Business Cycles

Benjamín Villena Roldán
CEA, Universidad de Chile

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Recognizing frictions

- Unemployment is out of the picture so far.
- So are unfilled job positions (vacancies), too.
- For unemployed workers, it takes time and effort to find a suitable job.
- For firms, it takes time and effort to find a suitable worker.
- One way to deal with this is to introduce a matching function.
- Most of our analysis comes from Pissarides (2000)

Matching function (1)

- Matching function can be understood as a "production function" technology whose output is paired filled jobs.
- It is a convenient "black box" which captures the fact that only some unemployed workers and unfilled vacancies meet appropriate matches $m = M(u, v)$, with $M_1, M_2 > 0$ for all $u, v \geq 0$
- If all jobs are identical, the probability of filling a vacancy is $q = m/v$
- If all workers are identical, the probability of finding a job is $(v/u)q = m/u$

Matching function (2)

- Usually assumed constant returns to scale, which makes it possible to write the probabilities in terms of $\theta = v/u$, called labor market tightness.

$$M(u,v)/v = M(u/v,1) = M(1/\theta,1) = q(\theta)$$
$$M(u,v)/u = (v/u)(M(u,v)/v) = \theta q(\theta) = p(\theta)$$

- Note $q'(\theta) < 0$ and $p'(\theta) > 0$.
- Search or congestion externalities: if one worker more applies, he decreases the chance of finding a job for others.
- Posting an additional vacancy reduces the probability of filling vacancies for other firms.
- Empirically successful functional form for matching function is Cobb-Douglas: $M(u,v) = Au^{\phi}v^{1-\phi}$

Matching function (3)

- For simplicity, a job is destroyed with exogenous probability λ each period
- Dynamic unemployment determination is

$$u' = (1 - \theta q(\theta))u + \lambda(1 - u)$$

- Steady state equilibrium: inflow = outflow so that u doesn't change

$$u^\star = \frac{\lambda}{\lambda + p(\theta)}$$

- This is called the "Beveridge curve", a negative relationship between u and v.

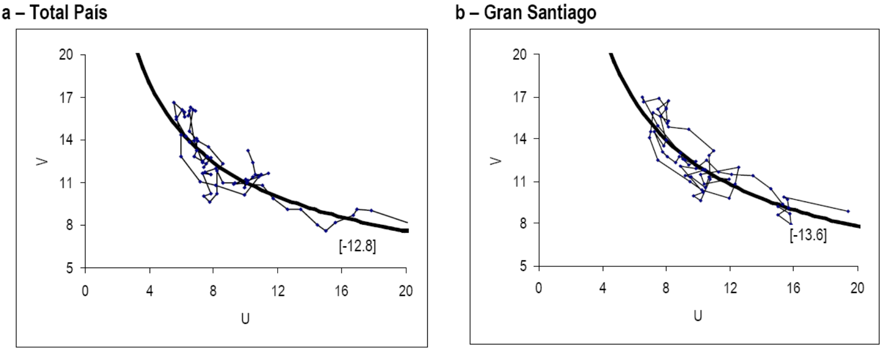
$$\begin{aligned}\frac{\partial u}{\partial v} &= -\frac{\lambda}{(\lambda + p(\theta))^2} p'(\theta) \frac{\partial \theta}{\partial v} \\ &= -\frac{u}{\lambda + p(\theta)} \frac{p'(\theta)}{u} = -\frac{p'(\theta)}{\lambda + p(\theta)} < 0\end{aligned}$$

Beveridge Curve in Chile (1)

- Beveridge Curve has been profusely studied in developed economies
- In Chile, Belani, García, and Pastén (2002) measured vacancies using Job wanted adds in newspapers and estimated Beveridge curves for Chile.
- "Well behaved" curves were estimated for the whole country, Santiago, Antofagasta, Valparaíso-Viña del Mar.
- For Concepción-Talcahuano and Temuco results show instability in the mid 90s.
- Vacancies lead employment and GDP in Chile.

Beveridge Curve in Chile (2)

Gráfico 1 – Curva de *Beveridge*:* 1986.I 2002.II
(series desestacionalizadas)



Job creation (1)

- Value of being unemployed U

$$U = b + \beta[\theta q(\theta)W + (1 - \theta q(\theta))U]$$

- What is b? Value of leisure, nonlabor income, unemployment benefit, home production
- Value of being employed W

$$W = w + \beta[\lambda U + (1 - \lambda)W]$$

Job creation (2)

- Value of posting a vacancy V

$$V = -c + \beta[q(\theta)J + (1 - q(\theta))V]$$

- Value of filling a job J

$$J = y - w + \beta[\lambda V + (1 - \lambda)J]$$

- Equilibrium: free-entry condition assumes there is ∞ potential entrants and no entry cost $V = 0$.
- In equilibrium the marginal benefit from hiring a worker equals the expected cost of finding him.

$$J = \frac{y - w}{1 - \beta(1 - \eta)} = \frac{c}{\beta q(\theta)}$$

Wage Determination (1)

- How is wage w determined?
- Bilateral monopoly situation: each match generates a surplus. How do agents split it?
- Several approaches. Most traditional is Nash bargaining.
- Each party has an outside option or threatening point which is the least amount to get.
- Nash proves that there is a unique solution which is
  - Pareto efficient: no surplus is wasted.
  - Independent of Irrelevant Alternatives: If x is the solution in the choice set X and is still available in $X_0 \subset X$, then x must be the solution in X_0.

Wage Determination (2)

- Threatening points / outside options:
  - Being unemployed U is the best alternative for the worker.
  - Posting vacancies V is the best alternative for the employer.
- Nash solution yields following properties:

$$w^\star = \underset{w}{\operatorname{argmax}} \left\{ (W(w) - U)^\alpha (J(w) - V)^{1-\alpha} \right\}$$

Wage Determination (3)

- First-order conditions

$$\alpha\left(\frac{J-V}{W-U}\right)^{1-\alpha}\frac{\partial W}{\partial w}+(1-\alpha)\left(\frac{W-U}{J-V}\right)^{\alpha}\frac{\partial J}{\partial w}=0$$

- which yields

$$\alpha(J-V)=(1-\alpha)(W-U)$$

- Using Free entry condition and that $r=\beta^{-1}-1$, we can write $J-V$ as

$$J-V=(1+r)\frac{y-w}{r+\lambda}$$

Wage Determination (4)

- Since every employer and worker bargain in the same way, the wage paid in the current job is the same as the one paid in a prospective job. Hence,

$$W - U = w - b + \beta(1 - \lambda - \theta q(\theta))(W - U)$$
$$W - U = (1 + r)\frac{w - b}{r + \lambda + \theta q(\theta)}$$

Wage Determination (5)

- This finally yields an equation for wage w

$$w = \left(\frac{(1-\alpha)(r+\lambda)}{r+\lambda+\theta q(\theta)} \right) b + \left(1 - \frac{(1-\alpha)(r+\lambda)}{r+\lambda+\theta q(\theta)} \right) y$$

- Some conclusions:
  - Wage is weighted average of productivity and nonlabor income.
  - The larger the α, the larger the wage w.
  - The larger the job finding rate $p(\theta)$, the larger the wage, i.e. greater θ increases wages.
  - The larger $r + \lambda$, the smaller the wage.

Competitive Equilibrium (1)

- An equilibrium in a triple $(u^\star, \theta^\star, w^\star)$ such that
  - The unemployment rate remains unchanged (flow condition / Beveridge curve)

$$u^\star = \frac{\lambda}{\lambda + p(\theta^\star)} \tag{1}$$

  - The job creation condition is satisfied (Job creation equation)

$$y - w^\star = \frac{c(r+\lambda)}{q(\theta^\star)} \tag{2}$$

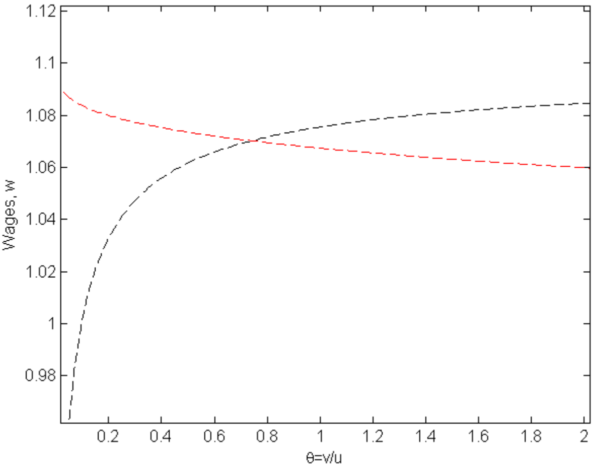
  - The wage is determined via Nash bargaining

$$w^\star = \left(\frac{(1-\alpha)(r+\lambda)}{r+\lambda+p(\theta^\star)}\right) b + \left(1 - \frac{(1-\alpha)(r+\lambda)}{r+\lambda+p(\theta^\star)}\right) y \tag{3}$$

Competitive Equilibrium (2)

- Using (2) and (3) we can solve for the pair $(\theta^\star, w^\star)$
- Graphical representation (plotting both curves) is useful to analyze comparative statics.

Competitive Equilibrium (3)



Competitive Equilibrium (4)

- Combining equations (2) and (3) we can obtain

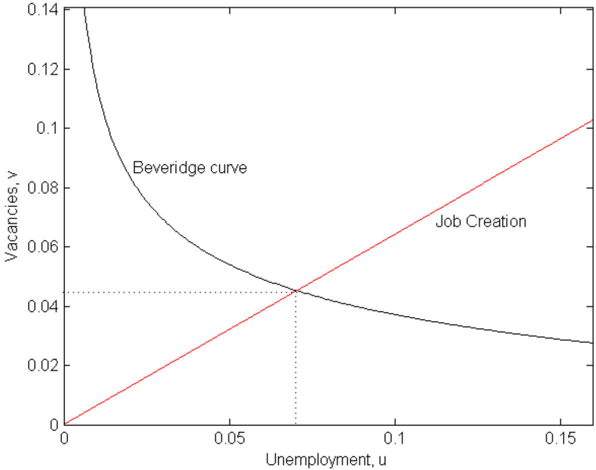
$$(1-\alpha)\left(\frac{y-b}{c}\right) = \frac{r+\lambda}{q(\theta^\star)} + \theta^\star \tag{4}$$

which depicts another Job creation locus in the space (v, u)

- Note that 4 defines a unique $\theta^\star$, i.e there is a unique relation $v = \theta^\star u$ consistent with the Beveridge curve
- With is result, we obtain the unemployment rate in equilibrium as

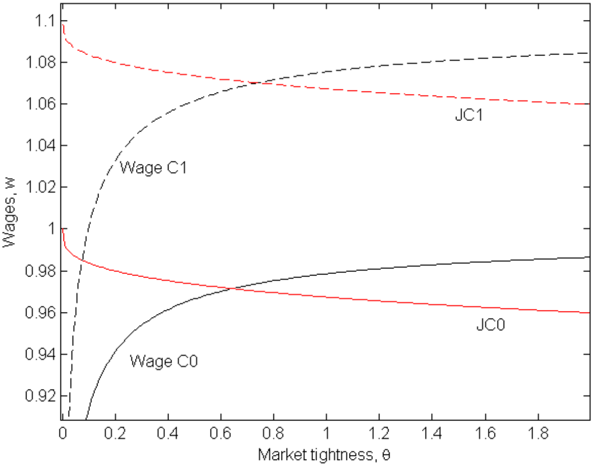
$$u^\star = \frac{\lambda}{\lambda + p(\theta^\star)}$$

Competitive Equilibrium (5)



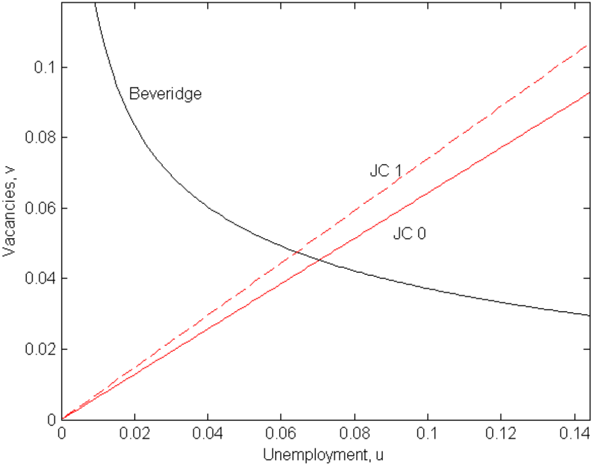
Competitive Equilibrium (6)

Job creation and Wage curves when y increases 10%



Competitive Equilibrium (7)

Beveridge curve and Job creation curves when y increases 10%



Business Cycles and Labor Search (1)

- One canonical example is Andolfatto (1996).
- His paper incorporates labor market search to standard RBC models.
- Why? It improves empirical performance in several regards
  - Standard RBC cannot explain unemployment nor Beveridge curve.
  - Hours worked fluctuations are greater than wage fluctuations.
  - Correlation between hours worked and productivity falls.
  - Output shows some endogenous persistence.

Business Cycles and Labor Search (2)

- Continuum of households who like consumption and leisure and face work and saving decisions.
- Worker spend time in working l hours if employed, in searching for jobs e hours if searching and in leisure.

$$U(c) + \phi_1 H(1 - l) + \phi_2 H(1 - e)$$

- Firms: Maximize profits, but labor becomes a "stock" of workers, since their accumulation is costly. Each job/firm needs one worker to produce.
- Productivity shocks follow a persistent stochastic process.

Business Cycles and Labor Search (2)

- There are n_t workers employed each period and $1 - n_t$ unemployed.
- Exogenous separation rate $0 < \sigma < 1$.
- v_t stands for the available umber of jobs in t (vacancies)
- κ flow cost of posting a vacancy.
- Matching technology $M(v, (1 - n)e) \leq \min(v, 1 - n)$, with e is worker search effort.
- Employment law-of-motion is

$$n_{t+1} = (1 - \sigma)n_t + M(v_t, (1 - n_t)e_t)$$

Social Planner Solution (1)

- In principle, as there are distortions (search frictions), Competitive solution may not be Pareto Optimal.
- Andolfatto (1996) strategy: Solve the Pareto optimal problem and try to "decentralize".
- The Planner faces the same matching frictions than does private agents
- Define state vector $s = (k, n, z)$. Why?

$$W(s) = \max_{c,l,k',n',v} \Big\{ U(c) + n\phi_1 H(1-l) + (1-n)\phi_2 H(1-e) + \beta E[W(s')|z] \Big\}$$

$$\text{s.t} \quad F(k, nl, z) + (1-\delta)k \geq k' + \kappa v + c \quad (\lambda)$$

$$(1-\sigma)n + M(v, (1-n)e) = n' \quad (\mu)$$

Social Planner Solution (2)

- First-order conditions

$$
\begin{aligned}
c: &\quad 0 = U_1(c) - \lambda \\
l: &\quad 0 = -n\phi_1 H'(1-l) + \lambda F_2(k, nl, z)n \\
k': &\quad 0 = \beta E[W_1'(s')|z] - \lambda \\
n': &\quad 0 = \beta E[W_2'(s')|z] - \mu \\
v: &\quad 0 = -\lambda\kappa + \mu M_1(v, (1-n)e)
\end{aligned}
$$

- Envelope conditions

$$
\begin{aligned}
W_1(s) &= \lambda(F_1(k, nl, z) + (1-\delta)) \\
W_2(s) &= \phi_1 H(1-l) - \phi_2 H(1-e) + \lambda F_2(k, nl, z)l \\
&\quad + \mu((1-\sigma) - eM_2(v, (1-n)e))
\end{aligned}
$$

Social Planner Solution (3)

- Equations characterizing optimal allocation

$$
\begin{aligned}
&U_1 = \beta E[(F_1 + 1 - \delta)U_1' | z] \quad \text{k-Euler} \\
&\phi_1 H_1(1 - l) = F_2 U_1 \quad \text{Hours supply} \\
&\mu = \beta E[\phi_1 H_1(1 - l') - \phi_2 H(1 - e) + U_1' F_2' l' \\
&\ + \mu'(1 - \sigma - eM_2') \quad \text{n-Euler} \\
&\mu = U_1 \kappa / M_1
\end{aligned}
$$

- These equations and resource constraint and employment law-of-motion characterize optimal allocation.
- Model calibration

Results Andolfatto (1996)

TABLE 1—CYCLICAL PROPERTIES: U.S. ECONOMY AND MODEL ECONOMIES

	U.S. economy $\sigma(y) = 1.58$			RBC economy $\sigma(y) = 1.22$			Search economy $\sigma(y) = 1.45$		
Variable (x)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Consumption	0.56	0.74	0	0.34	0.90	0	0.32	0.91	0
Investment	3.14	0.90	0	3.05	0.99	0	2.98	0.99	0
Total hours	0.93	0.78	+1	0.36	0.98	0	0.59	0.96	0
Employment	0.67	0.73	+1	0.00	0.00	0	0.51	0.82	+1
Hours/worker	0.34	0.66	0	0.36	0.98	0	0.22	0.66	0
Wage bill	0.97	0.76	+1	1.00	1.00	0	0.94	1.00	0
Labor's share	0.68	−0.38	−3	0.00	0.00	0	0.10	−0.62	−1
Productivity	0.64	0.43	−2	0.64	0.99	0	0.46	0.94	0
Real wage	0.44	0.04	−4	0.64	0.99	0	0.39	0.95	0

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between x and y. Column (3) is the phase shift in x relative to y: $-j$ or $+j$ corresponds to a lead or lag of j quarters.

Conclusions

- Search models are design to capture coexistence of unemployment and unfilled vacancies.
- Promising approach to model labor markets.
- New problems: Shimer (2005) shows that the cyclical volatility of market tightness is about 20 times larger than the amount predicted a standard search model.
- Partial solution to RBC problems, but also puts on the table many new issues.

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