Economic Growth Theory

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September 23, 2010

Quality ladder improvement (1)

- Instead of increasing the number of available products, there are quality improvements of the same goods.
- Quality innovator firm has monopoly power over its improved input.
- Lower quality products that are already created are put out of business. Also called “business stealing” effect.
- This is called “creative destruction” a term coined by Schumpeter.
- Adapted version of Aghion and Howitt (2005). This kind of model is also explained in Barro and Sala-i-Martin (2004) chapter 7 and Acemoglu (2009) chapter 14.

Quality ladder improvement (2)

- Firm i has the following technology

$$Y_i = L^{1-\alpha} \widetilde{X}_i^{\alpha}$$

- The input $\widetilde{X}_i$ is the quality-weighted amount of inputs created until the M vintage.
- The value $q > 1$ is the quality improvement obtained once research effort succeeds.

$$\widetilde{X} = \sum_{m=0}^{M} q^m X_{im}$$

Quality ladder improvement (3)

- The marginal return obtained from using a vintage m of X must equate the price of X_{im}

$$\partial Y_i / \partial X_{im} = \alpha L_i^{1-\alpha} q^{\alpha m} X_{im}^{\alpha-1} = P_m$$

- Hence, the demand for input X_{im} is

$$X_{im} = L \left(\frac{\alpha q^{\alpha m}}{P_m} \right)^{\frac{1}{1-\alpha}}$$

Quality ladder improvement (3)

- Since top innovator has monopolistic power, he set prices to maximize profits

$$\pi_M = (P_M - \eta)X_m = L\alpha q^{\frac{\alpha M}{1-\alpha}}(P_M^{-\frac{\alpha}{1-\alpha}} - P_M^{-\frac{1}{1-\alpha}})$$

where η is the cost of transforming one unit of consumption into input X.

- From FOC, we get $P_M^{\star} = \eta/\alpha$
- In equilibrium production is

$$X_M = L\alpha^{\frac{2}{1-\alpha}} q^{\frac{\alpha M}{1-\alpha}} \eta^{-\frac{1}{1-\alpha}}$$

Quality ladder improvement (4)

- The obtained profit is

$$\begin{aligned}\pi^{\star}_{M} &= L(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} q^{\frac{\alpha M}{1-\alpha}} \eta^{-\frac{\alpha}{1-\alpha}} \\ &= \psi L q^{\frac{\alpha M}{1-\alpha}}\end{aligned}$$

- What happens to the previous innovators?
- Since the price set by the M-th innovator is η/α, the $M-1$-th innovator cannot charge more than $\eta/\alpha q$, i.e. the same price per unit of quality.
- The $M-1$ innovator exits the market if $P_{M-1} = \eta/\alpha q$ is lower than the cost η, i.e. if $\alpha q \geq 1$.

Quality ladder improvement (5)

- If $\alpha q < 1$, the $(M-1)$-th innovator might still survive. But assume M-th innovator engage in Bertrand competition: the M firm will set the price low enough to make the $M-1$ indifferent between staying and exiting.
- In this case $P_M = \eta/\alpha q$. The M-th firm still makes positive profits because $P_M - \eta = \frac{\eta(1-\alpha q)}{\alpha q} > 0$
- In both cases, all the innovators exit the market except the last one: creative destruction.
- Total product is

$$Y = L\alpha^{\frac{2\alpha}{1-\alpha}}\eta^{-\frac{1}{1-\alpha}}q^{\frac{\alpha M}{1-\alpha}}$$

Quality ladder improvement (6)

- Research technology: Each firm pays a cost κ_M which gives the firm a fixed probability of generating a quality improvement of size q.
- Crucial distinction between discrete and continuous time.
- First case, there is a positive probability that various researchers discover the $M+1$-th quality at the same time.
- In continuous time at each moment of time the prob that two or more discoveries are made is negligible.
- Let's assume that the number of discoveries is a Poisson process with arrival rate μ.

A quick detour

- A Poisson process is a stochastic process in which events occur continuously and independently of one another. Continuous analogue of Bernoulli discrete process.
- Do you remember exponential distribution? It arises when there is a continuous probability of that an event occur in any time interval.

$$F(T) = Prob\{\text{An event occurs before time T}\} = 1 - e^{-\mu T}$$

- The density of T is $F'(T) = \mu e^{-\mu T}$
- The probability that the event occurs between T and $T + dt$ is $\approx \mu e^{-\mu T} dt$.
- If we set $T = 0$ the probability of the event when $dt \to 0$ is μdt.

Research Incentives (1)

- Let's use asset valuation equations. The expected flow of engaging in research is

$$rR_M = -\kappa_M + \mu_M(\underbrace{S_{M+1} - R_M}_{\text{Expected capital gain}})$$

- Likewise, the expected flow of discovering the $M+1$-th innovation is

$$rS_{M+1} = \pi^{\star}_{M+1} + \mu_M(\underbrace{R_{M+1} - S_{M+1}}_{\text{Expected capital loss}})$$

Research Incentives (2)

- In equilibrium, $R_m = 0$ for all $m = 1, 2, \ldots$ due to free entry.
- Thus we have that

$$S_{M+1} = \frac{\pi^{\star}_{M+1}}{r + \mu}$$

- Replacing into the previous equation we get

$$\kappa_M (r + \mu_M) = \mu_M \pi^{\star}_{M+1}$$

- In equilibrium, the interest rate increases when μ increases

$$r_M = \mu(\pi^{\star}_{M+1} / \kappa_M - 1)$$

Research Incentives (2)

- Intuition? The larger the μ_M, the larger the prob of succeeding and also the riskier the flow generated.
- Can r_M be negative? What happens in that case?
- Improving quality may typically involve increasingly higher costs. A simplifying assumption is $\kappa_M = \kappa_0 q^{\frac{\alpha M}{1-\alpha}}$. This prevents r from being increasing over time.

$$r_M = r = \mu_M(\psi L q^{\frac{1}{1-\alpha}}/\kappa_0 - 1)$$

- Exposition in Barro and Sala-i-Martin (2004)[ch 7] assumes that $\mu_M = Z_M \phi(M)$, that is the probability of success is proportional to overall R&D expenditure and decreases in M ($\phi' > 0$ and $\phi'' < 0$).

Research Incentives (3)

- Notice we have a scale effect again. Research project return depends on the "size" of the economy L.
- On the consumer side, using CRRA preferences we obtain the typical Euler equation

$$c'/c = (\beta(1 + r_M))^{1/\sigma}$$

- Notice the growth rate of consumption may be stochastic! If there is a discovery, growth rate may change.

Research Incentives (4)

- Remember that

$$Y = L\alpha^{\frac{2\alpha}{1-\alpha}}\eta^{-\frac{1}{1-\alpha}}q^{\frac{\alpha M}{1-\alpha}} = \xi q^{\frac{\alpha M}{1-\alpha}}$$

- To compute the discrete growth rate of the output during a period of length T, we use the fact that the number of innovations between two different dates follows a Poisson distribution with mean μT

Quality ladder model results (1)

- Hence the expected output in the next period of length T is

$$\begin{aligned}
E[Y'] &= \sum_{j=0}^{\infty} \frac{(\mu T)^j e^{-\mu T}}{j!} \xi q^{\frac{\alpha(M+j)}{1-\alpha}} \\
&= \xi q^{\frac{\alpha M}{1-\alpha}} \sum_{j=0}^{\infty} \frac{(\mu T q^{\frac{\alpha}{1-\alpha}})^j e^{-\mu T}}{j!} \\
&= Y e^{\mu T (q^{\frac{\alpha}{1-\alpha}} - 1)} \sum_{j=0}^{\infty} \frac{(\mu T q^{\frac{\alpha}{1-\alpha}})^j e^{-\mu T q^{\frac{\alpha}{1-\alpha}}}}{j!} \\
&= Y e^{\mu T (q^{\frac{\alpha}{1-\alpha}} - 1)}
\end{aligned}$$

- Thus, the expected growth rate is $E[Y']/Y = e^{\mu T (q^{\frac{\alpha}{1-\alpha}} - 1)}$

Quality ladder model results (2)

- We can obtain constant growth rate if we introduce a large number of inputs so that average discovery rate stays constant. See Barro and Sala-i-Martin (2004) chapter 7.
- If there is a large number of inputs in Schumpeterian competition, by the Law of Large Numbers

$$\lim_{N\to\infty} Y'/Y = E[Y']/Y = e^{\mu T(q^{\frac{\alpha}{1-\alpha}}-1)}$$

Quality ladder model results (3)

- Since the budget constraint is

$$Y = C + X + Z = C + \sum_{n=1}^{N} x_n + \sum_{n=1}^{N} \frac{\mu}{r+\mu} \pi^{\star}_{M_n+1}$$

where Z stands for the research expenditure $\sum_{n=1}^{N} \kappa_{M_n}$

- As usual, we can show that

$$g_Y = g_C = g_X = g_Z = e^{\mu_M T\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}$$

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