

Economic Growth Theory

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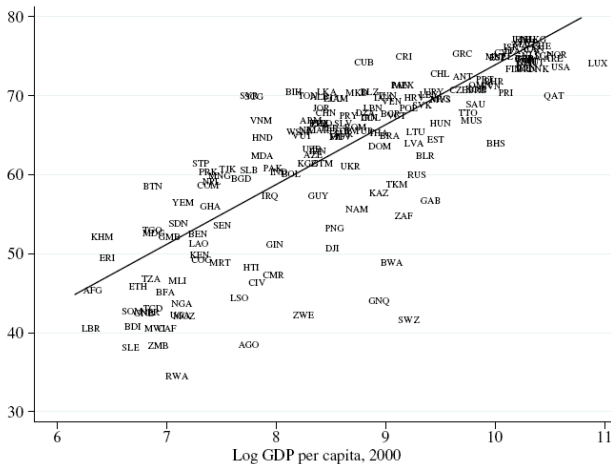
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Why growth?

- Large difference in per capita GDP levels across countries.
- Large differences in welfare associated.
- Income in the US is 30 times higher than it is in Ethiopia.
- What's driving such enormous differences?
- Obvious answer: persistent differences in growth rates.
- What does growth rate depend on? Very big question.

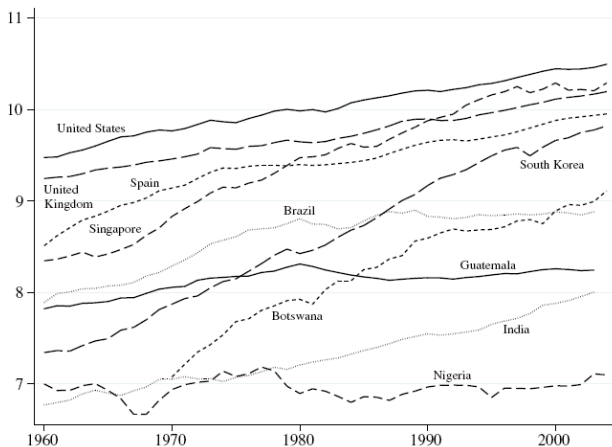
Life Expectancy vs GDP

Life expectancy, 2000 (years)



Levels of GDP by country

Log GDP per capita



Very trivial math

How many years are needed to duplicate GDP?

Growth rate	Years
0.1%	693.4
1%	69.6
2%	35.0
4%	17.6
5%	14.2
7%	10.2

Some data

Country	Population Millions	GDP per capita PPP level 2003	Growth 30y 1974-2003	Growth 20y 1984-2003
Argentina	38,741	10172	-0.13%	0.17%
Brazil	182,033	7204	1.04%	0.87%
Chile	15,665	12141	2.30%	3.76%
China	1,286,975	4970	7.54%	8.03%
Israel	6,117	20715	1.26%	1.73%
New Zealand	3,946	22197	0.97%	1.25%
Portugal	10,386	17333	2.21%	2.85%
Spain	42,144	20642	1.93%	2.60%
United States	292,617	34875	2.00%	1.95%

Relative GDP if we keep growth rate of 74-03

Country	US/country 2003	US/country 15 years	US/country 30 years	US/country 60 years
Argentina	3.43	4.71	5.90	10.14
Brazil	4.84	5.58	6.42	8.52
Chile	2.87	2.75	2.63	2.40
China	7.02	3.17	1.44	0.29
Israel	1.68	1.88	2.10	2.61
New Zealand	1.57	1.83	2.13	2.89
Portugal	2.01	1.95	1.89	1.77
Spain	1.69	1.71	1.72	1.76
United States	1.00	1.00	1.00	1.00

Relative GDP if we keep growth rate of 84-03

Country	US/country 2003	US/country 15 years	US/country 30 years	US/country 60 years
Argentina	3.43	4.47	5.82	9.88
Brazil	4.84	5.68	6.67	9.18
Chile	2.87	2.21	1.70	1.00
China	7.02	2.95	1.24	0.22
Israel	1.68	1.74	1.80	1.92
New Zealand	1.57	1.74	1.93	2.38
Portugal	2.01	1.77	1.55	1.19
Spain	1.69	1.54	1.40	1.15
United States	1.00	1.00	1.00	1.00

Years to catch-up

Country	Years US 03 growth 30	Years US 03 growth 20	Years to US growth 30	Years to US growth 20
Argentina	na	717.5	na	na
Brazil	152.0	181.6	na	na
Chile	46.3	28.6	352.4	60.2
China	26.8	25.2	36.8	33.7
Israel	41.7	30.3	na	na
New Zealand	46.8	36.4	na	na
Portugal	31.9	24.9	333.5	80.2
Spain	27.4	20.4	na	82.7
United States	0.0	0.0	0.0	0.0

What is a Macroeconomic Model?

Krueger notes

- Reality is very complex object, human intelligence is quite limited.
- Deliberate simplification of reality.
- No model can take into account all features of reality.
- Economic entities (agents) that make decisions subject to constraints.
- Different schools: use of shortcuts to model certain decision-making processes.

Decision-makers

- Households: Welfare (utility) maximizers. Preferences over commodities and restrictions involving prices and endowments.
- Firms: Profit maximizers subject to technological constraints. Technology describes how inputs become outputs.
- They may be homogenous or heterogeneous in a deterministic or in a stochastic way.

Decision-makers (2)

- Government: Policy instruments: taxes, expenditure, money supply. Two main approaches:
 - Positive: Policy instruments are given, subject to budgetary constraint.
 - Normative: Government has an objective function, subject to budgetary constraints and optimization conditions of firms and households.

Interactions

- Information assumptions: Specify what information agents have when making decisions.
- Trading assumptions: Search costs.
- Equilibrium concept:
 - How agents interact with each other.
 - Price-takers \implies Competitive Equilibrium.
 - Strategic: individual behavior directly affects prices or outcomes of other agents (Game theory)

Solow model environment

- Solow (1956)
- Good exposition: Acemoglu (2009) chapter 2, Barro and Sala-i-Martin (2004) chapter 1
- Households: ad-hoc behavior. They consume $s \in (0, 1)$ of their income. Key difference with neoclassical growth model.
- Identical agents admits a “representative agent” representation.
- Technology: Constant-return-to-scale technology with capital and labor. Term A is usually called Total Factor Productivity or TFP.

$$Y = F(A, K, L) \text{ such that } \lambda Y = F(\lambda K, \lambda L)$$

Solow model environment, cont

- Only one good to be consumed or invested. “Corn” economy.
- $F_K > 0$, $F_{KK} < 0$, $F_L > 0$, $F_{LL} < 0$ for all $0 < K, L < \infty$
- Inada (1963) conditions

$$\lim_{K \rightarrow \infty} F_K = \lim_{L \rightarrow \infty} F_L = 0$$

$$\lim_{K \rightarrow 0} F_K = \lim_{L \rightarrow 0} F_L = \infty$$

- Canonical example: Cobb-Douglas $Y = AK^\alpha L^{1-\alpha}$.
- Capital depreciates at constant rate δ
- Capital law-of-motion $K_{t+1} = K_t(1 - \delta) + I_t$

Solow model environment, cont

- Income is either consumed or invested $Y = C + I$
- Saving = Investment (closed economy) so $sY = I$
- Therefore, the fundamental equation in this model is

$$K_{t+1} = sF(A_t, K_t, L_t) + (1 - \delta)K_t$$

- Due to constant returns to scale, model is expressed in *per capita* terms with $k = K/L$, $y = Y/L$ and $f(k) = F(K/L, 1)$.

$$k_{t+1} = sA_t f(k_t) + (1 - \delta)k_t$$

Markets

- Household own labor and labor supply is inelastic wrt price (wage).
- Households own capital and rent it to firms at price r .
- Firms are profit-maximizers so that they solve

$$\max_{K \geq 0, L \geq 0} \{F(A_t, K, L) - w_t L - r_t K\}$$

- Optimization yields:
 - Marginal labor productivity $F_L = \frac{\partial F}{\partial L}$ equals wage $w_t = f(k) - kf'(k)$
 - Marginal capital productivity $F_K = \frac{\partial F}{\partial K}$ equals rental price of capital $r_t = f'(k) - \delta$
- Euler's theorem: $Y_t = K_t r_t + L_t w_t$. Due to constant return, all the product is paid to productive factors.

Competitive equilibrium

- A competitive equilibrium in this economy is a sequence of quantities (capital stocks, output levels, consumption levels) $\{K_t, Y_t, C_t\}_{t=0}^{\infty}$ and prices (wages and rental rates of capital) $\{w_t, r_t\}_{t=0}^{\infty}$ such that, given an exogenous sequence of TFP and labor endowments $\{A_t, L_t\}_{t=0}^{\infty}$ and an initial capital stock K_0
 - Households save a constant share of output, i.e.
$$(1 - s)Y_t = C_t$$
 - Households accumulate capital according to
$$K_{t+1} = sY_t + (1 - \delta)K_t$$
 - Firms optimize, i.e. $F_L(t) = w_t$ and $F_K(t) = r_t$
 - Market clears: all labor is hired and all capital is rented at prices $\{w_t, r_t\}_{t=0}^{\infty}$.

Steady state

- All endogenous variables growing at a constant (but perhaps different) rate. Also known as “balanced growth path”. Most models have this kind of solution.
- Suppose $A_{t+1} = A_t$ and $L_{t+1} = (1 + n)L_t$.
- Capital per worker evolves according to

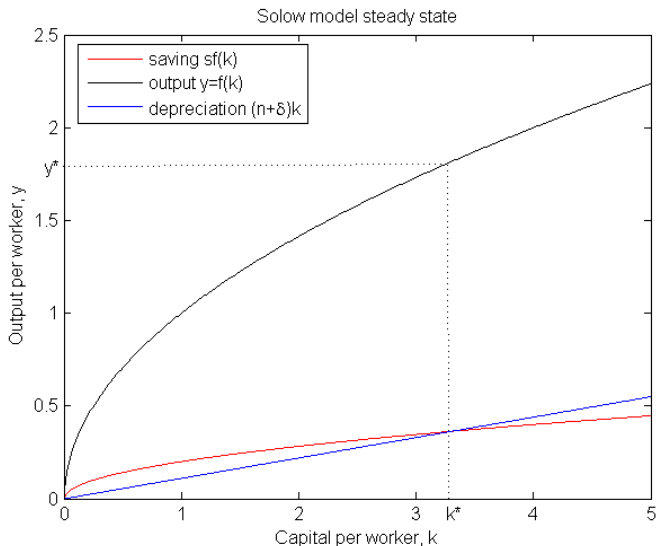
$$\frac{K_{t+1}}{L_{t+1}} \frac{L_{t+1}}{L_t} = s \frac{Y_t}{L_t} + (1 - \delta) \frac{K_t}{L_t}$$
$$k_{t+1}(1 + n) = sf(k_t) + (1 - \delta)k_t$$

- In steady state, $k_{t+1} = k_t = k^*$ so

$$sf(k^*) = k^*(n + \delta)$$

- Existence? Uniqueness?

Graphical Representation



Dynamic Efficiency - Golden Rule

- What is the saving rate that maximizes consumption?

$$c^* = (1 - s)f(k^*) = f(k^*) - (n + \delta)k^*$$

- Maximum steady-state consumption when

$$\partial c^* / \partial s = (f'(k^*) - (n + \delta)) \partial k^* / \partial s$$

- From Implicit Function Theorem, $\partial k^* / \partial s = \frac{f(k^*)}{s k^{*w^*}} > 0$.
- Then the “golden rule” implies $k^g = f'^{-1}(n + \delta)$.
- Hence $s^g = \frac{(n + \delta)f'^{-1}(n + \delta)}{f(f'^{-1}(n + \delta))}$
- If $s > s^g$ the economy is dynamically inefficient.

Transitional Dynamics (1)

- Equilibrium is the complete path, not just the steady state.
- Markets clear all the way towards the steady state.

$$k_{t+1}(1+n) = sf(k_t) + (1-\delta)k_t$$

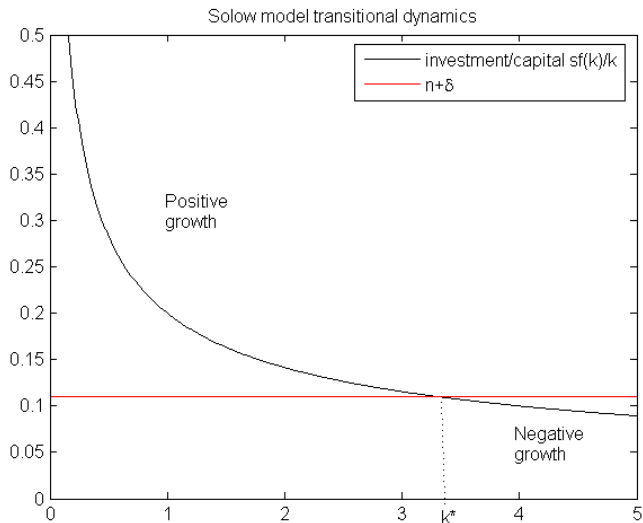
$$g_k \equiv \frac{k_{t+1} - k_t}{k_t} = \frac{1}{1+n} (sf(k_t)/k_t - (\delta + n))$$

- If time is continuous, there is a slight difference. See Acemoglu (2009), Barro and Sala-i-Martin (2004).
- Function $f(k)/k$ is decreasing because $\frac{d(f(k)/k)}{dk} = \frac{f'(k)k - f(k)}{k^2} = -\frac{w}{k^2} < 0 \quad \forall k$. As k increases, output growth rate declines.

Transitional Dynamics (2)

- Monotone convergence: If $k < k^*$, then $\frac{k'}{k} = (1+n)^{-1}(sf(k)/k + 1 - \delta) > (1+n)^{-1}(sf(k^*)/k^* + 1 - \delta) = 1$. If $k > k^*$, then $\frac{k'}{k} < 1$.
- Any bounded and monotone sequence has a unique limit.
- Intuition: Diminishing returns to capital kicks in.
- Steady state: Capital per capita saved exactly compensates the depreciated capital.

Transitional Dynamics (3)



Exogenous technological change (1)

- Technological progress in three ways:
- Hicks-neutral $\Rightarrow F(A_t, K_t, L_t) = AF(K_t, L_t)$
- Harrod-neutral (labor augmenting)
 $\Rightarrow F(A_t, K_t, L_t) = F(K_t, A_t L_t)$
- Solow-neutral $\Rightarrow F(A_t, K_t, L_t) = F(A_t K_t, L_t)$
- Suppose $A_{t+1} = (1 + a)A_t$ and labor-augmenting technological change.
- Only constant Harrod-neutral technological change is compatible with balanced growth in steady state. Formal proof in Jones and Scrimgeour (2008).

Exogenous technological change (2)

- Intuition: Asymmetry between K and L is that L cannot be accumulated.
- Consider $F(K/Y, L/Y) = 1$. K can grow at g_Y rate while L cannot. Hence, there must be a labor-augmenting technological change that restores balance.
- Define capital per effective unit of labor $\tilde{k}_t = K_t/(A_t L_t)$

$$\begin{aligned}
 K_{t+1} &= sF(K_t, A_t L_t) + (1 - \delta)K_t \\
 \frac{K_{t+1}}{A_{t+1} L_{t+1}} \frac{A_{t+1} L_{t+1}}{A_t L_t} &= sF\left(\frac{K_t}{A_t L_t}, 1\right) + (1 - \delta) \frac{K_t}{A_t L_t} \\
 \tilde{k}_{t+1}(1+n)(1+a) &= sf(\tilde{k}_t) + (1 - \delta)\tilde{k}_t
 \end{aligned}$$

- Steady state

$$\Rightarrow sf(\tilde{k}^*) = \tilde{k}^*(n + a + an + \delta) \approx \tilde{k}^*(n + a + \delta)$$

Convergence (1)

- Key implication is that the growth rate decreases as k increases because $g'_k = \left(\frac{sf(k)/k - \delta}{1+n}\right)' = -\frac{sw}{(1+n)k^2} < 0$
- Richer economies grow less, Poorer grow more.
- Economies with same $a, n, s, f(\cdot)$ and δ should achieve the same steady-state... testable prediction.
- Absolute/unconditional β -convergence. Not controlling for economies' characteristics. Data reject it.
- Some evidence of conditional β -convergence. "Similar" economies converge to similar per capita output.

Convergence (2)

- First notice that

$$g_{\tilde{y}} \approx d \log \tilde{y} = d \log(Y_t/L_t) - d \log A_t \approx g_y - a$$

- Moreover $d \log \tilde{y} \approx \alpha(\tilde{k}^*) d \log \tilde{k}$ with $\alpha(\tilde{k}^*) = \frac{\tilde{k}^* f'(\tilde{k}^*)}{f(\tilde{k}^*)} = \frac{d \log \tilde{y}}{d \log \tilde{k}}(k^*)$, the share of capital in output, or the capital-elasticity of output.
- Then, $g_y \approx a + \alpha(\tilde{k}^*) g_{\tilde{k}}$

Convergence (3)

- Using log-linear Taylor expansion around \tilde{k}^* (notice $\frac{dy}{d \log x} = x \frac{dy}{dx}$.)

$$\begin{aligned}
 g_{\tilde{k}} &= \frac{\tilde{k}'}{\tilde{k}} - 1 \\
 &\approx \frac{sf(\tilde{k}^*)/\tilde{k}^* - (n + a + an + \delta)}{(1 + n)(1 + a)} \\
 &\quad + \frac{s\tilde{k}^*}{(1 + n)(1 + a)} \frac{f'(\tilde{k}^*)\tilde{k}^* - f(\tilde{k}^*)}{\tilde{k}^{*2}} (\log \tilde{k} - \log \tilde{k}^*) \\
 &= - \left(\frac{n + a + an + \delta}{(1 + n)(1 + a)} \right) (1 - \alpha(\tilde{k}^*)) (\log \tilde{k} - \log \tilde{k}^*)
 \end{aligned}$$

- Hence, we conclude that

$$g_y \approx a - \left(\frac{n + a + an + \delta}{(1 + n)(1 + a)} \right) (1 - \alpha(k^*)) (\log y - \log y^*)$$

Continuous time

- Same conclusions, only math changes (Barro and Sala-i-Martin 2004)
- Notation derivative of X wrt to time is $\dot{X} \equiv dX/dt$.
- Hence

$$\begin{aligned}\dot{\tilde{k}} &= \frac{d(K/AL)}{dt} = \frac{(dK/dt)AL - (d(AL)/dt)K}{(AL)^2} \\ &= \frac{\dot{K}}{AL} - \frac{A(dL/dt) + L(dA/dt)}{AL} \tilde{k} = \frac{\dot{K}}{AL} - (n + a)\tilde{k}\end{aligned}$$

- Capital law-of-motion is

$$\dot{\tilde{k}} = \frac{\dot{K}}{AL} - (n + a)\tilde{k} = sf(\tilde{k}) - \delta\tilde{k}$$

- Basically, continuous time approach yields

$$g_y \approx a - (n + a + \delta)(1 - \alpha(k^*))(\log y - \log y^*)$$

σ-convergence (1)

- Does the cross-sectional dispersion of $\log y$ decreases over time? For a given country i , the Solow model predicts that $g_y \approx \log y_{i,t} - \log y_{i,t-1} \approx a - (n + a + \delta)(1 - \alpha(k^*))(\log y_{i,t} - \log y^*)$
- Rearranging the previous expression and adding a disturbance term ϵ we get

$$\log y_{i,t} = \tilde{a} + (1 - \tilde{b}) \log y_{i,t-1} + \epsilon_{i,t}$$

with $\tilde{a} = a + b \log y^*$ and $b = (n + a + \delta)(1 - \alpha(k^*))$

- Assuming that disturbance term is independent from $\log y$ we obtain

$$\sigma_t^2 = (1 - b)^2 \sigma_{t-1}^2 + \sigma_\epsilon^2$$

$$\text{with } \sigma_t^2 = I^{-1} \sum_{i=1}^I (\log y_{i,t} - \overline{\log y_{i,t}})^2$$

σ -convergence (2)

- Long run variance is

$$\sigma^{*2} = \frac{\sigma_{\epsilon}^2}{1 - (1 - b)^2}$$

- Faster β -convergence (higher b) implies lower cross-country log income dispersion.
- Larger disturbance variance imply larger cross-country log income dispersion.
- Dispersion evolves monotonically converging to steady-state value

$$\sigma_t^2 = \sigma^{*2} + (1 - b)^2(\sigma_{t-1}^2 - \sigma^{*2})$$

- If starting point is very low dispersion, we can have β -convergence and rising cross-country inequality at the same time.
- Hence σ -convergence $\Rightarrow \beta$ -convergence, but the other way

No convergence: Endogenous growth

- Simplest version is the linear technology model, i.e $y = Ak$.
- No decreasing return to capital leads to a continuous positive growth rate.
- Using a constant saving rate s and $a = 0$, we get that $g_k = sA - (n + \delta)$. Provided $sA > n + \delta$, a positive growth rate is sustained forever.
- No convergence since $\beta \approx (1 - \alpha)(n + a + \delta)$. Because $\alpha = 0$, then $\beta = 0$.
- Technologies that allow for transition and permanent growth $Y = AK + BK^\alpha L^{1-\alpha}$.
- What's behind this kind of technology?

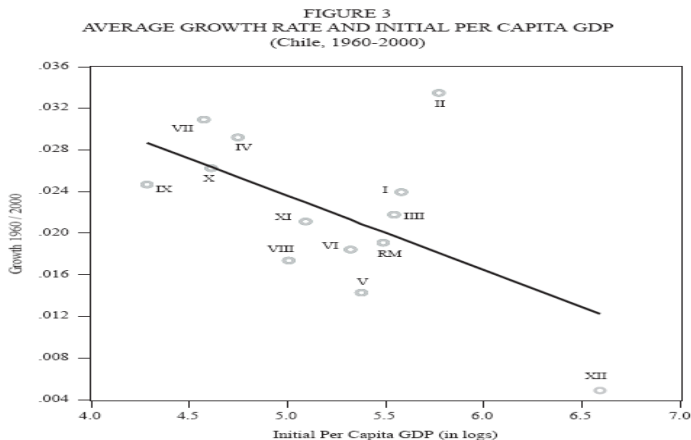
Convergence evidence for Chile (1)

- For Chile also Díaz and Meller (2004) find
 1. $\beta \approx 1.1\% - 2.1\%$. Half of the gap closes in 35-69 years. Similar to other regional studies in other countries.
 2. No clear evidence of σ -convergence.
 3. Income and wage convergence (CASEN measure) is faster than GDP convergence.
 4. Panel data estimation using IV yields much faster convergence.

Convergence evidence for Chile (2)

- For Chile Duncan and Fuentes (2006) report
 1. β absolute convergence across regions. Unit-root test rejection. $\beta \approx 1\%$
 2. Speed greatly increases if controlled for mining sector and/or education. Regions have different steady states because of different production functions.
 3. No clear evidence of σ -convergence.
 4. No significant multimodality, i.e. convergence clubs.

Convergence for Chilean regions Duncan and Fuentes (2006)



Regional convergence evidence

- For the US, Japan and Europe Barro and Sala-i-Martin (2004) (chapter 11) summarize findings of their papers.
- Within a country with similar tastes, technologies and institutions we should expect similar steady states.
- For US states, European regions and Japanese prefectures they find evidence for both absolute and conditional β -convergence.
- Convergence speed is surprisingly similar in US, European and Japanese economies.
- $\beta \approx 2\% - 3\%$ implying 25-35 years to close half of initial gap. It is not consistent with $\alpha = 1/3$ but with $\alpha = 3/4$.
- They also find some evidence for σ -convergence.

Convergence for US states (1)



Convergence for US states (2)

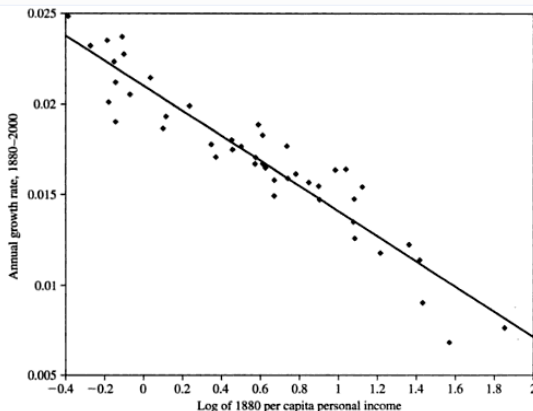


Figure 11.2

Convergence of personal income across U.S. states: 1880 personal income and 1880–2000 income growth. The average growth rate of state per capita income for 1880–2000, shown on the vertical axis, is negatively related to the log of per capita income in 1880, shown on the horizontal axis. Thus, absolute β convergence exists for the U.S. states.

Convergence for US states (3)

Period	Basic Equation		Equations with Regional Dummies		Equations with Structural Variables and Regional Dummies	
	$\hat{\beta}$	$R^2[\hat{\beta}]$	$\hat{\beta}$	$R^2[\hat{\beta}]$	$\hat{\beta}$	$R^2[\hat{\beta}]$
1880–2000	0.0172 (0.0024)	0.92 [0.0012]	0.0160 (0.0034)	0.95 [0.0010]	—	—
1880–1900	0.0101 (0.0022)	0.36 [0.0068]	0.0224 (0.0043)	0.62 [0.0054]	0.0268 (0.0051)	0.65 [0.0053]
1900–20	0.0218 (0.0031)	0.62 [0.0065]	0.0209 (0.0065)	0.67 [0.0062]	0.0270 (0.0077)	0.71 [0.0060]
1920–30	−0.0149 (0.0051)	0.14 [0.0132]	−0.0128 (0.0078)	0.43 [0.0111]	0.0209 (0.0119)	0.64 [0.0089]
1930–40	0.0129 (0.0033)	0.28 [0.0079]	0.0072 (0.0052)	0.34 [0.0078]	0.0147 (0.0083)	0.37 [0.0078]
1940–50	0.0502 (0.0058)	0.73 [0.0087]	0.0512 (0.0062)	0.88 [0.0059]	0.0304 (0.0065)	0.91 [0.0052]
1950–60	0.0193 (0.0039)	0.40 [0.0051]	0.0191 (0.0056)	0.52 [0.0047]	0.0305 (0.0053)	0.74 [0.0035]
1960–70	0.0286 (0.0039)	0.61 [0.0040]	0.0181 (0.0046)	0.73 [0.0034]	0.0196 (0.0061)	0.74 [0.0035]
1970–80	0.0186 (0.0049)	0.27 [0.0044]	0.0079 (0.0055)	0.44 [0.0040]	0.0057 (0.0068)	0.46 [0.0040]
1980–90	0.0036 (0.0085)	0.01 [0.0077]	0.0095 (0.0074)	0.57 [0.0052]	0.0029 (0.0070)	0.69 [0.0045]
1990–2000	0.0016 (0.0035)	0.01 [0.0035]	−0.0005 (0.0045)	0.07 [0.0035]	0.0029 (0.0050)	0.14 [0.0034]
Joint, 9 subperiods	0.0150 (0.0015)	—	0.0164 (0.0021)	—	0.0212 (0.0023)	—

Note: The regressions use nonlinear least squares to estimate equations of the form

$$(1/T) \cdot \log(y_{it}/y_{i,t-T}) = \alpha - [\log(y_{i,t-T})] \cdot [(1 - e^{-\beta T})/T] + \text{other variables}$$

Convergence for US states (4)

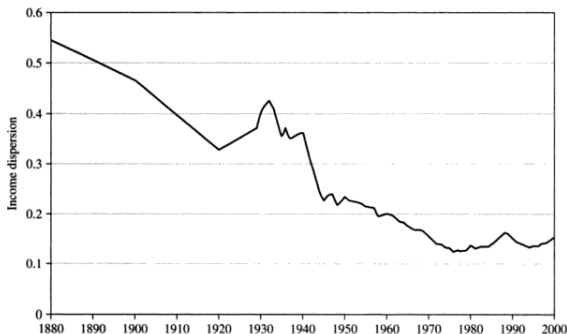


Figure 11.4

Dispersion of personal income across U.S. states, 1880–2000. The figure shows the cross-sectional standard deviation of the log of per capita personal income for 47 or 48 U.S. states or territories from 1880 to 2000. This measure of dispersion declined from 1880 to 1920, rose in the 1920s, fell from 1930 to the mid-1970s, rose through 1988, declined again through 1992, and then remained fairly flat.

Convergence for Japanese prefectures (1)



Convergence for Japanese prefectures (2)

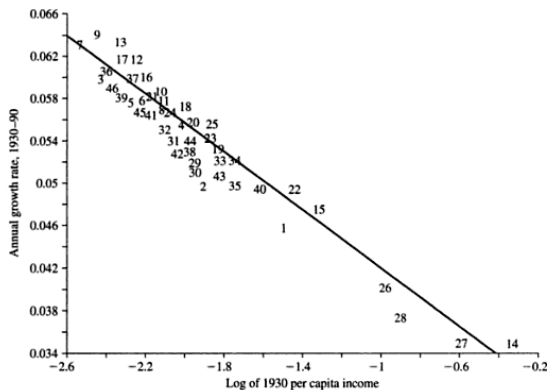


Figure 11.5

Convergence of personal income across Japanese prefectures: 1930 income and 1930-90 income growth. The growth rate of prefectural per capita income for 1930-90, shown on the vertical axis, is negatively related to the log of per capita income in 1930, shown on the horizontal axis. Thus absolute β convergence exists for the Japanese prefectures. The numbers shown identify each prefecture; see table 11.10.

Convergence for Japanese prefectures (3)

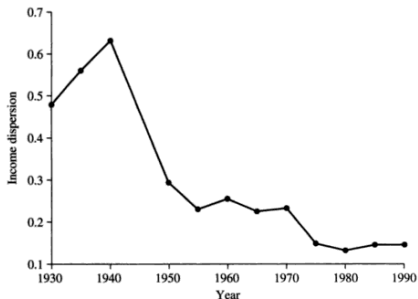
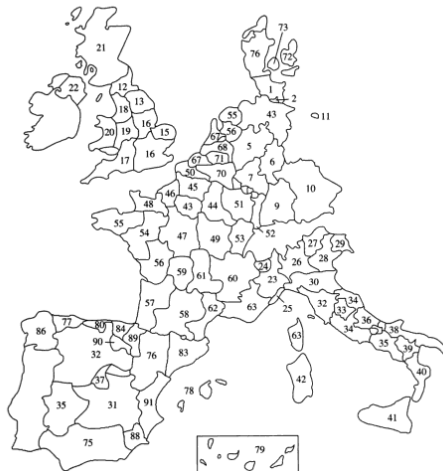


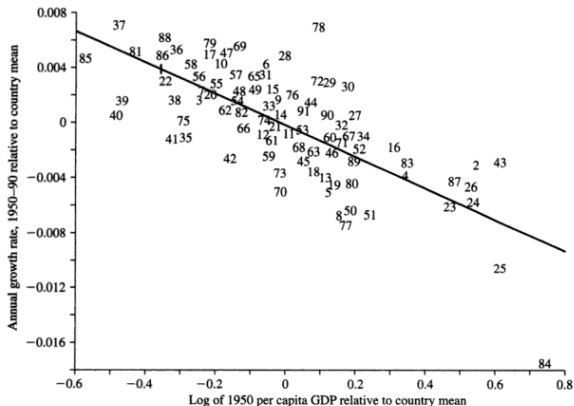
Figure 11.7

Dispersion of personal income across Japanese prefectures, 1930–90. The figure shows the cross-sectional standard deviation of the log of per capita personal income for 47 Japanese prefectures from 1930 to 1990. This measure of dispersion fell from the end of World War II until 1980.

Convergence for European regions (1)



Convergence for European regions (2)



Convergence for European regions (3)

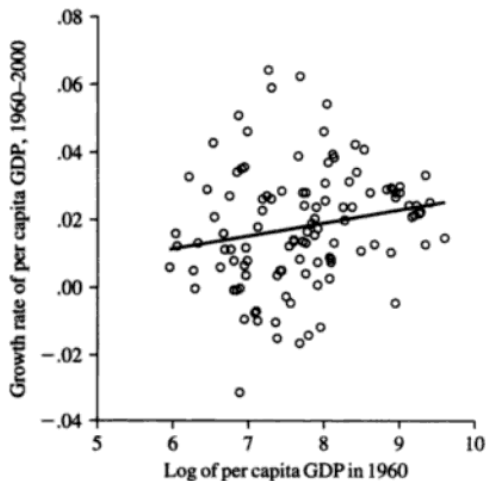
Table 11.3
Convergence Across European Regions

Period	(1) Equations with Country Dummies		(2) Equations with Sectoral Shares and Country Dummies	
	$\hat{\beta}$	$R^2[\hat{\sigma}]$	$\hat{\beta}$	$R^2[\hat{\sigma}]$
1950–60	0.018 (0.006)	0.83 [0.0099]	0.034 (0.009)	0.84 [0.0094]
1960–70	0.023 (0.009)	0.97 [0.0065]	0.020 (0.006)	0.97 [0.0064]
1970–80	0.020 (0.009)	0.99 [0.0079]	0.022 (0.007)	0.99 [0.0077]
1980–90	0.010 (0.004)	0.97 [0.0066]	0.007 (0.005)	0.97 [0.0064]
Joint, 4 subperiods	0.019 (0.002)	—	0.018 (0.003)	—
Likelihood-ratio statistic (p value)	4.9 (0.179)		8.6 (0.034)	

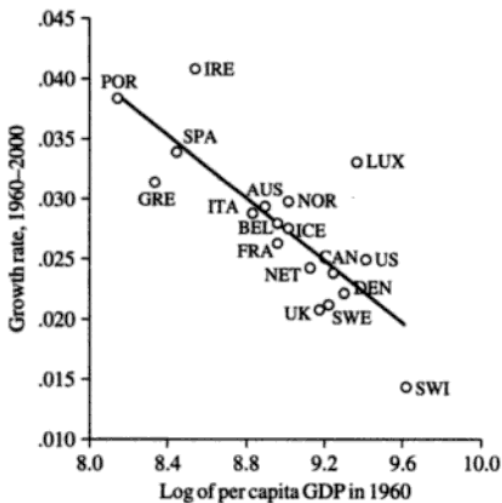
Cross-country convergence evidence

- Barro and Sala-i-Martin (2004) (chapter 12) summarize the evidence of these studies.
- Absolute β -convergence is rejected by the data using a complete cross-sectional sample of 114 countries (1960-2000).
- There is absolute convergence among OECD countries, a more homogenous sample.

Convergence in the world



Convergence for OECD economies



Cross-country convergence evidence

- Conditional convergence: attempt to control for factors that generate different steady states across economies.
- Typical equation is

$$T^{-1}(\log y_{i,t+T-1} - \log y_{i,t}) = a + \beta \log y_{i,t} + \gamma X_{i,t} + \epsilon_{i,t}$$

- Variables X_i try to capture differences in initial conditions that may generate different steady states.
- Simultaneity issues: use of lags of $X_{i,t}$ as instruments to avoid $E(X_{i,t}, \epsilon_{i,t}) \neq 0$
- $\beta \approx 2\% - 3\%$ implying 25-35 years to close half of initial gap. It is not consistent with $\alpha = 1/3$ but with $\alpha = 3/4$.
- Convergence only if other things equal

Convergence for OECD economies

Table 12.3

Basic Cross-Country Growth Regressions

(1)	(2)	(3)	(4)
Explanatory Variable	Coefficient	Coefficient for Low-Income Sample	Coefficient for High-Income Sample
Log of per capita GDP	-0.0248 (0.0029)	-0.0207 (0.0052)	-0.0318 (0.0049)
Male upper-level schooling	0.0036 (0.0016)	0.0056 (0.0045)	0.0020 (0.0016)
1/(life expectancy at age 1)	-5.04 (0.86)	-5.13 (1.18)	-1.28 (1.44)
Log of total fertility rate	-0.0118 (0.0050)	-0.0209 (0.0120)	-0.0211 (0.0054)
Government consumption ratio	-0.062 (0.023)	-0.102 (0.031)	-0.000 (0.031)
Rule of law	0.0185 (0.0059)	0.0237 (0.0099)	0.0223 (0.0063)
Democracy	0.079 (0.028)	0.044 (0.049)	0.105 (0.038)
Democracy squared	-0.074 (0.025)	-0.054 (0.052)	-0.080 (0.031)
Openness ratio	0.0054 (0.0048)	0.0169 (0.0113)	0.0061 (0.0046)
Change in terms of trade	0.130 (0.053)	0.181 (0.076)	0.036 (0.070)
Investment ratio	0.083 (0.024)	0.109 (0.035)	0.077 (0.027)
Inflation rate	-0.019 (0.010)	-0.019 (0.012)	-0.019 (0.009)
Constant	0.296 (0.034)	0.294 (0.052)	0.295 (0.052)
Dummy, 1975-85	-0.0078 (0.0026)	-0.0078 (0.0038)	-0.0066 (0.0032)
Dummy, 1985-95	-0.0128 (0.0034)	-0.0194 (0.0051)	-0.0052 (0.0040)
Number of observations	72, 86, 83	26, 38, 33	46, 48, 50
R-squared	.60, .49, .51	.78, .53, .65	.56, .56, .40

Controls in conditional convergence equation (1)

- Educational attainment: Schooling unadjusted for measures of quality. Positive impact for male, but unclear for female.
- 1/Life expectancy: Proxy for health capital (prob of dying).
- Government expenditures to GDP (not educ nor defense): Measure of level of distortions in the economy. Negative effect.
- Fertility rate: as suggested from Solow model, it negatively affects growth.
- Investment rate: as suggested from Solow model, it positively affects growth.

Controls in conditional convergence equation (2)

- Well functioning political and legal institutions eases growth.
- Democracy: Electoral rights. Institutional commitment to reduce expropriation risk of private capital.
- International Openness: $(\text{Export} + \text{Import})/\text{GDP}$. It reflects trade distortions and/or specialization that enhances productivity.
- Terms of trade $P_{\text{export}}/P_{\text{import}}$: If country is small, it reflects exogenous variation in level of income.
- Inflation rate: Measure of macroeconomic stability.

Controls in conditional convergence equation (3)

- Geography hypothesis: tropical areas show consistently lower income level. Explanation is the prevalence of infectious diseases. (Reverse causality?)
- Weber (1930) states that implicit cultural support of Protestants and Calvinists to pro-capitalism values such as hard work and thrift explains West and North Europe higher income.
- “Reversal of the Fortune” by Acemoglu, Johnson, and Robinson (2002) (also Acemoglu (2009) chapter 4).
- European colonists left better political institutions in originally empty or poor locations. Good economic performance in the future (US & Canada in particular).
- Abundant indigenous population and/or mineral resources lead to “exploitation” institutions, generating poor economic performance in the XX century.

Controls in conditional convergence equation (4)

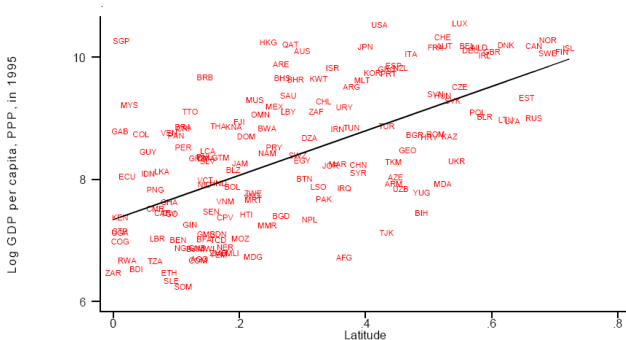


FIGURE 4.2. Relationship between latitude (distance of capital from the equator) and income per capita in 1995.

Controls in conditional convergence equation (5)

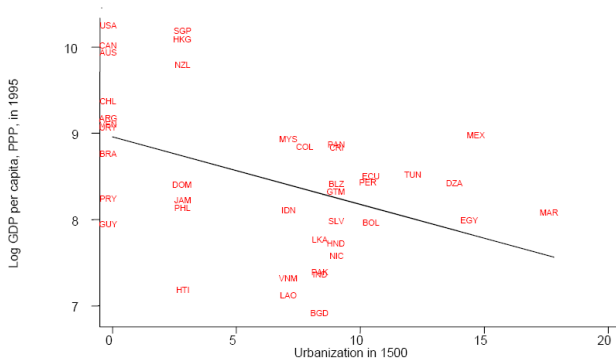


FIGURE 4.5. Reversal of Fortune: urbanization in 1500 versus income per capita in 1995 among the former European colonies.

Controls in conditional convergence equation (6)

Table 12.5

Additional Explanatory Variables in Cross-Country Growth Regressions

(1)	(2)	(3)	(4)	(5)
New Explanatory Variable	Coefficient	Additional New Variable	Coefficient	<i>p</i> Value ^d
Log of population	0.0004 (0.0009)			
Log of per capita GDP squared	−0.0035 (0.0020)			
Female upper-level schooling	−0.0034 (0.0041)			
Male primary schooling	−0.0011 (0.0025)	Female primary schooling	0.0007 (0.0024)	0.90
Male college schooling ^b	0.0105 (0.0093)	Male secondary schooling	0.0024 (0.0020)	0.075
Student test scores ^c	0.121 (0.024)			
Infant mortality rate	−0.001 (0.057)			
1/(life expectancy at birth)	−0.97 (2.52)			
1/(life expectancy at age 5)	0.90 (2.00)			
Malaria incidence	0.0019 (0.0045)			
Official corruption	0.0093 (0.0068)			
Quality of bureaucracy	0.0076 (0.0088)			
Civil liberties ^d	−0.045 (0.081)	Civil liberties squared	0.003 (0.070)	0.36
Sub-Saharan Africa dummy ^e	−0.0080 (0.0051)	Latin America dummy	0.0031 (0.0039)	0.011
East Asia dummy	0.0100 (0.0047)	OECD dummy	0.0004 (0.0054)	
Population share < 15	−0.070 (0.070)	Population share > 64	−0.080 (0.110)	0.61
Government spending on education	−0.057 (0.068)	Government spending on defense	0.064 (0.028)	0.069

Controls in conditional convergence equation (7)

Government spending on education	-0.057 (0.068)	Government spending on defense	0.064 (0.028)	0.069
Log of black-market premium	-0.0122 (0.0058)			
Private financial system credit	-0.0041 (0.0065)			
Financial system deposits	-0.002 (0.011)			
British legal structure dummy	-0.0018 (0.0044)	French legal structure dummy	0.0047 (0.0045)	0.10
Absolute latitude (degrees \div 100)	0.066 (0.027)	Latitude squared	-0.085 (0.044)	0.036
Landlocked dummy	-0.0088 (0.0032)			
Ethnic fractionalization	-0.0080 (0.0059)			
Linguistic fractionalization	-0.0084 (0.0050)			
Religious fractionalization	-0.0088 (0.0058)			
British colony dummy ^f	-0.0064 (0.0043)	French colony dummy	0.0003 (0.0053)	0.39
Spanish/Portuguese colony dummy	-0.0019 (0.0053)	Other colony dummy	-0.0055 (0.0075)	

Notes: Each new explanatory variable or group of new variables is added to the system shown in column 2 of table 12.3.

^a p value is for the test of the hypothesis that the coefficients of the new explanatory variables are jointly zero.

^b Upper-level male schooling is omitted. The p value for equality of college and secondary variables is 0.44.

^c Numbers of observations for this sample are 39, 45, and 44.

^d This system is only for the two periods 1975–85 and 1985–95.

^e The four regional dummy variables are entered together.

^f The four colony dummies are entered together.

Controls in conditional convergence equation (8)

- What variables do we include? Which ones should we exclude? Robustness issue.
- Sometimes a variable x_0 is not significant when x_1 is present, but it is if x_1 and x_2 are present.
- Econometric procedures to determine which variables really belong to the regression equation.
- List of possible variables is virtually infinite!
- Domestic credit, volatility of growth rate, exchange rate black market premium, financial repression, financial sophistication, income inequality, latitude, mining, religion, etc, etc.
- Incorporation of additional X_i variables is an *ad hoc* procedure. No clear theoretical reasons to include a particular X_i .

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