# Business Cycles

Benjamín Villena Roldán
CEA, Universidad de Chile

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# Home Production (1)

- Recessions are times in which people allocate time either unemployed or out of the labor force.
- One way to rationalize this evidence is by recognizing home production.
- Households produce nonmarket goods: meals, cleaning, repairing, child-caring, etc using
  - Labor effort at home
  - Household capital durable goods plus residential structure

# Home Production (2)

- Should we care? Definitely yes.
- Typical US married couple spend 25% of their discretionary time in unpaid work at home and 33% of their time working in the market (Juster & Stafford 1991)
- Household capital is 15% larger than business capital in the US (Eisner 1988)
- Contribution of home production to business cycle theory: alternative activity to stay out of the labor market during recessions.

# Home Production (3)

- A deeper question: what's leisure? what's housework?
- Aguiar and Hurst (2007) Leisure is any domestic activity in which you cannot pay for in order to obtain a comparable experience.
- Example: Cleaning vs. Watching movies. You can pay someone to clean for you, but you cannot pay someone to watch a movie for you...
- Example: Changing diapers vs. Playing with children.
- Example: Cooking for fun vs. Preparing a meal. Ambiguous?

# A model of Home Production (1)

- Standard model by Greenwood, Rogerson, and Wright (1993)
- Households maximize

$$\sum_{t=0}^{\infty} \beta^t (b \log C_t + (1-b) \log l_t)$$

- Composite consumption $C_t$ is composed by market and home goods

$$C_t = (a c_{Mt}^{\epsilon} + (1-a) c_{Ht}^{\epsilon})^{1/\epsilon}$$

- Leisure is the remaining time when market and home production time are allocated

$$l_t = 1 - h_{Mt} - h_{Ht}$$

# A model of Home Production (2)

- Market budget constraint is

$$
\begin{aligned}
c_{Mt} + k_{M,t+1} + k_{H,t+1} = &(1-\tau_l) w_t h_{Mt} + (1-\tau_k) r_t k_{M,t} \\
&+ (1-\delta_M) k_{M,t} + T_t
\end{aligned}
$$

- Home goods production function

$$
c_{Ht} = G(z_{Ht}, k_{Ht}, h_{Ht}) = k_{Ht}^{\eta} (z_{Ht} h_{Ht})^{1-\eta}
$$

- Notice that home production cannot be traded in the market not converted into market capital. Only market sector can produce capital!
- Market goods production function

$$
y_t = F(z_{Mt}, k_{Mt}, h_{Mt}) = k_{Mt}^{\theta} (z_{Mt} h_{Mt})^{1-\theta}
$$

# A model of Home Production (3)

- Market and Home technological shock follows AR(1) processes in logs

$$\log z_{M,t+1} = \rho_M \log z_{M,t+1} + \varepsilon_{Mt}$$
$$\log z_{H,t+1} = \rho_H \log z_{H,t+1} + \varepsilon_{Ht}$$

  where $(\varepsilon_{Mt}, \varepsilon_{Ht})$ are independently identically distributed (iid) random variables.

- Government revenue is redistributed via lump-sum transfer to households

$$T_t = \tau_l w_t h_{Mt} + \tau_k r_t k_{Mt}$$

# Home production: Solving the model (1)

- Recursive formulation

$$
\begin{aligned}
& V(k_M, k_H, z_M, z_H) \\
& = \max_{k'_M, k'_H, h_M, h_H} \{u(c_M, c_H, h_M, h_H) + \beta E[V(k'_M, k'_H, z'_M, z'_H)|z_M, z_H]\} \\
& \text{s.t } c_M + k'_M + k'_H = (1-\delta_M)k_M + (1-\delta_M)k_H + w(1-\tau_h)h_M \\
& + r(1-\tau_K)k_M + \delta_M \tau_K k_M + T \\
& c_H = g(h_H, k_H, z_H)
\end{aligned}
$$

- Balanced growth path with $z_{Mt} = z_{Ht} = \lambda^t$
- Using preferences represented by

$$
u(c_M, c_H, h_M, h_H) = (b/e)\log[ac_M^e + (1-a)c_H^e] + (1-b)\log[1-h_M-h_H]
$$

# Home production: Solving the model (2)

- First-order conditions and Envelope imply

$$h_M: \quad u_1 w(1-\tau_h) = -u_3$$

$$abc_M^{e-1} C^{-e} y(1-\theta)(1-\tau_h) = (1-b)h_M/(1-h_M-h_H) \tag{1}$$

$$h_H: \quad u_2 g_1 = -u_4$$

$$(1-a)bc_H^e C^{-e}(1-\eta) = (1-b)(1-h_H)/(1-h_M-h_H) \tag{2}$$

$$k_M: \quad u_1 = \beta u_1'(r'(1-\tau_K) + 1 - \delta_K + \tau_K \delta_K)$$

$$\theta(1-\tau_k)(y/k_M) = \lambda/\beta - 1 + \delta_M(1-\tau_K) \tag{3}$$

$$k_H: \quad u_1 = \beta(u_1'(1-\delta_H) + u_2' g_2')$$

$$\eta(1-\alpha)c_H^e C^{1-e} = ak_H(\lambda/\beta - 1 + \delta_H) \tag{4}$$

- Also holds that

$$\frac{x_M}{y} = \frac{k_M' - (1-\delta_M)k_M}{y} = \lambda - 1 + \delta_M \tag{5}$$

$$\frac{x_H}{y} = \frac{k_H' - (1-\delta_H)k_H}{y} = \lambda - 1 + \delta_H \tag{6}$$

# Home production: Solving the model (3)

Calibration

| Parameter | Value | Explanation |
|---|---|---|
| $\lambda$ | 1.005 | Quarter growth rate GDP |
| $\beta$ | 0.9898 | 6% annual return assets |
| $\tau_h$ | 0.25 | Labor income tax rate (est) |
| $\tau_k$ | 0.7 | Capital income tax rate (est) |
| $\rho_M$ | 0.95 | Persistence market shock |
| $\rho_H$ | 0.95 | Persistence home shock |
| $\sigma_M$ | 0.007 | SD market shock |
| $\sigma_H$ | 0.007 | SD home shock |

# Home production: Solving the model (4)

- Using equations 1-6 we can solve for the average following moments in the US

$$k_M/y = 4 \qquad k_H/y = 5 \quad x_M/y = 0.118 \qquad x_H/y = 0.135$$

- Averaging data 1971 and 1981, we find $h_M = 0.33$ and $h_H = 0.25$
- This info determines $\delta_M, \delta_H, \theta, \eta$ and two of the three preference parameters $a$, $b$, $e$.

$$\delta_M = 0.0247 \quad \delta_H = 0.0218$$
$$\theta = 0.29(\text{cons with NIPA}) \quad \eta = 0.32$$

# Home production: Solving the model (5)

- Substitution $e$ and shocks' correlation $\gamma$ are changed.
- Model 1: Minimizes role of home sector $e = 0$ elasticity subs between $c_M$ and $c_H$ is 1. Default $\gamma = 2/3$. Similar to standard RBC.
- Model 2: Set $e = 2/3$, implying higher substitution between $c_M$ and $c_H$.
- Model 3: Likelihood estimation McGrattan et al (1992) sets $e = 0.4$ and $\gamma = 0$.
- Model 4: CES home production

$$g(h_H, k_H, z_H) = (\eta k_H^\psi + (1-\eta)(h_H z_H)^\psi)^{1/\psi}$$

sets $\psi = -1/2$, $\gamma = 0.99$ and $e = 2/3$.

# Home production: Results

| | % S.D. of Market Output | Ratio of Standard Deviations: Variable Relative to Market Output: Total Investment | Market Consumption | Market Hours | Market Wage | Market Hours Relative to Market Wage | Correlation: Market Hours and Market Wage | Investment in Business and Household Capital |
|---|---|---|---|---|---|---|---|---|
| U.S. Time Series, 1947–87* | 1.96 | 2.61 | .54 | .78 | .73 | 1.06 | –.12 | .30 |
| **Models†** | | | | | | | | |
| 1. Standard: Home Production Minimized ($e=0$, $\gamma=2/3$) | 1.36 | 2.82 | .41 | .41 | .60 | .68 | .96 | –.09 |
| 2. Increased Willingness to Substitute Between Home and Market ($e=2/3$, $\gamma=2/3$) | 1.60 | 2.34 | .61 | .52 | .52 | 1.00 | .86 | –.82 |
| 3. Increased Incentive to Substitute Between Home and Market ($e=0.40$, $\gamma=0$) | 1.59 | 2.44 | .53 | .48 | .53 | .91 | .95 | –.75 |
| 4. More General Home Production Function and Highly Correlated Technology Shocks ($e=2/3$, $\gamma=0.99$, $\psi=-1/2$) | 1.21 | 2.95 | .38 | .39 | .62 | .63 | .95 | .50 |

# Home production: Conclusions

- Macroeconomics need to take into account a very important sector often neglected.
- Introducing home production helps to explain cyclical labor supply response.
- Results are sensitive to preferences and technological shocks.
- Unfortunately there is not much independent evidence for those parameters.
- Improving labor market results yields counterfactual negative correlation between home and market investment.

# Home production: micro estimates (1)

- Not considering housework time in micro data underestimates the labor supply wage elasticity.
- Consider Rupert, Rogerson, and Wright (2000) model. Households maximize

$$\sum_{t=1}^{T} \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt})$$

$$\text{s.t} \sum_{t=1}^{T} (1+r)^{-t} (c_{mt} + k_{nt+1} - (1-\delta) k_{nt}) \leq A_0 + \sum_{t=1}^{T} (1+r)^{-t} w_t h_{mt}$$

$$c_{nt} \leq g(h_{nt}, k_{nt})$$

$$h_{mt} + h_{nt} \leq H$$

- Get FOC with respect to $h_{mt}$

$$-\beta^t U_3(t) = \lambda (1+r)^{-t} w_t$$

$$\log(-U_3(t)) = \log \lambda - t \log \beta(1+r) + \log w_t$$

# Home production: micro estimates (2)

- Taking $U(c_{mt}, c_{nt}, h_{mt}, h_{nt}) = u(c_{mt}, c_{nt}) - \phi \exp(-\gamma(H - h_{mt} - h_{nt}))$ yields

$$\gamma(H - h_{mt} - h_{nt}) = \log(\lambda/\gamma\phi) - t \log \beta(1 + r) + \log w_t$$

- If you want to estimate labor supply you need a measure of $h_{nt}$

$$h_{mt} = \alpha_0 + \alpha_1 t + \alpha_2 w_t + \alpha_3 h_{nt}$$

# Home production: micro estimates (3)

- Little detour to study OLS bias
- Suppose you estimate the model $y = a_0 + a_1 x + u$, but the true model is $y = b_0 + b_1 x + b_2 z + u$
- Your OLS estimator for $a_1$ will be

$$\hat{a}_1 = \frac{\mathrm{cov}(x, y)}{\mathrm{var}(x)}$$

- Replacing the true model yields

$$\begin{aligned}\hat{a}_1 &= \frac{\mathrm{cov}(x, b_0 + b_1 x + b_2 z + u)}{\mathrm{var}(x)} \\ &= \frac{b_0\,\mathrm{cov}(x,1) + b_1\,\mathrm{cov}(x,x) + b_2\,\mathrm{cov}(x,z) + \mathrm{cov}(x,u)}{\mathrm{var}(x)} \\ &= b_1 + \underbrace{b_2 \frac{\mathrm{cov}(x,z)}{\mathrm{var}(x)}}_{\text{bias}}\end{aligned}$$

# Home production: micro estimates (4)

- Learning from $\alpha_2$ when $h_{nt}$ is omitted biases estimates.
- If $\text{cov}(w, h_n) > 0$ and $\alpha_3 < 0$, downward bias for $\alpha_2$.
- Standard labor supply model underestimates Frisch elasticity.
- Data shows that along the life cycle people work more at home when their wages peak.
- Examples: Buying a new house, having children, etc in the 30s-40s.
- Rupert, Rogerson, and Wright (2000) report elasticities 50% larger for home-production models with respect to non-home-production models.

Greenwood, J., R. Rogerson, and R. Wright (1993, Summer).
Putting Home Economics into Macroeconomics.
*Federal Reserve Bank of Minneapolis Quarterly Review 17*(3), 2–11.

Rupert, P., R. Rogerson, and R. Wright (2000).
Homework in labor economics: Household production and intertemporal substitution.
*Journal of Monetary Economics 46*(3), 557 – 579.