtained in the limit.

Next, we relax the assumption that voters are ex ante indifferent between the alternatives. Theorem 4 shows that the previous results hold as long as the asymmetry in prior beliefs and preferences is small or $C''(0)$ is close enough to zero. In equilibria with information acquisition, voters randomize between acquiring information and voting according to the signal received, or acquiring no information and voting for the alternative favored ex ante. The beliefs of voters, conditional on being decisive, are kept very close, so that very little information can change the behavior of voters at the booth. This, in turn, makes voters to be willing to acquire vanishingly little information.

Theorems 5 and 6, finally, deal with a situation in which voters' preferences are heterogeneous. In that case, the fraction of voters who acquire information goes to zero as the size of the electorate increases. This is reminiscent of the work by Feddersen and Pesendorfer [11]. Feddersen and Pesendorfer show that the fraction of "swing" voters who use their private information in order to decide whom to vote for declines to zero in large elections. Our model shows that the quality of information acquired by (individual) swing voters also declines to zero. However, the best alternative is chosen with probability strictly larger than $\frac{1}{2}$ as long as $C'(0) = C''(0) = 0$, and with probability approaching one as long as

$C'(0) = C''(0) = C'''(0) = 0$. These conditions are considerably more stringent than those that apply when voters have common preferences. Intuitively, we are a step closer to the traditional problem of public good provision in that most voters do not contribute to a better informed electorate.

Taken together, our results support the idea that elections serve the interests of the majority better than what the rational ignorance hypothesis would seem to indicate at first glance, at least if swing voters have common preferences. They suggest that models of public opinion that take into account the production of information by the media, interest groups, and the like, can be enriched by considering the aggregate implications of voters investing some small (but positive) effort in costly information processing.

Our model is related to the literature on information aggregation in elections inspired by Condorcet's jury theorem [15,3,11,14,9]. This literature typically assumes that there is some information dispersed among the voters, while in our paper the distribution of information arises endogenously through the actions of voters. As a consequence, we obtain that larger electorates are beneficial for society in some circumstances, but not in others. (Lack of information aggregation in large elections is obtained also by Yariv [21] in a context in which private signals carry less information as the electorate grows, and by Razin [19] in a context in which voters use their vote as a message to influence the policy of the winning candidate.)

Recently, Mukhopadhaya [16] and Persico [17] have proposed other models of endogenous information in collective decision making. In their models, the quality of the signal is given; voters can either acquire or not acquire information. As a consequence, in their models it is not possible to have arbitrarily large numbers of voters acquiring arbitrarily poor information. Persico, in particular, is concerned with the optimal design of committees, i.e. the optimal selection of committee size and voting rule in situations in which large elections are inefficient, while we concern ourselves with the endogenous production and aggregation of information in situations in which large elections may be asymptotically efficient.

2. The model

We analyze an election with two alternatives, A and B. There are $2n + 1$ voters ($i = 1, \ldots, 2n + 1$). A voter's utility depends on the chosen alternative $d \in \{A, B\}$, the state $z \in \{z_A, z_B\}$, and the quality of information acquired by the voter before the election $x \in [0, \frac{1}{2}]$. Acquiring information of quality x has a utility cost given by $C(x)$, so the utility of a voter can be written as

$$U(d, z) - C(x).$$

At the beginning of time, nature selects the state. The prior probability of state z_A is $q_A \in (0, 1)$ and the prior probability of state z_B is $q_B = 1 - q_A$. Voters are uncertain about the realization of the state. After the realization of the state, each voter must decide the quality of her information. After deciding on x, the voter receives a signal $s \in \{s_A, s_B\}$. The probability of receiving signal s_A in state z_A is equal to the probability of receiving signal s_B in state z_B and is given by $\frac{1}{2} + x$. That is, the likelihood of receiving the "right" signal is increasing in the quality of information acquired by the voter; if the voter acquires no information the signal is uninformative. Signals are private information.

The election takes place after voters receive their signals. A voter can either vote for A or vote for B. (That is, there are no abstentions.) The alternative with most votes is chosen.

We assume

$$U(A, z_A) - U(B, z_A) = r_A > 0 \text{ and } U(B, z_B) - U(A, z_B) = r_B > 0.$$

That is, A is the "right" alternative in state z_A and B is the "right" alternative in state z_B.

The cost function C is strictly increasing, strictly convex, and twice continuously differentiable on $(0, \frac{1}{2})$. We assume that $C(0) = 0$, so that acquiring no information is costless. Note that $C'(0) \in [0, \infty)$. If $C''(x)$ grows unboundedly as x goes to zero, we use the notation $C''(0) = \infty$. Thus, $C''(0) \in [0, \infty]$.

We may prefer to think of the probability of receiving the right signal as a function $\frac{1}{2}+m(e)$ of some underlying "effort level" by the voter with utility cost $\Psi(e)$. In that case, the quality of information x is defined as $m(e)$, and the cost of information C is defined as $\Psi \circ m^{-1}$. C satisfies the assumptions of the model if m is concave and Ψ is convex (one of them strictly so), and both are strictly increasing and twice continuously differentiable on $(0, \overline{e})$ for some $\overline{e} > 0$, with $\Psi(0) = m(0) = 0$ and $m(\overline{e}) \leqslant \frac{1}{2}$. Intuitively, the requirements on C are satisfied if the marginal productivity of a voter's effort in the acquisition of information is constant or diminishing, and the marginal cost of effort is increasing. Note that we can always recover the effort level associated to a given information quality by using the one-to-one relation $e = m^{-1}(x)$.

After describing the environment, we turn now to the description of strategies and the definition of equilibrium in the model. A pure strategy is a triple (x, v_A, v_B), where $x \in [0, \frac{1}{2}]$ specifies a quality of information, $v_A \in \{A, B\}$ specifies which alternative to vote for after receiving signal s_A, and $v_B \in \{A, B\}$ specifies which alternative to vote for after receiving signal s_B. A mixed strategy for voter i is a probability distribution α_i over the set of pure strategies.

A *voting equilibrium* $\overline{\alpha}$ ($\alpha_i = \alpha$ for all i) is a symmetric Nash equilibrium. An *equilibrium with information acquisition* is a voting equilibrium such that the equilibrium distribution assigns positive probability to the set of pure strategies with $x > 0$.

Obviously, there are at least two equilibria without information acquisition: for every voter to adopt the pure strategy $(0, A, A)$ with probability one, and for every voter to adopt the pure strategy $(0, B, B)$ with probability one. In either case, the probability that a single voter is decisive is zero, so it is a best response to acquire no information and vote for the alternative favored by every other voter. In fact, as long as $q_A r_A \neq q_B r_B$ or $C'(0) = 0$, there is no other equilibrium without information acquisition. If voters adopt any other mixed strategy without information acquisition, the probability that a single voter is decisive is positive and the same in both states. But then, if, say, $q_A r_A > q_B r_B$, the pure strategy $(0, A, A)$ has a higher payoff than any other pure strategy with $x = 0$. If, instead, $q_A r_A = q_B r_B$ and $C'(0) = 0$, it is a best response for a voter to acquire some information with probability one. (See the proof of Theorem 1.) We focus on equilibria with information acquisition in the remainder of the paper.

3. Rational ignorance

For the ease of presentation, from here to Section 5 we consider the case in which $q_A r_A = q_B r_B$, so that neither alternative is favored by prior beliefs and preferences. The following theorem states that $C'(0) = 0$ is a necessary and sufficient condition for the existence of an equilibrium with information acquisition in a large election, and characterizes this equilibrium.

Theorem 1. (i) *If* $C'(0) = 0$ *and* $q_A r_A = q_B r_B$, *there is an equilibrium with information acquisition, and it is unique within the class of equilibria with information acquisition. The equilibrium strategy gives probability one to the pure strategy* (x^*, A, B), *where* x^* *solves*

$$2\binom{2n}{n}\left(\tfrac{1}{4} - x^2\right)^n q_A r_A = C'(x). \tag{1}$$

(ii) *If* $C'(0) > 0$, *there is some* $\overline{n}$ *such that for every* $n \geqslant \overline{n}$ (*holding the other parameters of the model constant*) *there is no equilibrium with information acquisition.*

(The proof of this and other results in the paper is in the Appendix.) Intuitively, if a pure strategy with information acquisition is played with positive probability in equilibrium, then it must equate the marginal cost with the marginal benefit of acquiring information. The marginal benefit of acquiring information, in turn, is equal to the sum of the probabilities of being decisive, conditional on each state being realized, multiplied by the utility gain in choosing the right alternative. A voter is decisive if n other voters vote for A and n other voters vote for B. Since the probability of this event converges uniformly to zero as the size of the electorate increases for any symmetric strategy profile, it follows that there cannot be an equilibrium with information acquisition for n large enough if $C'(0) > 0$. Therefore, $C'(0) = 0$ is a necessary condition for the existence of equilibria with information acquisition in large elections.

To check that $C'(0) = 0$ is also sufficient whenever $q_A r_A = q_B r_B$, consider a symmetric strategy profile in which every voter adopts a pure strategy (x, A, B) (with $x > 0$) with probability one. Eq. (1) equates the marginal benefit with the marginal cost of acquiring information for that strategy profile. In particular, the probability of being decisive for that profile is the same for both states and is given by

$$\binom{2n}{n}\left(\tfrac{1}{2} + x\right)^n\left(\tfrac{1}{2} - x\right)^n = \binom{2n}{n}\left(\tfrac{1}{4} - x^2\right)^n.$$

As explained in the proof, the other equilibrium requirement is that the pure strategy with information acquisition played with positive probability in equilibrium must have a higher or equal payoff than the pure strategies of acquiring no information and voting for a fixed alternative. This requirement is easily verified if $q_A r_A = q_B r_B$.

As described in the previous section, we can consider C as composed from two more primitive functions, Ψ and m, representing respectively the utility cost of effort and the gain in information quality as a function of effort. Since $C'(x) = \Psi'(e)/m'(e)$ for $x = m(e)$, it follows that $C'(0) = 0$ iff either $\Psi'(0) = 0$ or $m'(0) = \infty$. Thus, Theorem 1 states that there is a equilibrium with information acquisition in large elections if and only if the marginal

Table 1
The quadratic example $C(x) = 5x^2$; $q_A r_A = q_B r_B = \frac{1}{2}$

Electorate size	Information quality	Probability of right decision	Aggregate cost
1	0.10000	0.60000	0.05000
11	0.02432	0.56558	0.03253
101	0.00786	0.56293	0.03121
1001	0.00249	0.56265	0.03107
10001	0.00079	0.56262	0.03105
100001	0.00025	0.56262	0.03105

Table 2
The cubic example $C(x) = 33\frac{1}{3}x^3$; $q_A r_A = q_B r_B = \frac{1}{2}$

Electorate size	Information quality	Probability of right decision	Aggregate cost
1	0.10000	0.60000	0.03333
11	0.04845	0.62913	0.04170
101	0.02632	0.70220	0.06139
1001	0.01330	0.80018	0.07859
10001	0.00613	0.89002	0.07688
100001	0.00258	0.94868	0.05729

utility cost of effort is arbitrarily small for small effort, or if the marginal productivity of a voter's effort in the acquisition of information is arbitrarily large for small effort. Intuitively, we can think of the voter as having access to a wealth of useful information in exchange for exercising a little effort in paying attention.

Let x_n represent the value of x^* for a given n. Since the left-hand side of Eq. (1) converges to zero as n goes to infinity for any sequence of $x \in [0, \frac{1}{2}]$, x_n converges to zero as n goes to infinity. Note that in the equilibrium with information acquisition the probability of reaching the right decision is the same in both states and is given by

$$\sum_{m=n+1}^{2n+1} \binom{2n+1}{m} (\tfrac{1}{2} + x_n)^m (\tfrac{1}{2} - x_n)^{2n+1-m}.$$

Though x_n goes to zero, this probability may not go to $\frac{1}{2}$ since the number of terms in the summation increases with n. That is, the strong version of the rational ignorance hypothesis may fail even if the weak version holds, as suggested by the examples below.

Consider this *quadratic example*: $C(x) = 5x^2$ and $q_A r_A = q_B r_B = \frac{1}{2}$. The second column of Table 1 gives us the values of x_n for different electorate sizes $(2n + 1)$, including the case of a single decision-maker. The third column gives us the probability of reaching the right decision. Though this probability decreases with n, it seems to converge to 0.56262. The aggregate cost of information acquisition, $(2n + 1)C(x_n)$, also decreases with n and it seems to converge around 0.031.

Now consider this *cubic example*: $C(x) = 33\frac{1}{3}x^3$ and $q_A r_A = q_B r_B = \frac{1}{2}$, illustrated by Table 2. In contrast with the previous example, the probability of reaching the right decision seems to approach one. Though initially increasing, the aggregate cost is eventually decreasing in the electorate size. The next section reveals that these examples are, in fact, representative of general results.

4. Information acquisition in large elections

In this section we analyze the limiting properties of the sequence of equilibria with information acquisition obtained by increasing the size of the electorate.

4.1. Information aggregation

We investigate here the limit probability of reaching the right decision. If $C''(0) = c \in (0, \infty)$, let k be the solution to

$$2\sqrt{2}\,\phi(2\sqrt{2}k)\,q_A r_A = kc. \tag{2}$$

(We use ϕ to denote the standard normal density and Φ to denote the standard normal distribution function.) Note that k is strictly decreasing in c and grows unboundedly as c goes to zero. For the sake of completeness, we define $k = \infty$ if $C''(0) = 0$ and $k = 0$ if $C''(0) = \infty$, and we use the convention $\Phi(\infty) = 1$. As we will see below, k is an indicator of the information held by the electorate in large elections. We have

Theorem 2. *Assume $C'(0) = 0$ and $q_A r_A = q_B r_B$. Along the sequence of equilibria with information acquisition, the probability of choosing the right alternative converges to $\Phi(2\sqrt{2}k)$. In particular, it converges to one if $C''(0) = 0$.*

The first part of the proof of Theorem 2 establishes that $n^{1/2}x_n$ goes to k as n goes to infinity. To provide an intuition, we can obtain from Eq. (1), using a Taylor approximation in the right-hand side and a Stirling approximation in the left-hand side,

$$2q_A r_A \pi^{-1/2} n^{-1/2} (1 - 4x_n^2)^n \approx x_n C''(0).$$

Rearranging,

$$2q_A r_A \pi^{-1/2} [1 - 4(n^{1/2}x_n)^2/n]^n \approx n^{1/2} x_n C''(0).$$

Using an approximation (for large n and fixed $n^{1/2}x_n$) for the term in brackets on the left-hand side we get

$$2q_A r_A \pi^{-1/2} \exp\{-4(n^{1/2}x_n)^2\} \approx n^{1/2} x_n C''(0).$$

If $C''(0) = 0$, this expression cannot hold unless $n^{1/2}x_n$ goes to $+\infty$; otherwise the right-hand side goes to zero and the left-hand side remains bounded away from zero. If $C''(0) = c \in (0, \infty)$, $k \in (0, \infty)$ can be obtained directly from the expression above.

With respect to the rest of the proof, note that a voter votes for the right alternative with probability equal to $\frac{1}{2} + x_n$. Thus, the expected number of votes for the right alternative of $(2n + 1)(\frac{1}{2} + x_n)$ and the variance is $(2n + 1)(\frac{1}{4} - x_n^2)$. A normal approximation to the probability of the right alternative winning the election would give us

$$\Phi\left(-\frac{n - (2n + 1)(\frac{1}{2} + x_n)}{\sqrt{(2n + 1)(\frac{1}{4} - x_n^2)}} \right).$$

Taking limits, we get $\Phi(2\sqrt{2}k)$. A "naive" application of the central limit theorem as described is not really appropriate because the distribution representing the decision of a given voter changes with the electorate size. Instead, we use a normal approximation result for finite samples, the Berry–Esseen theorem.

We can interpret Eq. (2) as a "first order condition" at infinity if $C'(0) = 0$, $C''(0) = c \in (0, \infty)$ and $q_A r_A = q_B r_B$. To see this, if every voter acquires some information, the expected utility of voter i (up to a positive affine transformation) is approximately

$$\Phi\left(\frac{(2n+1)\overline{x}}{\sqrt{(2n+1)(\frac{1}{4})}}\right)(2q_A r_A) - cx_i^2/2,$$

where $\overline{x}$ is the average quality acquired by the $2n+1$ voters and x_i the information quality acquired by voter i. Maximizing this expression with respect to x_i we get

$$2(2n+1)^{-1/2}\,\phi(2(2n+1)^{1/2}\overline{x})(2q_A r_A) = x_i c,$$

or for large n,

$$\sqrt{2}\,\phi(2\sqrt{2}n^{1/2}\overline{x})(2q_A r_A) \approx n^{1/2} x_i c.$$

Eq. (2) follows from $k \approx n^{1/2} x_i = n^{1/2}\overline{x}$.

To illustrate Theorem 2, note that for the quadratic example we can calculate k as 0.055723. In fact, in this example $5000^{1/2} x_{5000}$=0.055721 and $50000^{1/2} x_{50000}$=0.055723. Moreover, the limit probability of society making the right decision is 0.56262. Thus, Theorem 2 provides an excellent approximation for the quadratic example with 10,000 voters or more. For the cubic example, $k = +\infty$, and the limit probability of society making the right decision is one. With 100,000 voters we are still some way off the limit.

In terms of the decomposition of C in Ψ and m, we have that

$$C''(x) = (\Psi''(e)m'(e) - \Psi'(e)m''(e))/(m'(e))^3$$

for $x = m(e)$. Thus, the condition for the society making the right decision with probability one in the limit, $C'(0) = C''(0) = 0$, is obtained if $m''(e)$ and $\Psi''(e)$ are uniformly bounded and, in addition, either $\Psi'(0) = \Psi''(0) = 0$ or $m'(0) = \infty$.

A result related to Theorem 2 is that elections with information acquisition will tend to be very close. Define the *winning margin* to be a random variable representing the difference between the number of votes for the winner and the number of votes for the loser, divided by $2n+1$. We have

Proposition 1. *Assume $C'(0) = 0$ and $q_A r_A = q_B r_B$. For any $\kappa > 0$, the probability that the winning margin is larger than κ converges to zero along the sequence of equilibria with information acquisition as the size of the electorate increases.*

Intuitively, the mean of the distribution of the percentage of votes for the right alternative is $\frac{1}{2} + x_n$, which converges to $\frac{1}{2}$ from above. If $C''(0) = 0$, however, the distribution of the percentage of votes for the right alternative concentrates very fast around its central terms as n goes to infinity, so that the probability that this percentage is larger than $\frac{1}{2}$ goes to one.

4.2. The cost of information

We turn now to the cost of information acquisition in large elections. The following theorem gives us an estimate of the aggregate cost of information acquisition ($(2n+1)C(x_n)$) in large elections.

Theorem 3. *Assume $C'(0) = 0$ and $q_A r_A = q_B r_B$. If $C''(0) = c \in (0, \infty)$, then the aggregate cost converges to ck^2 along the sequence of equilibria with information acquisition as the size of the electorate increases. If either $C''(0) = 0$ or $C''(0) = \infty$, the aggregate cost converges to zero.*

To see this, note that

$$\lim_{n\to\infty} \{(2n+1)C(x_n)\} = 2 \lim_{n\to\infty} \{(n^{1/2}x_n)^2\, C(x_n)/x_n^2\}.$$

If $C''(0) = c \in (0, \infty)$, the statement of the theorem follows from $n^{1/2}x_n \to k$ (from the proof of Theorem 2) and $C(x_n)/x_n^2 \to c/2$ (by L'Hôpital's rule).

Substituting $z = ck^2$ in Eq. (2), we get

$$2q_A r_A \pi^{-1/2} \exp\{-z/c\} = \sqrt{zc}.$$

It is simple to check that $z \to 0$ if either $c \to 0$ or $c \to +\infty$. This shows that the aggregate cost of information is near 0 in a large election if $C''(0)$ is very small or very large. An argument in the Appendix (similar to the proof of Theorem 2) shows that in fact the aggregate cost converges to 0 if either $C''(0) = 0$ or $C''(0) = \infty$.

To illustrate this result, in the quadratic example we can compute $ck^2 = 0.03105$, so that we have an excellent approximation with 10,000 voters or more. In the cubic example, the aggregate cost should go to zero; with 100,000 voters we are still some way off.

In terms of the decomposition of C in Ψ and m, we can write the aggregate cost of information acquisition as $(2n + 1)\Psi(e_n)$, where e_n is the equilibrium effort level. Since e_n must satisfy the relation $e_n = m^{-1}(x_n)$, we get

$$\begin{aligned}(2n+1)\Psi(e_n) &= (2n+1)\Psi(m^{-1}(x_n)) \\ &= (2n+1)(\Psi \circ m^{-1})(x_n) \\ &= (2n+1)C(x_n).\end{aligned}$$

In other words, to determine the aggregate cost of information acquisition for any electorate size we do not need to know the functions Ψ and m separately; we only need to know the composite function $C = \Psi \circ m^{-1}$.

5. Efficiency and design

In this section we deal with normative issues such as the asymptotic efficiency of equilibria with information acquisition and the optimal size of the electorate.

5.1. Efficiency of large elections

In this section we investigate whether the equilibrium with information acquisition is efficient in the limit. For any (symmetric or asymmetric) strategy profile α^n, the utilitarian social welfare is, up to a positive affine transformation,

$$V^{\alpha^n} = (2n+1)(P_A^{\alpha^n} q_A r_A + P_B^{\alpha^n} q_B r_B) - C_T^{\alpha^n},$$

where $P_d^{\alpha^n}$ is the probability of choosing alternative d in state z_d under the strategy profile α^n, and $C_T^{\alpha^n}$ is the total expected cost invested in information acquisition by society members. Note that $V^{\alpha^n} \leqslant (2n+1)(q_A r_A + q_B r_B)$.

Let α^{n*} be the information acquisition equilibrium strategy profile. We say that the equilibrium with information acquisition is *asymptotically efficient* if for every $\varepsilon > 0$ there is some finite m such that for all $n \geqslant m$,

$$V^{\alpha^{n*}} / V^{\alpha^n} \geqslant 1 - \varepsilon$$

for all strategy profiles $\alpha^n = (\alpha_1, \ldots, \alpha_{2n+1})$.

From Theorems 2 and 3, we know that along the sequence of equilibria with information acquisition the probability of choosing the right alternative converges to one and the aggregate cost converges to zero as n goes to infinity if $C'(0) = C''(0) = 0$ and $q_A r_A = q_B r_B$. Thus, under those conditions, $V^{\alpha^{n*}}/[(2n+1)(2q_A r_A)] \to 1$, so the equilibrium with information acquisition is asymptotically efficient.

If $C''(0) > 0$, however, the equilibrium with information acquisition is not asymptotically efficient. To see this, note that

$$V^{\alpha^{n*}}/[(2n+1)(2q_A r_A)] \to \Phi(2\sqrt{2}k) < 1.$$

Let α^{no} represent the optimal symmetric profile. We have

Proposition 2. *If* $C'(0) = 0$, $C''(0) < \infty$, *and* $q_A r_A = q_B r_B$,

$$V^{\alpha^{no}}/[(2n+1)(2q_A r_A)] \to 1.$$

The proof is similar to those of Theorems 2 and 3, and establishes that the probability of choosing the right alternative converges to one, and the average cost (as opposed to the aggregate cost) converges to zero, along the sequence of optimal symmetric profiles. The asymptotic inefficiency of the equilibrium with information acquisition follows from the proposition if $0 < C''(0) < \infty$.

If $C''(0) = \infty$, we can compare the information acquisition equilibrium profile with the asymmetric profile in which only the first voter is asked to acquire some positive amount of information ρ and vote according to the signal received, while n voters vote for A and n voters vote for B regardless of the signal. The ratio of social welfare under the information acquisition equilibrium profile to social welfare under the asymmetric profile described converges to $(\frac{1}{2})/(\frac{1}{2} + \rho) < 1$. Thus, the equilibrium with information acquisition is asymptotically inefficient if $C''(0) = \infty$. Of course, a similar argument shows that equilibria without information acquisition are never asymptotically efficient.

For instance, consider the quadratic example. Social welfare in the information acquisition equilibrium with 100,001 voters is given by 56,263. The optimal symmetric profile would prescribe x around 0.007, which yields a social welfare of 99,976. Now consider the cubic example. Social welfare in the information acquisition equilibrium is given by 94,869. The optimal symmetric profile would prescribe x around 0.008 which yields a social welfare of 99,999, very close to the upper bound, 100,001. Both in the quadratic and the cubic example, any equilibrium without information acquisition would yield a social welfare of 50000.5.

5.2. *The optimal size of the electorate*

Consider a society with N members, where N is an odd number. If the society gets to choose the size of the electorate $1 \leqslant 2n+1 \leqslant N$, anticipating that the voters will play the information acquisition equilibrium, what would be the optimal choice? We now let $n=0$ represent the choice of a single decision-maker.

Since per capita social welfare converges to its upper bound $q_A r_A + q_B r_B$ as the size of the electorate increases whenever $C'(0) = C''(0) = 0$ and $q_A r_A = q_B r_B$, it follows that under those conditions for a large society the optimal size of the electorate is either the size of the society or near it. That is, universal suffrage is at least nearly optimal. Moreover, under the (quite mild) condition $C'(\frac{1}{2}) > 2q_A r_A$, guaranteeing that a single decision-maker would not acquire perfect information, social welfare under universal suffrage is larger than under delegation to a small committee for a large society. In the cubic example, for instance, a society with 100,001 members would find that social welfare is increasing in participation in elections.

Universal suffrage, however, is not necessarily nearly optimal if $C'(0) = 0$ and $C''(0) > 0$. In the quadratic example, for instance, a society with 100,001 members would be better off by delegating the decision on a single person rather than holding elections. If we replace the cost function in this example with

$$C(x) = \begin{cases} 5x^2 & \text{if } x \leqslant 0.02, \\ -0.0008 + 0.12x - x^2 + 100x^3 & \text{if } x \geqslant 0.02, \end{cases}$$

then the society would be better off by delegating the decision on a committee with eleven members. If we further replace the cost function with

$$C(x) = 5x^2 + 3000x^3,$$

then the society would be better off by holding elections with universal suffrage. That is, by changing the shape of the cost function away from 0 we can get any electorate size to be optimal.

Finally, if $C'(0) > 0$, universal suffrage cannot be optimal or nearly optimal in a large society as long as $C'(0) < 2q_A r_A$; that is, as long as a single-decision maker is willing to acquire some information. A sufficient condition for a single-decision maker to be optimal in a large society is $q_A r_A \leqslant C'(0) < 2q_A r_A$; in this case the free-riding problem is so severe that committee members would not acquire any information. If $C'(0) < q_A r_A$, then, as in the previous case, the optimal size of the electorate depends on the shape of the cost function away from 0.

6. Robustness

In this section we show that our previous results hold under certain conditions if prior beliefs and preferences are asymmetric. Relaxing the common preference assumption of the model, however, is more problematic.

6.1. Asymmetric preferences

In this section we relax the assumption that $q_A r_A = q_B r_B$. As it turns out, if $C''(0) = 0$ a (mixed strategy) information acquisition equilibrium exists in large elections, and perfect information aggregation and zero aggregate costs are obtained in the limit regardless of any asymmetry in preferences (and prior beliefs). However, if $C''(0) > 0$, an information acquisition equilibrium in a large election exists if and only if the asymmetry in preferences is moderate.

If $q_A r_A \neq q_B r_B$, adopting with probability one the pure strategy that equates the marginal benefit and cost of acquiring information (as in Section 3) cannot be an equilibrium for n large enough. For this strategy profile, beliefs about the states, conditional on being pivotal, are equal to prior beliefs, and hence independent of n. It is easy to check that this establishes a lower bound, independent of n, on the information that a voter is willing to acquire. Intuitively, if prior beliefs favor voting for A, a voter will acquire costly information only if after receiving a signal favoring B, her posterior beliefs about the states change enough to induce the voter to vote for B. However, the information acquired by each voter must converge to zero as n increases. The solution to this difficulty consists in allowing voters to randomize between the pure strategy of voting for the alternative favored by preferences and the pure strategy of acquiring information and voting according to the signal received. Under this mixed strategy, beliefs about the states conditional on being decisive remain close enough to make it worth acquiring vanishingly little information.

The difficulty alluded above when $q_A r_A \neq q_B r_B$ is a simple illustration of a classic result by Radner and Stiglitz [18] on the nonconcavity of the value of information. Radner and Stiglitz show that it is unprofitable to acquire "very little" information when there is a decision that is favored by prior beliefs, and posterior beliefs and the payoff from the corresponding optimal decision vary continuously with the amount of information. In our setup, however, what matters for voters are not their prior beliefs over the states, but their beliefs about the states conditional on being decisive. This gives us a way around the nonconcavity of the value of information.

If $C''(0) = c \in (0, \infty)$ and $q_A r_A \neq q_B r_B$, let $(\hat{k}, \hat{h})$ be the solution to the system

$$\sqrt{2}\,\phi(\sqrt{2}(2k' + h'))\,q_A r_A + \sqrt{2}\,\phi(\sqrt{2}(2k' - h'))\,q_B r_B \quad = \quad k'c, \tag{3.1}$$

$$\phi(\sqrt{2}(2k' + h'))\,q_A r_A \quad = \quad \phi(\sqrt{2}(2k' - h'))\,q_B r_B, \tag{3.2}$$

$$k' > |h'/2|, \tag{3.3}$$

if a solution exists. As shown in the Appendix, the system (3.1)–(3.3) has a solution if $q_A r_A$ and $q_B r_B$ are close enough or c is close enough to zero, and the solution is unique. Moreover, if $(\hat{k}, \hat{h})$ exists, $\hat{h} > 0$ iff $q_A r_A > q_B r_B$, and $\hat{h} < 0$ iff $q_A r_A < q_B r_B$. If $C''(0) > 0$ and $q_A r_A = q_B r_B$, we let $(\hat{k}, \hat{h}) = (k, 0)$, where k is as defined in Section 4.

As shown in the proof of Theorem 4, if $(\hat{k}, \hat{h})$ exists and $q_A r_A > q_B r_B$, there is an equilibrium with information acquisition in large elections in which voters adopt the pure strategy $(0, A, A)$ with probability δ_n, and the pure strategy $(\hat{x}_n, A, B)$ with probability $1 - \delta_n$. Moreover, $n^{1/2}\hat{x}_n$ converges to $\hat{k}$, and $n^{1/2}\delta_n$ converges to $\hat{h}$. (Similarly, if $q_A r_A < q_B r_B$, the pure strategy $(0, B, B)$ is adopted with probability δ_n, and $n^{1/2}\delta_n$ converges to $-\hat{h}$.)

Theorem 4. (i) *If $C'(0) = C''(0) = 0$, there is some $\overline{n}$ such that for every $n \geqslant \overline{n}$, there is an equilibrium with information acquisition. Moreover, along a sequence of equilibria with information acquisition, the probability of choosing the right alternative in either state converges to one and the aggregate cost of information converges to zero.*

(ii) *If $C'(0) = 0$, $C''(0) = c > 0$, and $(\hat{k}, \hat{h})$ exists, there is some $\overline{n}$ such that for every $n \geqslant \overline{n}$, there is an equilibrium with information acquisition. Moreover, along a sequence of equilibria with information acquisition, the probability of choosing the right alternative converges to $\Phi(\sqrt{2}(2\hat{k}+\hat{h}))$ in state z_A and to $\Phi(\sqrt{2}(2\hat{k}-\hat{h}))$ in state z_B, and the aggregate cost of information converges to $c\hat{k}^2$.*

(iii) *If $C''(0) = \infty$, or if $C''(0) = c \in (0, \infty)$ and there is no solution to the system* (3.1)–(3.2), *there is some $\overline{n}$ such that for every $n \geqslant \overline{n}$, there is no equilibrium with information acquisition.*

As in Section 4.1, we can interpret Eq. (3.1) as a "first-order condition" at infinity, equating the marginal benefit with the marginal cost of information at the limit for a given voter. Eq. (3.2) makes the voter indifferent in the limit between voting for A or voting for B, in the absence of additional information coming from the signal. This is necessary to keep voters acquiring arbitrarily little information. Inequality (3.3) allows us to show that if there is a "limit" equilibrium, given by a solution to (3.1)–(3.2), then there is actually an information acquisition equilibrium for large n. [1]

Though the equilibria with information acquisition described in case (ii) are asymptotically inefficient, they yield a higher social welfare than any equilibria without information acquisition. If, say, $q_A r_A > q_B r_B$, this is the case in a large election if

$$\Phi(\sqrt{2}(2\hat{k} + \hat{h}))\, q_A r_A + \Phi(\sqrt{2}(2\hat{k} - \hat{h}))\, q_B r_B > q_A r_A$$

or, using (3.2) and symmetry of the normal distribution,

$$\phi(-\sqrt{2}(2\hat{k} + \hat{h}))/\Phi(-\sqrt{2}(2\hat{k} + \hat{h})) > \phi(\sqrt{2}(2\hat{k} - \hat{h}))/\Phi(\sqrt{2}(2\hat{k} - \hat{h})),$$

which is satisfied because the normal hazard rate is strictly decreasing.

Note that, when preferences are asymmetric, adopting a rule other than simple majority may alleviate the need to randomize between acquiring or not information. For instance, suppose that $q_A r_A > q_B r_B$. If the voting rule requires less than $n+1$ votes to choose A, then the ratio of the probability of being pivotal in state z_A to the probability of being pivotal in state z_B would fall below $\frac{1}{2}$ for any given symmetric pure strategy profile (x, A, B). This is

[1] There is gap between the cases covered by (ii) and (iii) in Theorem 3, so that in principle it may be possible to relax (3.3) and preserve existence of an equilibrium with information acquisition.

potentially important if $C'(0) = 0$ and $C''(0) > 0$, as in this case information acquisition equilibria under simple majority are asymptotically inefficient or may even fail to exist.

6.2. Heterogenous preferences

In this section we modify the model to allow for heterogeneous preferences among voters. In particular, we assume that, for each voter i, $U_i(A, z_A) = u_i$, where u_i is a random variable uniformly distributed over [0, 1]. The remaining preference parameters are given by $U_i(B, z_B) = 1 - u_i$ and $U_i(A, z_B) = U_i(B, z_A) = 0$. The random variables $\{u_i\}$ are independently distributed (from each other, from the distribution of the state, and from the distribution of signals about the state). The utility of voter i can be written now as $U_i(d, z) - C(x)$. We keep the assumptions about the cost of information acquisition from Section 2, and we add that C is thrice continuously differentiable. Each voter decides how much information to acquire after learning privately the realization of the preference parameter u_i. As in Section 2, the election takes place after voters receive their signals about the state $z \in \{z_A, z_B\}$. For simplicity, we assume $q_A = q_B = \frac{1}{2}$.

An *action* in the model with heterogenous preferences is defined as a triple (x, v_A, v_B), where $x \in [0, \frac{1}{2}]$ specifies a quality of information, $v_A \in \{A, B\}$ specifies which alternative to vote for after receiving signal s_A, and $v_B \in \{A, B\}$ specifies which alternative to vote for after receiving signal s_B. A *strategy* for voter i is now a (measurable) mapping

$$\sigma_i(u_i) : [0, 1] \to [0, \tfrac{1}{2}] \times \{A, B\} \times \{A, B\},$$

specifying an action for every realization of the preference parameter u_i. (For simplicity, we omit considering strategies that allow for randomizing over actions.) An *equilibrium* $\overline{\sigma}$ ($\sigma_i = \sigma$ for all i) is a symmetric Nash equilibrium. An *equilibrium with information acquisition* is an equilibrium such that the distribution over actions (induced by the distribution of preferences and by the equilibrium mapping) assigns positive probability to the set of actions with $x > 0$.

We have

Theorem 5. *In the model with heterogeneous preferences, if $C'(0) = 0$, there is an equilibrium with information acquisition, and it is unique (a.e.) within the class of equilibria with information acquisition. The equilibrium mapping is (a.e.)*

$$\sigma(u_i) = \begin{cases} (0, B, B) & \text{if } u_i < \frac{1}{2} - \tilde{x}_n + C(\tilde{x}_n)/C'(\tilde{x}_n), \\ (0, A, A) & \text{if } u_i > \frac{1}{2} + \tilde{x}_n - C(\tilde{x}_n)/C'(\tilde{x}_n), \\ (\tilde{x}_n, A, B) & \text{otherwise,} \end{cases}$$

where $\tilde{x}_n$ satisfies

$$\binom{2n}{n} \left(\tfrac{1}{4} - 4(\tilde{x}_n - C(\tilde{x}_n)/C'(\tilde{x}_n))^2 \tilde{x}_n^2 \right)^n = 2C'(\tilde{x}_n). \qquad (4)$$

Finally, if $C'(0) > 0$, there is some $\overline{n}$ such that for $n \geqslant \overline{n}$ there is no equilibrium with information acquisition.

Intuitively, the probability of being decisive is too small for a voter with a strong bias in favor of A or in favor of B to be willing to acquire any costly information.

Since $\tilde{x}_n$ converges to zero as n goes to infinity, the fraction of voters who acquire information $(2\tilde{x}_n - 2C(\tilde{x}_n)/C'(\tilde{x}_n))$ converges to zero. We may wonder whether "good" information aggregation results are possible with a vanishing fraction of voters acquiring vanishingly little information. The answer to this question is yes, but under more restrictive conditions than in the model with common preferences.

If $C'(0) = C''(0) = 0$ and $C'''(0) = \tilde{c} \in (0, \infty)$, let $\tilde{k}$ be the solution to

$$2\sqrt{2}\,\phi(2\sqrt{2}\tilde{k}) = (3/2)\tilde{k}\tilde{c}.$$

If $C'(0) = C''(0) = C'''(0) = 0$, let $\tilde{k} = \infty$. If $C'''(0) = \infty$, let $\tilde{k} = 0$. We have

Theorem 6. *Assume $C'(0) = 0$. In the model with heterogeneous preferences, along the sequence of equilibria with information acquisition, the probability of choosing the right alternative converges to $\Phi(2\sqrt{2}\tilde{k})$. In particular, it converges to one if $C''(0) = C'''(0) = 0$. Moreover, the expected aggregate cost of information converges to $\tilde{c}\tilde{k}^2/4$ if $C''(0) = 0$ and $C'''(0) = \tilde{c} < \infty$, and it converges to 0 otherwise.*

The proof of Theorem 6 hinges on the fact that, with a large n, the fraction of voters who acquire information is approximately $4\tilde{x}_n/3$. Thus, the probability of choosing the right alternative depends on $4\tilde{x}_n^2 n^{1/2}/3$. The limit of this expression is precisely $\tilde{k}$.

Theorems 5 and 6 are reminiscent of similar results by Feddersen and Pesendorfer [11]. In a model with free information and heterogeneous preferences, they show that a vanishing fraction of voters takes into account their private information when casting a vote. However, perfect information aggregation is obtained in the limit. In our model, not only the fraction of swing voters but also the information acquired by each swing voter goes to zero. Thus, perfect information aggregation is obtained only under special assumptions with respect to the cost function. If those assumptions fail, the equilibrium with information acquisition is asymptotically inefficient.

In terms of the decomposition of C in Ψ and m, we have

$$\begin{aligned} C'''(x) = {} & (\Psi'''(e)m'(e) - \Psi'(e)m'''(e))/(m'(e))^4 \\ & -3m''(e)(\Psi''(e)m'(e) - \Psi'(e)m''(e))/(m'(e))^5 \end{aligned}$$

for $x = m(e)$. Thus, the assumption for perfect information aggregation, $C'(0) = C''(0) = C'''(0)$, is obtained if the second and third derivatives of Ψ and m are uniformly bounded and, in addition, either $\Psi'(0) = \Psi''(0) = \Psi'''(0) = 0$ or $m'(0) = \infty$. From this perspective, perfect information aggregation can be obtained with heterogeneous preferences if the marginal productivity of a voter's effort in the acquisition of information is arbitrarily large for small effort.

7. Final remarks

In a setting in which acquiring political information is costly, we have shown that the electorate as a whole may be much better informed than individual voters. In some circumstances, a result analogous to Condorcet's jury theorem is upheld: increasing the size of the electorate improves social welfare even taking into account the voters' costs. In some other circumstances, though, Condorcet's contention about the superiority of larger electorates fails.

In the environment we study, a small deviation from rationality by voters—ignoring completely the effects of a single opinion—would have important negative effects on the responsiveness of collective decision making to the interests of the majority. Akerlof [1] has approached the issue of rational ignorance from that perspective. However, deviations from strictly rational beliefs may be as likely to occur in the direction of overestimating the importance of a single opinion as in the direction of underestimating it. Voters may derive some satisfaction from the belief that their vote counts for more than it actually does, and overinvest in political information for that reason.

We have represented information acquisition by voters as a strictly individual endeavor. If voters can communicate their information to others before the election, if different voters have access to the same sources of information, or if sources of information compete for subscribers, strategic considerations will differ from those in the current framework in nontrivial ways. There is clearly a need for more formal research on the issue of endogenous production and aggregation of information in large elections, perhaps in connection with the recent interest in pre-election communication by privately informed voters [4,13], and our individualistic framework is meant as a first step.

Appendix

For any symmetric strategy profile α and for any given voter, we define $P_\alpha(\text{piv}|z)$ as the probability that n other voters vote for A and n other voters vote for B in state $z \in \{z_A, z_B\}$. $P_\alpha(\text{piv}|z)$ represents the probability that a single vote is pivotal (i.e. decisive) in state z. Letting $p_\alpha(d|z)$ denote the probability that a voter votes for alternative d in state z, as induced by the strategy α, we have

$$P_\alpha(\text{piv}|z) = \frac{(2n)!}{n!n!}\left(p_\alpha(A|z)p_\alpha(B|z)\right)^n .$$

Since $p_\alpha(A|z)p_\alpha(B|z)$ is bounded above by $\frac{1}{4}$, $P_\alpha(\text{piv}|z)$ is bounded above by $(2n)!/(2^{2n}n!n!)$, which converges to zero as n goes to infinity. Thus, the probability of being decisive in either state converges uniformly to zero for any sequence of symmetric strategy profiles as n goes to infinity. We use this fact throughout the proofs.

Proof of Theorem 1. It is easy to show that, for any $x > 0$, the pure strategies (x, A, A), (x, B, B), and (x, B, A) are strictly dominated. Thus, for any pure strategy played in equilibrium with positive probability, $x > 0$ implies $v_A = A$ and $v_B = B$.

Now, suppose that every voter other than i adopts the strategy α. Then, the expected utility for voter i of adopting any pure strategy (x_i, A, B) for $x_i \geqslant 0$ is given by

$$G_\alpha(x_i) = \left[P_\alpha(\text{piv}|z_A)q_A r_A + P_\alpha(\text{piv}|z_B)q_B r_B\right]\left(\tfrac{1}{2} + x_i\right) - C(x_i)$$

plus a term that does not depend on the action chosen by i. Note that this expected utility is a strictly concave function of x_i. Thus, it is maximized by a unique choice of x_i, say x'. It follows that a best-responding voter plays at most one pure strategy with information acquisition with positive probability, the pure strategy (x', A, B). Moreover, the expected payoff of the pure strategy $(0, A, B)$ is a convex combination of the expected payoffs of $(0, A, A)$ and $(0, B, B)$. It follows that if $x' > 0$, then (x', A, B) has a higher payoff than either $(0, A, A)$ or $(0, B, B)$. Moreover, if the expected payoffs of $(0, A, A)$ and $(0, B, B)$ are not equal, one of these pure strategies has a higher payoff than $(0, A, B)$ and $(0, B, A)$. Thus, if a best-responding voter plays an strategy with information acquisition, the support of this strategy is either (I) $\{(x', A, B)\}$, (II) $\{(x', A, B), (0, A, A)\}$, or (III) $\{(x', A, B), (0, B, B)\}$.

Since the term in brackets in the definition of G_α converges to zero for any sequence of symmetric strategy profiles (and is independent of x_i), x' must be equal to 0 for n large enough if $C'(0) > 0$. This proves part (ii) of the theorem.

Now consider a possible information acquisition equilibrium strategy of type (I). Since the term in brackets in G_α is equal to zero if every voter adopts the pure strategy $(\frac{1}{2}, A, B)$, it follows that the information acquired by every voter satisfies $0 < x^* < \frac{1}{2}$. Thus, the solution to the problem of maximizing $G_\alpha(x_i)$ is interior, so that x^* must satisfy

$$P_\alpha(\text{piv}|z_A)q_A r_A + P_\alpha(\text{piv}|z_B)q_B r_B = C'(x^*).$$

Moreover, if every voter adopts the pure strategy (x^*, A, B) with probability one, we get

$$P_\alpha(\text{piv}|z) = \binom{2n}{n}(\tfrac{1}{2} + x^*)^n(\tfrac{1}{2} - x^*)^n = \binom{2n}{n}(\tfrac{1}{4} - (x^*)^2)^n$$

for $z = z_A, z_B$. Using these two equations we obtain the equilibrium condition

$$\binom{2n}{n}(\tfrac{1}{4} - (x^*)^2)^n(q_A r_A + q_B r_B) = C'(x^*).$$

Note that x^* exists and is unique for every n as long as $C'(0) = 0$. Additionally, the pure strategy (x^*, A, B) must yield a higher payoff than the pure strategies $(0, A, A)$ and $(0, B, B)$. That is,

$$G_\alpha(x^*) \geqslant \max\{P_\alpha(\text{piv}|z_A)q_A r_A, P_\alpha(\text{piv}|z_B)q_B r_B\}.$$

Using the two equilibrium conditions we obtain

$$x^* - \frac{1}{2}\frac{|q_A r_A - q_B r_B|}{q_A r_A + q_B r_B} \geqslant \frac{C(x^*)}{C'(x^*)}. \tag{5}$$

This inequality is always satisfied for $q_A r_A = q_B r_B$, so that if $C'(0) = 0$ and $q_A r_A = q_B r_B$ there is a unique equilibrium of type (I) and it is as described in part (i) of the theorem.

Consider a possible information acquisition equilibrium strategy of type (II). Let δ be the probability that $(0, A, A)$ is played under the proposed strategy and let (x, A, B) the strategy with information acquisition that is played with probability $(1 - \delta)$. Then if every voter other than i adopts the proposed strategy,

$$P_\alpha(\text{piv}|z_A) = \binom{2n}{n}((\tfrac{1}{2}+x)(1-\delta)+\delta)^n((\tfrac{1}{2}-x)(1-\delta))^n$$
$$= \binom{2n}{n}(\tfrac{1}{4}-(x+\delta(\tfrac{1}{2}-x))^2)^n$$

and

$$P_\alpha(\text{piv}|z_B) = \binom{2n}{n}((\tfrac{1}{2}-x)(1-\delta)+\delta)^n((\tfrac{1}{2}+x)(1-\delta))^n$$
$$= \binom{2n}{n}(\tfrac{1}{4}-(x-\delta(\tfrac{1}{2}+x))^2)^n.$$

The equilibrium conditions are

$$P_\alpha(\text{piv}|z_A)q_Ar_A + P_\alpha(\text{piv}|z_B)q_Br_B = C'(x) \tag{6}$$

(for n large enough) and

$$G_\alpha(x) = P_\alpha(\text{piv}|z_A)q_Ar_A \geqslant P_\alpha(\text{piv}|z_B)q_Br_B. \tag{7}$$

It is easy to check that $P_\alpha(\text{piv}|z_A) < P_\alpha(\text{piv}|z_B)$, so that the last inequality cannot be satisfied if $q_Ar_A = q_Br_B$. It follows that if $q_Ar_A = q_Br_B$, there cannot be an equilibrium of type (II). A similar argument holds with respect to type (III). □

Proof of Theorem 2. The first part of the proof shows that $n^{1/2}x_n$ goes to k as n goes to infinity. Letting $y_n = n^{1/2}x_n$ we get from Eq. (1)

$$\binom{2n}{n}\left(\tfrac{1}{4}-y_n^2/n\right)^n(2q_Ar_A) = C'(n^{-1/2}y_n).$$

Using the mean value theorem for C' and rearranging slightly we have

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}}\left(1-4y_n^2/n\right)^n(2q_Ar_A) = y_n\,C''(\xi_n) \tag{8}$$

for some ξ_n between zero and $n^{-1/2}y_n$.

Note that

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}} \to \pi^{-1/2}$$

(from Stirling's formula) and

$$0 < \left(1-4y_n^2/n\right)^n < 1$$

(because $0 < y_n < n^{1/2}$).

Now consider the case $C'(0) = C''(0) = 0$. Suppose that along some subsequence y_n converges to a finite $L \geqslant 0$. Then, along the subsequence the right-hand side of Eq. (8)

converges to zero. However, the left-hand side converges to a positive number, as can be seen from the fact that

$$\left(1 - 4y_n^2/n\right)^n \to \exp\{-4L^2\}$$

[10, Theorem 4.2, p. 94]. Thus, if $C'(0) = C''(0) = 0$, y_n diverges to $+\infty$.

Consider the case $C'(0) = 0$ and $C''(0) = c < \infty$. Suppose that along some subsequence y_n converges to a finite $L \geqslant 0$. Following the steps of the previous case, we get that L must satisfy $2q_A r_A \pi^{-1/2} \exp\{-4L^2\} = Lc$ or, equivalently, $L = k$. It remains to show that along no subsequence y_n diverges to $+\infty$. To see this, note that the right-hand side of Eq. (8) grows without bound if y_n goes to infinity, while for any positive ε, the left-hand side is smaller than $(\pi^{-1/2} + \varepsilon)(2q_A r_A)$ for n large enough.

Finally, consider the case $C'(0) = 0$ and $C''(0) = \infty$. If along some subsequence y_n converges to a finite $L > 0$ or diverges to $+\infty$, the right-hand side of Eq. (8) grows without bound, while the left-hand side is bounded by the argument above. Thus, if $C'(0) = 0$ and $C''(0) = \infty$, y_n converges to 0.

The second part of the proof uses a probabilistic argument to establish the desired result. Suppose the state is z_A (similar calculations hold if the state is z_B). Given the equilibrium strategy described in Theorem 1(i), the event of a given voter voting for A in state z_A corresponds to a Bernoulli trial with probability of success $\frac{1}{2} + x_n$. For $n = 1, 2, \ldots$ and $i = 1, \ldots, 2n+1$ define the random variables

$$V_i^n = \begin{cases} \frac{1}{2} - x_n & \text{if voter } i \text{ votes for } A, \\ -\frac{1}{2} - x_n & \text{if voter } i \text{ votes for } B. \end{cases}$$

For each n, the random variables V_i^n are iid. Moreover,

$$E(V_i^n) = 0,$$
$$E((V_i^n)^2) = \tfrac{1}{4} - x_n^2,$$

and

$$E(|V_i^n|^3) = 1/8 - 2x_n^4.$$

Let F_n stand for the distribution of the normalized sum

$$(V_1^n + \cdots + V_{2n+1}^n)/\sqrt{E((V_i^n)^2)(2n+1)}.$$

Note that A loses the election if it obtains n or fewer votes, that is, if

$$V_1^n + \cdots + V_{2n+1}^n + (2n+1)(\tfrac{1}{2} + x_n) \leqslant n$$

or equivalently

$$V_1^n + \cdots + V_{2n+1}^n \leqslant -\tfrac{1}{2} - (2n+1)x_n.$$

Then, the probability of A winning the election is $1 - F_n(J_n)$, where

$$J_n = \frac{-\frac{1}{2} - (2n+1)x_n}{\sqrt{E((V_i^n)^2)(2n+1)}}.$$

Using the Berry–Esseen theorem [12, p. 542]; [10, p. 106], for all w,

$$|F_n(w) - \Phi(w)| \leqslant \frac{3E(|V_i^n|^3)}{E((V_i^n)^2)^{3/2}\sqrt{2n+1}}.$$

The right-hand side of the equation above converges to zero as n goes to infinity, so we obtain an increasingly good approximation using the normal distribution even though the distribution of V_i^n changes with n. Thus,

$$\lim_{n\to\infty} |F_n(J_n) - \Phi(J_n)| = 0.$$

If $\lim_{n\to\infty} n^{1/2}x_n = k < \infty$, then J_n converges to $-2\sqrt{2}k$. Since Φ is continuous,

$$\lim_{n\to\infty} |\Phi(J_n) - \Phi(-2\sqrt{2}k)| = 0.$$

Thus, the probability of A winning converges to $1 - \Phi(-2\sqrt{2}k) = \Phi(2\sqrt{2}k)$.

If $n^{1/2}x_n$ goes to infinity with n, then J_n goes to $-\infty$. Thus, for arbitrarily large L, the probability of A winning the election is larger than $1 - F_n(-L)$ for n large enough. Using the normal approximation above we can see that the probability of A winning must go to one. $\square$

Proof of Proposition 1. Suppose the state is z_A (similar calculations hold if the state is z_B). Using the notation of the proof of Theorem 2, the number of votes for A is given by

$$V_1^n + \cdots + V_{2n+1}^n + (2n+1)(\tfrac{1}{2} + x_n).$$

Then, the winning margin is

$$\left|\frac{2\left(\sum_{i=1}^{2n+1} V_i^n + (2n+1)(\frac{1}{2} + x_n)\right) - (2n+1)}{2n+1}\right|$$

or equivalently,

$$2\left|\frac{1}{2n+1}\sum_{i=1}^{2n+1} V_i^n + x_n\right|.$$

Therefore, the probability that the winning margin is smaller or equal to κ is equal to $F_n(D_n) - F_n(I_n)$, where

$$D_n \frac{(2n+1)(\kappa/2 - x_n)}{\sqrt{E((V_i^n)^2)(2n+1)}}$$

and

$$I_n \frac{(2n+1)(-\kappa/2 - x_n)}{\sqrt{E((V_i^n)^2)(2n+1)}}.$$

Note that D_n goes to $+\infty$ and I_n goes to $-\infty$ with n. Following the last steps of the proof of Theorem 2, we have that the probability that the winning margin is smaller or equal to κ must go to one. □

Proof of Theorem 3. In the text for the case $C''(0) \in (0, \infty)$. Consider the case $C''(0) = \infty$. As in Eq. (8) in the proof of Theorem 2, we can write

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}}\left(1 - 4y_n^2/n\right)^n (2q_A r_A) = n^{1/2}C'(x_n).$$

Recall that, using Stirling's formula, $(2n)!n^{1/2}/(n!n!2^{2n}) \to \pi^{-1/2}$. Also, since in this case $y_n \to 0$, $\left(1 - 4y_n^2/n\right)^n \to 1$ [10, Theorem 4.2, p. 94]. Thus, $n^{1/2}C'(x_n)$ converges to $2q_A r_A \pi^{-1/2}$. Since $x_n nC'(x_n) = y_n n^{1/2}C'(x_n)$, we get that $x_n nC'(x_n)$ converges to zero. Using $C(x_n) \leqslant x_n C'(x_n)$, we get that $nC(x_n)$ converges to zero. The statement of the theorem follows.

Now consider the case $C''(0) = 0$. As in Eq. (8) in the proof of Theorem 2, we can write

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}}\, y_n \left(1 - 4y_n^2/n\right)^n (2q_A r_A) = x_n nC'(x_n).$$

Since in this case $y_n \to \infty$, we claim that $\lim_{n\to\infty} y_n \left(1 - 4y_n^2/n\right)^n = 0$. To see this, using Lemmas 4.3 and 4.4 in Durrett [10, p. 94],

$$\left|\left(1 - 4y_n^2/n\right)^n - \exp(-4y_n^2)\right| \leqslant 16y_n^4/n^3.$$

The claim follows from $y_n \exp(-4y_n^2) \to 0$ and $16y_n^5/n^3 \to 0$ as $n \to \infty$. Recall that, using Stirling's formula, $(2n)!n^{1/2}/(n!n!2^{2n}) \to \pi^{-1/2}$. Thus, $x_n nC'(x_n)$ converges to zero. The statement of the theorem follows from $C(x_n) \leqslant x_n C'(x_n)$. □

Proof of Proposition 2. Letting x_n^o be the amount of information acquired by voters in the optimal symmetric profile, we have

$$2\binom{2n}{n}\left(\tfrac{1}{4} - (x_n^o)^2\right)^n (2n+1)q_A r_A = C'(x_n^o). \tag{9}$$

That is, the optimal amount of information is what the voters would acquire if they internalize the gains of choosing the right alternative for the entire society. Letting $y_n^o = n^{1/2}x_n^o$ and rewriting the equation above,

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}}\left(1 - 4y_n^{o2}/n\right)^n (2q_A r_A) = y_n^o\; C''(\xi_n)/(2n+1)$$

for some ξ_n between zero and $n^{-1/2}y_n^o$. Since the right-hand side of this equation converges to zero, an argument similar to the first part of the proof of Theorem 2 establishes that $y_n^o \to \infty$. Thus, the probability of choosing the right alternative converges to one along the sequence of optimal symmetric profiles.

Rewriting Eq. (9) again, we have

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}}\, y_n^o \left(1 - 4y_n^{o2}/n\right)^n (2q_A r_A) = y_n^o n^{1/2}\; C'(x_n^o)/(2n+1).$$

An argument similar to the proof of Theorem 3 shows that the left-hand side converges to zero. Thus, $x_n^o nC'(x_n^o)/(2n+1) \to 0$. Using $x_n^o C'(x_n^o) \geqslant C(x_n^o)$, we get that $nC(x_n^o)/(2n+1) \to 0$ which implies $C(x_n^o) \to 0$. $\square$

Existence of a solution to (3.1)–(3.3). System (3.1)–(3.2) is equivalent to

$$2\pi^{-1/2}\exp\{-4(k'+h'/2)^2\}q_A r_A = k'c, \tag{10.1}$$

$$2\pi^{-1/2}\exp\{-4(k'-h'/2)^2\}q_B r_B = k'c \tag{10.2}$$

or

$$h' = \frac{1}{8k'}\ln\left(\frac{q_A r_A}{q_B r_B}\right),$$
$$2\pi^{-1/2}\exp\left\{-4\left(k' - \frac{1}{16k'}\ln\left(\frac{q_A r_A}{q_B r_B}\right)\right)^2\right\} q_B r_B = k'c.$$

If, say, $q_A r_A > q_B r_B$, this system has a solution satisfying $h' < 2k'$ iff

$$64(q_B r_B)^2/(\pi c^2) > \ln((q_A r_A)/(q_B r_B)), \tag{11}$$

and the solution (if it exists) is unique. Thus, if $q_A r_A > q_B r_B$, the inequality (11) is necessary and sufficient for the existence of a solution to (3.1)–(3.3). A similar condition is easily obtained if $q_A r_A < q_B r_B$. $\square$

Proof of Theorem 4. We continue the argument from the proof of Theorem 1. If $q_A r_A \neq q_B r_B$, inequality (5) cannot be satisfied for n large enough since x^* is positive but converges to zero as n goes to infinity. Thus, if $q_A r_A \neq q_B r_B$ there is no equilibrium of type (I) for large n. We consider the case $q_A r_A > q_B r_B$ in the remainder of the proof, as the case $q_A r_A < q_B r_B$ can be dealt with similarly. It is easy to check that there cannot be an equilibrium of type (III) if $q_A r_A > q_B r_B$ because under any strategy of type (III), $P_\alpha(\text{piv}|z_A) > P_\alpha(\text{piv}|z_B)$. It remains to find out conditions under which there is an equilibrium of type (II) for large n.

Using the equilibrium conditions (6) and (7) for $q_A r_A > q_B r_B$ we obtain

$$\binom{2n}{n}\left(\tfrac{1}{4} - (x + \delta(\tfrac{1}{2} - x))^2\right)^n = ((\tfrac{1}{2} + x)C'(x) - C(x))/(q_A r_A), \tag{12.1}$$

$$\binom{2n}{n}\left(\tfrac{1}{4} - (x - \delta(\tfrac{1}{2} + x))^2\right)^n = ((\tfrac{1}{2} - x)C'(x) + C(x))/(q_B r_B). \tag{12.2}$$

(Note that the inequality in (7) is strictly satisfied if $q_A r_A > q_B r_B$.) Let $H = \delta/x$ and $\gamma(n) = \binom{2n}{n}4^{-n}$. ($\gamma(n)$ is strictly decreasing in n and goes to zero as n goes to infinity.) Rewriting (12.1)–(12.2),

$$\gamma(n)(1 - 4[(1+H/2)x - Hx^2]^2)^n = ((\tfrac{1}{2} + x)C'(x) - C(x))/(q_A r_A), \tag{13.1}$$

$$\gamma(n)(1 - 4[(1-H/2)x - Hx^2]^2)^n = ((\tfrac{1}{2} - x)C'(x) + C(x))/(q_B r_B). \tag{13.2}$$

The expressions in the RHS of (13.1) and (13.2) are strictly increasing in x, while those in the LHS are decreasing in x for every H. It is easy to check that for any given $H \geqslant 0$ and for n large, there is a unique $x_n^I(H)$ and a unique $x_n^{II}(H)$ solving respectively Eqs. (13.1) and (13.2). Moreover, $x_n^I(H)$ and $x_n^{II}(H)$ are positive, continuous in H, and converge to zero as n goes to infinity.

Note that

$$((\tfrac{1}{2}+x)C'(x)-C(x))/((\tfrac{1}{2}-x)C'(x)+C(x))$$

converges to one from above as x goes to zero. Thus, the RHS of (13.1) is smaller than the RHS of (13.2) for x close to zero. Since the LHS of (13.1) and (13.2) are equal for every x if $H=0$, we get $x_n^I(0) > x_n^{II}(0)$ for large n.

We claim first that, under the conditions stated in part (i) of the theorem, for every $H>0$ there is some $\overline{n}$ such that $x_I(H) < x_{II}(H)$ for $n \geqslant \overline{n}$. It follows that there is a sequence of solutions to (12.1)–(12.2) (i.e. a sequence of information acquisition equilibria) such that along that sequence δ/x converges to zero. To establish the claim it is sufficient to prove that for any $H>0$, for n large enough, the RHS is larger than the LHS of (13.1), evaluating them at $x_n^{II}(H)$. That is,

$$\left(\frac{1-4[(1+H/2)x-Hx^2]^2}{1-4[(1-H/2)x-Hx^2]^2}\right)^n < \frac{q_B r_B}{q_A r_A}\,\frac{(\frac{1}{2}+x)C'(x)-C(x)}{(\frac{1}{2}-x)C'(x)+C(x)}, \tag{14}$$

where $x=x_n^{II}(H)$. Letting $y_n(H)=n^{1/2}x_n^{II}(H)$ and using the mean value theorem, we get from (13.2)

$$\begin{aligned}&\gamma(n)n^{1/2}(1-4[(1-H/2)y_n(H)/n^{1/2}-H(y_n(H))^2/n]^2)^n\\&\quad= y_n(H)(\tfrac{1}{2}-\xi_n)C''(\xi_n)/(q_B r_B)\end{aligned}$$

for some ξ_n between zero and $n^{-1/2}y_n^H$. Following the steps of the first part of the proof of Theorem 2, we get $y_n(H)\to\infty$. Using Lemmas 4.3 and 4.4 in Durrett [10, p. 94], the LHS of (14) is approximately $\exp\{-8H(y_n(H))^2\}$ for large n. Thus, it converges to 0. Since the RHS of (14) is bounded below by $(q_A r_A)/(q_B r_B)$, (14) is satisfied for large n. This establishes the claim.

Now let $\hat{x}_n$ and δ_n denote the information acquired by each voter and the probability of playing the pure strategy $(0, A, A)$ according to a sequence of equilibria with information acquisition in case (i) of the theorem. Using (12.1) and following the steps of the first part of the proof of Theorem 2, we get $\hat{x}_n n^{1/2}\to\infty$. A minor variation on the second part of the proof of Theorem 2 establishes that the probability of choosing the right alternative converges to one. Similarly, a minor variation on the proof of Theorem 3 establishes that the aggregate cost converges to zero.

Second, we claim that, under the conditions stated in part (ii) of the theorem, $x_n^I(2) < x_n^{II}(2)$ for large n, so that there is an equilibrium satisfying $x>\delta/2$. To see this, letting $k_{II}=\lim_{n\to\infty} y_n(2)$ from (13.2) we get $k_{II}=2\pi^{-1/2}q_B r_B/c$. Since, for $H=2$, the LHS of (14) converges to $\exp\{-16k_{II}^2\}$ and the RHS is bounded below by $(q_A r_A)/(q_B r_B)$, (14) is satisfied for large n if Eq. (11) is satisfied, that is, if (3.1)–(3.3) has a solution.

Now let $\hat{x}_n$ and δ_n denote the information acquired by each voter and the probability of playing the pure strategy $(0, A, A)$ according to a sequence of information acquisition equilibria satisfying $\delta_n/\hat{x}_n < 2$ in case (ii) of the theorem. Using (12.1)–(12.2) and following the steps of the first part of the proof of Theorem 2, we get $\hat{x}_n n^{1/2}\to k'$ and $\delta_n n^{1/2}\to h'$.

Minor variations on the proofs of Theorems 2 and 3 establish the statement about social welfare in part (ii) of the theorem.

To prove part (iii) of the theorem, we proceed by contradiction. If $C''(0) = c \in (0, \infty)$ and there is a sequence $\{\hat{x}_n, \delta_n\}$ that solves (12.1)–(12.2) for arbitrarily large n, then along that sequence we must get $\hat{x}_n n^{1/2} \to k''$ and $\delta_n n^{1/2} \to h''$, where k'', h'' solve (10.1)–(10.2). But this system is equivalent to (3.1)–(3.2). Finally, if $C''(0) = \infty$, from (12.1) we get $\hat{x}_n n^{1/2} \to 0$. But using (12.1)–(12.2) we get

$$\frac{\exp\{-4((1-\delta_n)\hat{x}_n n^{1/2} + \delta_n n^{1/2}/2)^2\}}{\exp\{-4((1-\delta_n)\hat{x}_n n^{1/2} - \delta_n n^{1/2}/2)^2\}} \to \frac{q_B r_B}{q_A r_A},$$

which is impossible unless $q_A r_A = q_B r_B$. $\square$

Proof of Theorem 5. It is straightforward to show that there is no equilibrium with information acquisition for large n if $C'(0) > 0$. Assume $C'(0) = 0$. It is easy to show that a best-responding voter will put probability zero on the set of actions $\{(x, v_A, v_B) \neq (x, A, B)$ and $x > 0\}$. Thus, if all other voters put positive probability on the set of actions with $x > 0$, the probability that a given voter is decisive, conditional on either state, will be positive. But then, if $C'(0) = 0$, a best-responding voter will put probability zero on the actions $(0, A, B)$ and $(0, B, A)$. The reason is that if the realization of u_i is such that the voter is indifferent between $(0, A, A)$ and $(0, B, B)$, then it will pay the voter to acquire some information. Thus, if $C'(0) = 0$, an information acquisition equilibrium mapping can put positive probability only on the actions $(0, A, A)$ and $(0, B, B)$ and on the set $\{(x, A, B) : x > 0\}$. It is easy to check that an information acquisition equilibrium mapping must order the action $(0, B, B)$ for $u_i < \underline{u}$ and $(0, A, A)$ for $u_i > \overline{u}$ for some pair $\underline{u}, \overline{u}$ satisfying $0 < \underline{u} < \overline{u} < 1$. We claim that $\underline{u} = 1 - \overline{u}$. Suppose, e.g., $\underline{u} > 1 - \overline{u}$. Then the probability that a voter is decisive in state z_A would be larger than the probability the voter is decisive in state z_B. But then if the voter is indifferent between acquiring information or playing $(0, A, A)$ if $u_i = \overline{u}$, then the voter should prefer to acquire information if $u_i \in (1 - \overline{u}, \underline{u})$. Since $\underline{u} = 1 - \overline{u}$, the probability of being decisive in an equilibrium with information acquisition is the same in both states. Denoting this probability $P_\sigma(\text{piv})$ we get that for $u_i \in (\underline{u}, \overline{u})$, the information quality chosen in equilibrium $\tilde{x}$ must maximize

$$(\tfrac{1}{2})P_\sigma(\text{piv})(\tfrac{1}{2} + x) - C(x).$$

(Note that the specific realization of u_i drops from the objective function because the probability of being decisive is the same in both states.) For a voter to be indifferent between acquiring information and playing $(0, A, A)$ if $u_i = \overline{u}$, it is necessary that

$$(\overline{u}/2)P_\sigma(\text{piv}) = (\tfrac{1}{2})P_\sigma(\text{piv})(\tfrac{1}{2} + \tilde{x}) - C(\tilde{x})$$

or equivalently,

$$\overline{u} = \tfrac{1}{2} + \tilde{x} - 2C(\tilde{x})/P_\sigma(\text{piv}).$$

Thus, $\tilde{x} < \frac{1}{2}$. But then it must satisfy the first-order condition

$$(\tfrac{1}{2})P_\sigma(\text{piv}) = C'(\tilde{x}).$$

Hence,

$$\overline{u} = \tfrac{1}{2} + \tilde{x} - C(\tilde{x})/C'(\tilde{x})$$

and

$$\begin{aligned} P_\sigma(\mathrm{piv}) &= \binom{2n}{n} \left(1 - \overline{u} + (\overline{u} - \underline{u})(\tfrac{1}{2} + \tilde{x})\right)^n \left(\underline{u} + (\overline{u} - \underline{u})(\tfrac{1}{2} - \tilde{x})\right)^n \\ &= \binom{2n}{n} \left(\tfrac{1}{4} - 4(\tilde{x}_n - C(\tilde{x}_n)/C'(\tilde{x}_n))^2 \tilde{x}_n^2\right)^n. \end{aligned}$$

Eq. (4) follows. It is simple to verify that (4) has a unique solution for each n. □

Proof of Theorem 6. Using Theorem 5, the probability that a voter votes for the right alternative is equal to

$$\tfrac{1}{2} + 2\tilde{x}_n(\tilde{x}_n - C(\tilde{x}_n)/C'(\tilde{x}_n))$$

or

$$\tfrac{1}{2} + 2\tilde{x}_n^2(1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n))).$$

As in the proof of Theorem 2, we are interested in calculating the limit of

$$2\tilde{x}_n^2 n^{1/2}(1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n))).$$

Let $Q = 1 - \lim_{x \downarrow 0} C(x)/(xC'(x))$. Using L'Hôpital's rule, it is easy to check that $Q \in [0, 1]$ if $C'(0) = 0$, $Q \in [\frac{1}{2}, 1]$ if in addition $C''(0) = 0$, and $Q = 2/3$ if in addition $C'''(0) = \tilde{c} \in (0, \infty)$. Using Eq. (4) as in the first part of the proof of Theorem 2, we get $\tilde{x}_n^2 n^{1/2} \to 0$ if $C'(0) = 0$ and $C''(0) > 0$, or if $C'(0) = C''(0) = 0$ and $C'''(0) = \infty$. Similarly, $\tilde{x}_n^2 n^{1/2} \to \infty$ if $C'(0) = C''(0) = C'''(0) = 0$. Finally, if $C'(0) = C''(0) = 0$ and $C'''(0) = \tilde{c} \in (0, \infty)$, from (4) we can get that the limit $\tilde{L}$ of $2Q\tilde{x}_n^2 n^{1/2}$ must satisfy $\pi^{-1/2} \exp\{-4\tilde{L}^2\} = (3/4)\tilde{L}\tilde{c}$, or equivalently $\tilde{L} = \tilde{k}$. The probability of choosing the right alternative can be obtained following the steps of the second part of the proof of Theorem 2.

With respect to the aggregate cost of information, suppose that $C'(0) = C''(0) = 0$ and $C'''(0) = \tilde{c} \in (0, \infty)$. Note that, using Theorem 5, the probability that a voter acquires information is given by $2\tilde{x}_n(1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n)))$. Thus, the expected aggregate cost in the limit is given by

$$\begin{aligned} &\lim_{n\to\infty} \{(2n + 1)(2\tilde{x}_n)(1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n)))C(\tilde{x}_n)\} \\ &\quad = \lim_{n\to\infty} \{(2n^{1/2}\tilde{x}_n^2)^2(1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n)))C(\tilde{x}_n)/\tilde{x}_n^3\}. \end{aligned}$$

Using

$$2\tilde{x}_n^2 n^{1/2}(1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n))) \to \tilde{k} \quad \text{and} \quad 1 - C(\tilde{x}_n)/(\tilde{x}_n C'(\tilde{x}_n)) \to 2/3$$

(from the previous paragraph) and

$$C(\tilde{x}_n)/\tilde{x}_n^3 \to \tilde{c}/6,$$

(by L'Hôpital's rule) we get that the expected aggregate cost converges to $\tilde{c}\tilde{k}^2/4$, as stated in the theorem. Other cases can be dealt with following the steps of the proof of Theorem 3. □

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