Business Cycles

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Extensive vs Intensive margins

- Several problems can be interpreted as difficulties to understand time allocation problems.
- RBC model requires very high labor supply elasticity.
- How to reconcile micro and macro evidence in this case?
- Hansen (1985) comes up with a solution
- There are fixed costs of working (commuting, specific clothes, etc).
- The relevant decision is to work or not to work.
- Individuals spend $n = \bar{n}$ hours while working and $n = 0$ while not working.
- Intensive vs. extensive margins (hours worked while working vs labor force participation)

Indivisible Labor Hansen (1985) (1)

- One problem: to have a competitive solution with representative agents, we need a convex decision set.
- Solution is to "convexify" choice set. A job is a contract that delivers $\bar{n}$ hours of work with probability x and 0 hours with probability $1 - x$ paid at wage w.
- Households choose x as well as capital accumulation.
- Formally, the utility function can be written as

$$\begin{aligned} u(c, x; \bar{n}) &= x(\log c + \theta \log(1 - \bar{n})) + (1 - x)(\log c + \theta \log 1) \\ &= \log c + x\theta \log(1 - \bar{n}) \end{aligned}$$

Indivisible Labor Hansen (1985) (2)

- The recursive formulation is

$$V(k,z) = \max_{c,k',x} \{\log c + x\theta \log(1-\bar{n}) + \beta V(k',z')\}$$
$$\text{s.t} \quad wx\bar{n} + Rk + (1-\delta)k \geq k' + c$$

- Firms hire productive factors to maximize profits

$$\max_{\tilde{k},\tilde{x}} \{zF(\tilde{k},\phi\bar{n}) - R\tilde{k} - w\tilde{x}\bar{n}\}$$

Indivisible Labor Hansen (1985) (3)

- The representative agent's decision on hours worked can be written as

$$n = x\bar{n} \quad \to \quad x = n/\bar{n}$$

- Substituting into utility function, we get preferences that are linear in n

$$u(c, n; \bar{n}) = \log c + n\frac{\theta \log(1-\bar{n})}{\bar{n}} = \log c - Bn$$

with $B = -\frac{\theta \log(1-\bar{n})}{\bar{n}} > 0$

Indivisible Labor Hansen (1985) (4)

- Regardless of the microeconomic labor elasticity, the aggregate intertemporal wage elasticity is ∞. Why?
- Let's assume that

$$V(k,z) = \max_{c,k',n} \left\{ \log c - \theta \frac{n^{1+\phi}}{1+\phi} + \beta E[V(a',z')|z] \right\}$$

$$\text{s.t } c + a' \leq a(1+r) + wn \quad (\text{Lagrange mult } \upsilon)$$

- First-order condition for n

$$-\theta n^{\phi} + \upsilon w = 0 \quad \rightarrow \quad n = (w\upsilon/\theta)^{1/\phi}$$

Indivisible Labor Hansen (1985) (5)

- Intertemporal substitution of hours worked can be defined as

$$\frac{\partial \log(n'/n)}{\partial \log(w'/w)} = \frac{1}{\phi}$$

  with υ kept constant.
- For linear specification we need $\phi = 0$ which means that the representative agent does not want to smooth leisure. Hours move much more in this scenario.

Indivisible Labor Hansen (1985) (6)

Table 1

Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

	Quarterly U.S. time series[a] (55,3–84,1)		Economy with divisible labor[b]		Economy with indivisible labor[b]	
Series	(a)	(b)	(a)	(b)	(a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)	1.76 (0.21)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)	0.51 (0.08)	0.87 (0.04)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)	5.71 (0.70)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)	0.47 (0.10)	0.05 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)	1.35 (0.16)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)	0.50 (0.07)	0.87 (0.03)

[a] The U.S. time series used are real GNP, total consumption expenditures, and gross private domestic investment (all in 1972 dollars). The capital stock series includes nonresidential equipment and structures. The hours series includes total hours for persons at work in non-agricultural industries as derived from the *Current Population Survey*. Productivity is output divided by hours. All series are seasonally adjusted, logged and detrended.

Macro Fluctuations and Time Allocation (1)

- Hall (1997) develops a simple framework to understand the sources of business cycle fluctuations.
- Candidates of driving forces of business cycles
    - Productivity - technological shocks
    - Government purchases
    - Preference shifts
- Hall (1997) finds that most of the fluctuations of business cycles are caused from people's time allocation in booms and recessions.

Macro Fluctuations and Time Allocation (2)

- The model is very simple

$$u(C, N) = X \log C - \frac{N^{1+\phi}}{1+\phi}$$

where x is a random weight of consumption utility and ϕ is the elasticity of the marginal value of time

- Detour: Frisch elasticity is the wage-elasticity of labor supply holding constant the marginal utility of wealth. To understand this, let's use the following simple model

$$V(a, z) = \max_{c, a', n} \left\{ x \log c - \frac{n^{1+\phi}}{1+\phi} + \beta E[V(a', z')|z] \right\}$$
$$\text{s.t } c + a' \leq a(1+r) + wn \quad \text{(Lagrange mult } \upsilon\text{)}$$

Macro Fluctuations and Time Allocation (3)

- FOC and envelope conditions

$$\begin{aligned} \text{c:} \quad & x/c - \upsilon = 0 \\ \text{a':} \quad & \beta E[V_1(a', z')] - \upsilon = 0 \\ \text{n:} \quad & n^{\phi} - \upsilon w = 0 \\ \text{Env:} \quad & V_1(a, z) = \upsilon(1 + r) \end{aligned}$$

- $\lambda = x/c$ is the stochastic marginal utility of wealth.
- Intratemporal / Labor supply equation becomes

$$n^{\phi} = \upsilon w \quad \rightarrow \frac{\partial \log n}{\partial \log w} = \frac{1}{\phi}$$

Macro Fluctuations and Time Allocation (4)

- Using previous setup labor supply equation is written as

$$cn^{\phi}/x = w$$

- Firms produce with technology $Y = Z\alpha^{-1}N^{\alpha}K^{1-\alpha}$
- Competitive equilibrium $\partial Y/\partial N = Z(K/N)^{1-\alpha} = w$, Hence

$$cn^{\phi}/x = z(k/n)^{1-\alpha}$$

- Reexpressing $\tilde{x} = \log X$, the equilibrium is determined by the three following equations

$$\begin{aligned}
\phi\tilde{n} &= \tilde{z} + (1-\alpha)(\tilde{k} - \tilde{n}) + \tilde{x} - \tilde{c} \\
\tilde{y} &= \tilde{z} + \alpha\tilde{n} + (1-\alpha)\tilde{k} - \tilde{\alpha} \\
\tilde{c} &= \tilde{y} - v - g
\end{aligned}$$

Macro Fluctuations and Time Allocation (5)

- In the last equality $v = -\log(1 - V/(Y - G))$ and $g = -\log(1 - G/Y)$ where V stands for investment plus next exports; G is government expenditure, so that $Y = C + V + G$ (check)
- Conditioning on v and replacing previous equations we obtain

$$\tilde{n} = \lambda(\tilde{x} + v + g + \tilde{\alpha})$$

where $\lambda = (1 + \phi)^{-1}$

- This equation expresses n as a function of taste, investment and government shocks.
- Atemporal determination of labor.

Macro Fluctuations and Time Allocation (6)

- Similarly, shocks can be measured using data

$$\begin{aligned}\tilde{z} \equiv& \tilde{y} - \alpha\tilde{n} - (1-\alpha)\tilde{k} + \tilde{\alpha} \\ \tilde{x} \equiv& \tilde{c} - \tilde{y} + (1+\phi)\tilde{n} - \tilde{\alpha} \\ v \equiv& \tilde{y} - \tilde{c} - g\end{aligned}$$

- Hall follows an empirical, time-series approach to capture intertemporal channels that affect labor in equilibrium through investment.

$$\begin{aligned}v =& \beta_x(L)x + \beta_z(L)z + \beta_g(L)g + u \\ n =& \lambda\left(x + \beta_x(L)x + \beta_z(L)z + g + \beta_g(L)g + u\right)\end{aligned}$$

- Assumption

$$E(ux) = E(uz) = E(ug) = 0$$

Macro Fluctuations and Time Allocation (7)

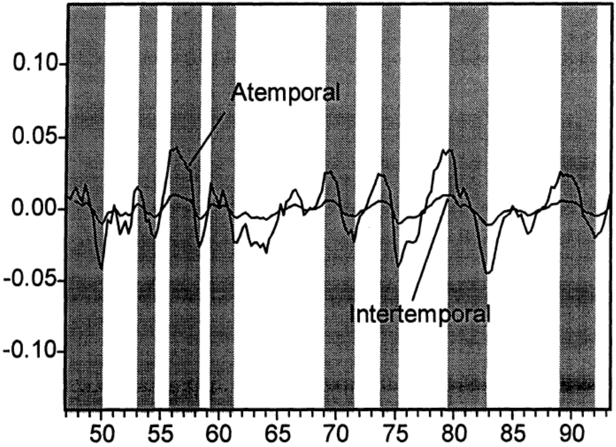


FIG. 15.—Contribution of marginal-rate-of-substitution shift to high-frequency movements in hours.

Macro Fluctuations and Time Allocation (8)

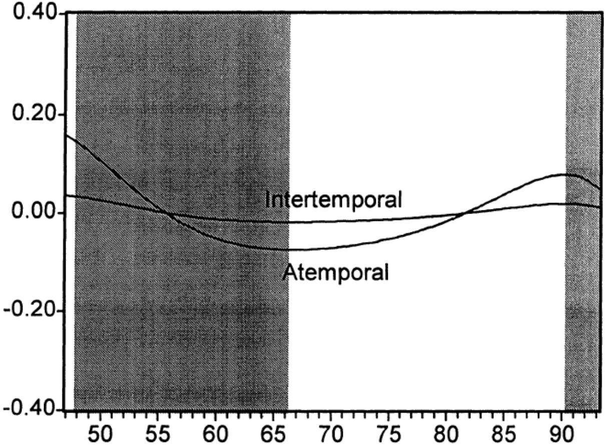


FIG. 16.—Contribution of marginal-rate-of-substitution shift to medium-frequency movements in hours.

Macro Fluctuations and Time Allocation (9)

Table 1
Contributions to Variance of Hours of Work

Component	Explanation	Covariance Contribution (% of Total Variance)	SD (%)	Covariance Contribution (% of Total Variance)	SD (%)
n	Hours of work	100.0	2.25	100.0	6.16
λx	Atemporal effect of preference shift	84.6	1.97	105.0	6.49
$\lambda\beta_x(L)x$	Intertemporal effect of preference shift	20.3	.48	25.2	1.55
$\lambda\beta_z(L)z$	Intertemporal effect of technology shift	−1.5	.50	−34.6	2.22
λg	Atemporal effect of government purchases	−1.7	.58	−7.3	1.00
$\lambda\beta_g(L)g$	Intertemporal effect of government purchases	2.0	.46	7.7	1.03
λu	Effect of residual driving forces	−3.6	.35	4.0	.53

Macro Fluctuations and Time Allocation (10)

Lessons:

- Primary driving force of the business cycle looks like an shift of the intertemporal marginal rate of substitution between c and $1 - n$: “labor wedge”
- Investment fluctuates too little $\rightarrow$ intertemporal transmission mechanism is weak.
- Government purchases explains very little of fluctuations.
- Challenge is to explain intratemporal allocation of time in the business cycle.

Business Cycles Accounting (1)

- Idea developed by Chari, Kehoe, and McGrattan (2007).
- Using a standard Dynamic Stochastic General Equilibrium model to explain the sources underlying business cycle fluctuations.
- General results for the US and Chile (Simonovska and Soderling 2008) in line with Hall (1997) findings: Fluctuations are mainly due to changes in the "labor wedge"
- Main aspect of cyclical fluctuations: time allocation and labor markets.

Business Cycles Accounting (2)

- Benchmark prototype economy: Households

$$\max_{\{c_t, x_t, n_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} u(c_t, 1-n_t) P_t$$

$$\begin{aligned} s.t. \quad & c_t + (1+\tau_{xt}) x_t = r_t k_t + (1-\tau_{nt}) w_t n_t \\ & P_{t+1} k_{t+1} = P_t((1-\delta) k_t + x_t) \\ & c_t, x_t \geq 0 \quad \text{in all states} \end{aligned}$$

- Representative firm

$$\max_{\{K_t, N_t\}_{t=0}^{\infty}} \{F(K_t, Z_t N_t) - r_t K_t - w_t N_t$$

Business Cycles Accounting (3)

- Finally there are fiscal expenditures and net exports

$$P_t(c_t + x_t) + G_t = F(K_t, Z_t N_t)$$

- Aggregate variables are consistent with individual decisions

$$P_t k_t = K_t$$
$$P_t n_t = N_t$$

- Labor wedge is $1 - \tau_{nt}$.
- Investment wedge is $1/(1 + \tau_{xt})$.
- Efficiency wedge is z_t.
- Income accounting wedge is G_t.

Business Cycles Accounting (3)

- Chari, Kehoe, and McGrattan (2007) point is that distortions can be mapped into the different wedges.
- Examples: Financial frictions in which one sector can get funds at higher rate will generate an inefficiency that resembles an efficiency wedge.
- Sticky price economy generate labor wedges.
- Several frictions –departures from perfectly competitive markets with zero tax rates– will be generate an effect that can be understand as some kind of taxation.

Business Cycles Accounting (4)

- Accounting method: using optimality FOC and the data, compute implied wedges.
- Then, simulate the model using one wedge at a time.
- Examine how good is the fit to the actual data.

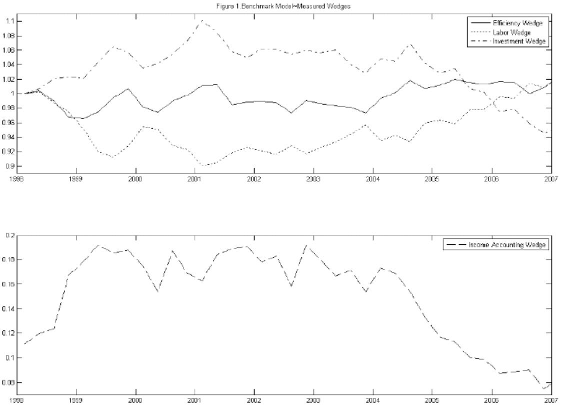
BCA Results for Chile (1)

From Simonovska and Soderling (2008)

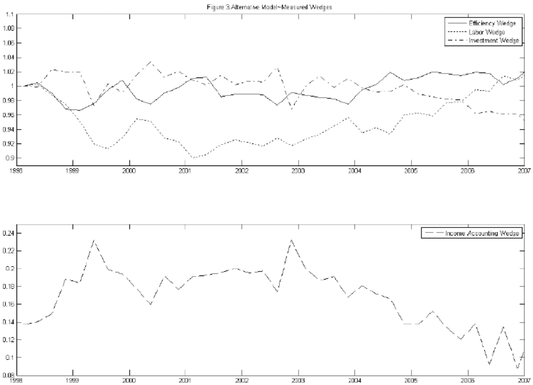
Table 1: Parameter Estimates for Benchmark Economy

Parameter	Value	Source (Assume Chile is in SS in 1998Q1)
θ	0.3000	Bergoeing et al. (2001)
δ	0.0125	Bergoeing et al. (2001)
β	0.9939	Calibration
ψ	3.3631	Calibration
g_n	0.400%	Match 1.6% annual growth rate of population
g_z	0.500%	Assume 2% annual TFP growth rate

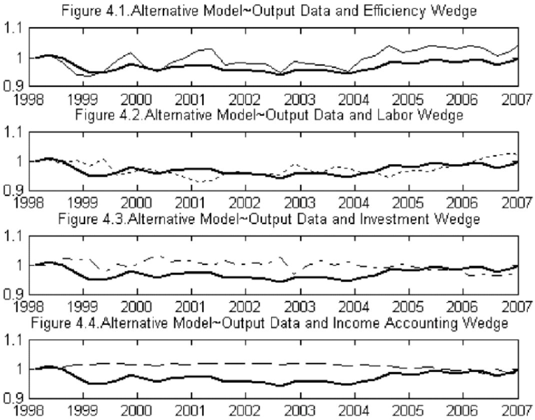
BCA Results for Chile (2)



BCA Results for Chile (3)



BCA Results for Chile (4)



Lessons from Business Cycles Accounting

- Conclusions: Efficiency and Labor wedge best explains aggregate variables in Chile during 1998-2007 Simonovska and Soderling (2008).
- As in the US, there is no too much role for investment wedge.
- This evidence suggests to look at labor market to explain business cycles.
- Theories that can be mapped as labor or efficiency wedges are the most fruitful.
- Unfortunately, the methodology does not identify specific causes behind wedges.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007).
Business Cycle Accounting.
Econometrica 75(3), 781–836.

Hall, R. E. (1997).
Macroeconomic Fluctuations and the Allocation of Time.
Journal of Labor Economics 15(1), S223–S250.

Hansen, G. D. (1985).
Indivisible labor and the business cycle.
Journal of Monetary Economics 16(3), 309 – 327.

Simonovska, I. and L. Soderling (2008, March).
Business Cycle Accounting for Chile.
IMF Working Papers 08/61, International Monetary Fund.