Business Cycles

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Model performance: Propagation (1)

- Propagation: Technological shocks are amplified due to the response of labor supply.
- In the RBC model (King and Rebelo 1999) we have
  - Income/Wealth effect (IE): If the worker is more productive, his income increases and he gets more normal goods, in particular leisure → hours worked decrease.
  - Substitution effect (SE): If the worker is more productive, leisure becomes more expensive → hours worked increase.
- In dynamic economies we also have a new intertemporal channel: workers substitute leisure today for leisure tomorrow when facing a temporarily high wage.

Model performance: Propagation (2)

- Productivity shocks have a transitory and a permanent component
- Transitory shock generate a large substitution effect $\rightarrow$ hours worked increase.
- Permanent shock generate a large income/wealth effect $\rightarrow$ hours worked decrease.
- Since Solow residuals are very persistent, wealth effect cancels out most of the substitution effect.
- In the long run, hours are approx stable in the US.
- We assume preferences such that the income and substitution effects cancel out in response to a permanent shock.

Model performance: Propagation (3)

- Additional income due to increase in z is mostly saved if the shock is transitory because household likes smooth consumption.
- Investments jumps after a transitory shock and so does K.
- Higher K raises marginal productivity of labor, which increases w in the future.
- K starts to decline to generate higher consumption and leisure.
- Almost no inner propagation through intertemporal factors.
- Only through very persistent shocks we get realistic persistence in time series.

Impulse-Response Analysis

- What is the path that follows an endogenous variable after the economy is hit by a productivity shock?

$$\begin{aligned}\hat{k}_{t+1} &= B_1\hat{z}_t + B_0\hat{k}_t \\ &= B_1\hat{z}_t + B_0(B_1\hat{z}_{t-1} + B_0\hat{k}_{t-1}) \\ &= B_1\hat{z}_t + B_0B_1\hat{z}_{t-1} + B_0^2(B_1\hat{z}_{t-2} + B_0\hat{k}_{t-2})\end{aligned}$$

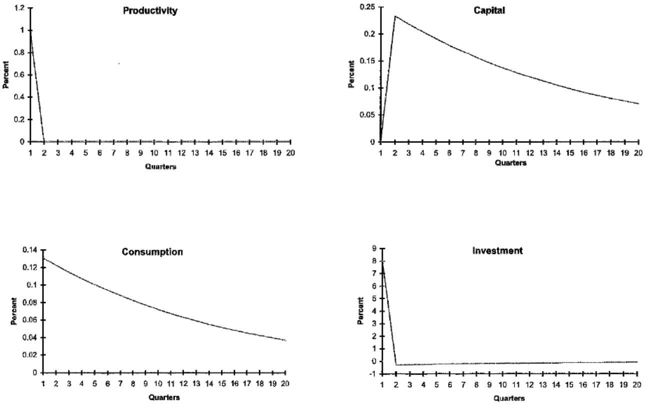
- Since in $t = 0$ economy was in steady state, $\hat{k}_0 = 0$. Hence,

$$\hat{k}_{t+1} = B_1\sum_{j=0}^{t} B_0^j\hat{z}_{t-j}$$

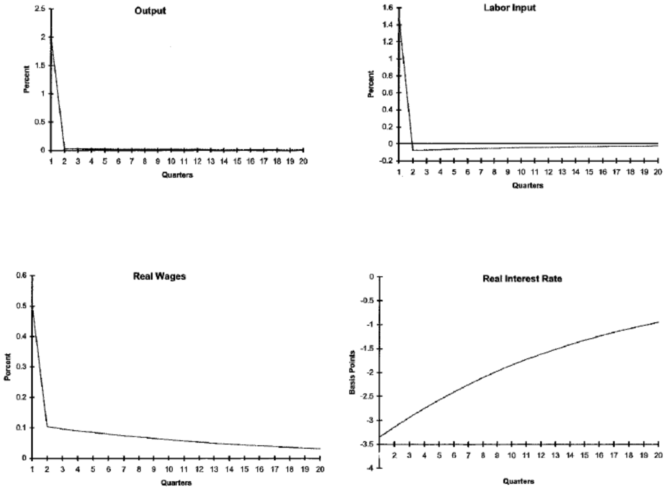
- For instance, temporary shock yields

$$\hat{k}_{t+1} = B_1B_0^t\hat{z}_0$$

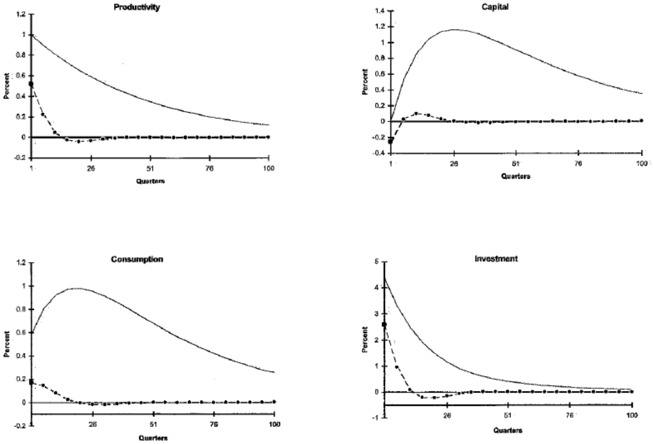
Response to temporal shock (1)



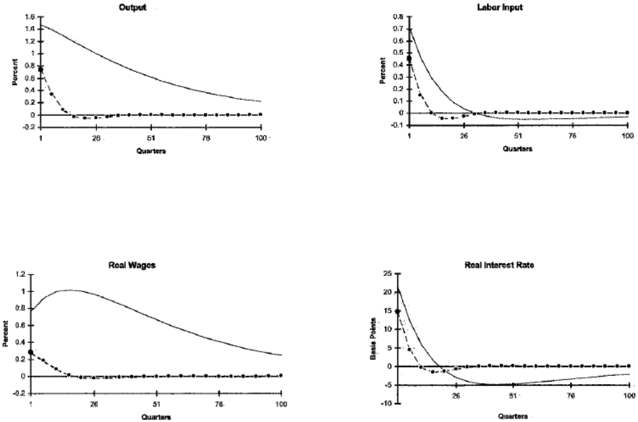
Response to temporal shock (2)



Response to persistent shock (1)



Response to persistent shock (2)



Main problems of RBC approach: Nature of shocks

- Reliance on technology shocks as a source of business cycle fluctuation. What is a technological regress?
- Remember that Solow residuals is the “measure of our ignorance”
- Conventional measures of Solow residual imply high chance of technological regress.
- When introduced cyclical physical capacity utilization, Solow residual seems more resonable but it is not as volatile or persistent as needed for RBC theory.
- Main weakness of RBC theory.

Main problems of RBC approach: Labor supply

- Amplification-propagation mechanism relies in high intertemporal wage-elasticity of the labor supply.
- Calibration done implies very high labor supply wage elasticity

$$1 - n = \frac{\theta c}{w} \quad \rightarrow \quad \log(1-n) = \log\theta + \log c - \log w$$

$$\frac{\partial \log(1-n)}{\partial \log w} = -1 \quad \rightarrow \quad \frac{\partial \log n}{\partial \log w} \approx \frac{1-n^\star}{n^\star} = \frac{0.8}{0.2} = 4$$

- Micro estimations show individual wage elasticity is much lower, in the range 0-0.5
- Problem: data shows highly procyclical hours worked with nearly acyclical real wages.

Pending agenda

- To minimize the reliance on technology shocks, we need more powerful endogenous amplification mechanisms.
- Lack of endogenous persistence. Virtually all the persistence in model-generated series comes from the assumed persistence in the technological shock.

Slight Methodological Detour

- We have seen that RBC models predict dynamic responses of endogenous variables (impulse-response).
- How can we obtain an empirical counterpart of these dynamic responses?
- There is a set of “atheoretical” econometric models that may become useful in this regard: Vector Autoregressions (VAR) proposed by Sims (1980)
- We can interpret them as a summary of the joint dynamic behavior of several macroeconomic variables.

Univariate autorregresive models (AR) (1)

- The simplest dynamic atheoretical model we can think of is called AR(1)

$$y_t = \alpha + \rho y_{t-1} + \epsilon_t$$

  where $E[\epsilon_t] = 0$ and $E[\epsilon_t^2] = V[\epsilon_t^2] < \infty$ for all t
- The model is said to be stable $\rho \in (-1, 1)$.
- If $|\rho| = 1$ we say the equation has a unit root, is a non-stationary dynamic equation.
- In contrast, we consider $|\rho| < 1$ a stationary process. In business cycle theory we mainly care about these since cycles are fluctuations around a trend.

Univariate autorregresive models (AR) (2)

- We can talk about an AR(2) model, too

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t$$

- ... or even an AR(p) model

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_p y_{t-p} \epsilon_t$$

- We can analyze dynamic stability of an AR(2) just by writing the problem as

$$\begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix}$$

or more compactly $Y_t = PY_{t-1} + \varepsilon$

Univariate autorregresive models (AR) (2)

- We can assess the stability of the dynamic process via the Spectral Decomposition of a square matrix of size $n \times n$

$$P = MDM^{-1} \quad \text{with } D = diag(\lambda_1, ..., \lambda_n)$$

where λ_i are n different eigenvalues and M is a matrix containing eigenvectors.

- Using this result we can establish that $P^k = MD^kM^{-1}$.
- If we have a shock $\epsilon_0 > 0$ and we want to assess its impact in $t > 0$, we can substitute recursively

$$\begin{aligned} Y_1 &= PY_0 + \epsilon_0 \\ Y_2 &= PY_1 = P(PY_0 + \epsilon_0) = P^2Y_0 + P\epsilon_0 \\ Y_3 &= P^3Y_0 + P^2\epsilon_0 \\ Y_{t+1} &= P^{t+1}Y_0 + P^t\epsilon_0 \end{aligned}$$

Univariate autorregresive models (AR) (3)

- The long run behavior of this process depends on the eigenvalues λ_i
- Example: consider $y_t = 0.5y_{t-1} - 0.2y_{t-2} + \epsilon_t$
- Matrix $P = \begin{bmatrix} 0.5 & -0.2 \\ 1 & 0 \end{bmatrix}$.
- Compute eigenvalues solving $|P - \lambda I| = 0$

$$- \lambda(0.5 - \lambda) + 0.2 = \lambda^2 - 0.5\lambda + 0.2 = 0$$
$$\lambda = \frac{0.5 \pm \sqrt{0.5^2 - 4 \cdot 0.2}}{2} \quad \rightarrow \lambda = 0.25 \pm 0.37i$$

- The equation is stable if the modulus of all the roots is lower than 1, i.e. for $\lambda = a + bi$ we need $\sqrt{a^2 + b^2} < 1$. This condition is clearly satisfied in this case.

Univariate autorregresive models (AR) (4)

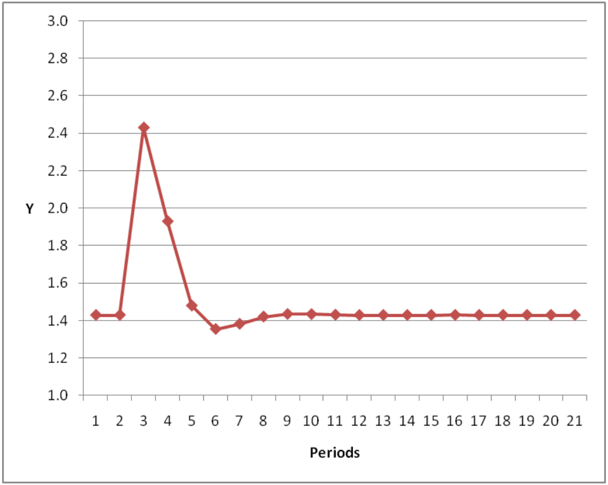


Figure: Dynamic AR(2) model $y_t = 0.5y_{t-1} - 0.2y_{t-2} + \epsilon_t$

Univariate autorregresive models (AR) (5)

- Models usually represented by lag operator defined as $Lx_t = x_{t-1}$. Therefore $L^p x_t = x_{t-p}$. The advantage is that the operator can be used as a standard algebra as long as the processes is stable (or invertible). That is

$$\begin{aligned} y_t &= \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \ldots + \rho_p y_{t-p} + \epsilon_t \\ y_t(1 - \rho_1 L - \rho_2 L^2 - \ldots - \rho_p L^p) &= \alpha + \epsilon_t \\ \theta(L) y_t &= \alpha + \epsilon_t \end{aligned}$$

- The long run mean of the process is given by $\mu = \alpha\theta(1)^{-1}$

Estimating AR processes

- The model can be consistently estimated via OLS under the following assumptions:
  - Conditional mean independence of error, $E[\epsilon_t | y_{t-1}, ..., y_{t-p}] = 0$ (no error autocorrelation)
  - Finite variance conditional variance $E[\epsilon_t^2 | y_{t-1}, ..., y_{t-p}] < \infty$
  - True underlying process stable, i.e. all roots lie in the unit circle (otherwise we have a unit root situation, with different asymptotic theory)
- We can estimate a stationary dynamic process of a single variable.
- Using Gaussian errors, we can estimate via max likelihood (Hamilton 1994, ch 5).

Vector autorregresive models (VAR) (1)

- After log-linearizing and RBC or any dynamic macro model we usually obtain a multivariate linear dynamic model.
- Its empirical counterpart is called a Vector Autorregression (VAR), an autoregressive process of a vector in which the dynamic behavior of a variable affects all the others.
- Originally, the VAR was a way to approach somewhat arbitrary assumptions to construct empirical models.
- In a VAR, all variables can be regarded as endogenous; it is an agnostic exercise.
- Today VARs are viewed as a useful empirical tool to summarize the dynamic (linear) relations among multiple variables.

Vector autorregresive models (VAR) (2)

- A VAR(q) process of r variables is

$$Y_t = P_1 Y_{t-1} + P_2 Y_{t-2} + ... + P_q Y_{t-q} + U_t$$

with $Y_t = [y_{1t} y_{2t} ... y_{rt}]'$, $P_i = \begin{bmatrix} \rho_{11,i} & \rho_{12,i} & \cdots & \rho_{1r,i} \\ \rho_{21,i} & \rho_{22,i} & \cdots & \rho_{2r,i} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{r1,i} & \rho_{r2,i} & \cdots & \rho_{rr,i} \end{bmatrix}$ and $U_t = [u_{1t} u_{2t} ... u_{rt}]'$ and $i = 1, 2, ..., q$

Vector Autorregresive models (VARs) (3)

- We can study stability in a similar way as we did with AR(p) processes.
- Define $\mathcal{Y}_t = [Y_t \, Y_{t-1} \ldots Y_{t-q+1}]'$ and $\mathcal{Y}_{t-1} = [Y_{t-1} \, Y_{t-2} \ldots Y_{t-q}]'$
- Define $\mathcal{P} = \begin{bmatrix} P_1 & P_2 & \ldots & P_q \\ I_r & 0_r & \ldots & 0_r \\ 0_r & I_r & \ldots & 0_r \\ \ldots & \ldots & \ldots & \ldots \\ 0_r & 0_r & \ldots & I_r \end{bmatrix}$
- ... and obtain the eigenvalues of $\mathcal{P}$
- Check that all eigenvalues have a modulus lower than 1 to ensure stability.

Vector Autorregresive models (VARs) (3)

- How can we estimate these models from the data?
- We have the following dynamic system of r variables with q lags

$$y_{1,t} = \alpha + \sum_{i=1}^{q} \rho_{11,i} y_{1,t-i} + ... + \sum_{i=1}^{q} \rho_{1r,i} y_{r,t-i} + u_{1t}$$

$$y_{2,t} = \alpha + \sum_{i=1}^{q} \rho_{21,i} y_{1,t-i} + ... + \sum_{i=1}^{q} \rho_{2r,i} y_{r,t-i} + u_{2t}$$

$$...$$

$$y_{r,t} = \alpha + \sum_{i=1}^{q} \rho_{r1,i} y_{1,t-i} + ... + \sum_{i=1}^{q} \rho_{rr,i} y_{r,t-i} + u_{rt}$$

Vector Autorregresive models (VARs) (4)

- Let's state the needed assumptions
  - Conditional mean independence of error, $E[\epsilon_t | y_{t-1}, ..., y_{t-p}] = 0$ (no error autocorrelation)
  - Finite variance conditional variance $E[\epsilon_t^2 | y_{t-1}, ..., y_{t-p}] < \infty$
  - True underlying process stable, i.e. all roots lie in the unit circle (otherwise we have a unit root situation, with different asymptotic theory)

Vector Autorregresive models (VARs) (4)

- We can estimate each equation separately via OLS.
- Since all of the equations have the same regressors and previous properties are satisfied, we cannot do better by considering errors from other equations. Technically, OLS and SUR (Seemingly Unrelated Equations) are the same here.
- With Gaussian errors, equation-by-equation OLS becomes fully efficient ML.
- How to choose the lag length q?

Vector Autorregresive models (VARs) (5)

- The model is atheoretical, so we basically rely on automatic criterion:
    - Set a maximum length $\overline{q}$. Common choices $\overline{q} = 4$ for annual data; $\overline{q} = 8, 12$ for quarterly data; $\overline{q} = 12, 24$ for monthly data.
    - For each $q = 1, ..., \overline{q}$, compute a measure of one-step predictive accuracy, penalized by the size of q such as BIC or AIC (Bayesian or Akaike information criterion). See Hayashi (2000, p. 398) or Lütkepohl (2005, ch. 3)
    - Select the model with the lowest BIC or AIC. We should also test no autocorrelation of residuals.
- Stata and E-Views among other statistical packages have build-in routines to estimate these models.

Vector Autorregresive models (VARs) (6)

- How do we use VARs?
- One of the main applications is the impulse-response function, which displays the implied dynamic response of the system to a shock.
- This is the same thing we did when analyzing log-linearized dynamic approximations.
- In general we can write the VAR(q) as (with mean zero)

$$Y_t = P_1 Y_{t-1} + P_2 Y_{t-2} + ... + P_q Y_{t-q} + U_t$$

or more succinctly as

$$\mathcal{Y}_t = \mathcal{P}\mathcal{Y}_{t-1} + \mathcal{U}_t$$

Vector Autorregresive models (VARs) (7)

- Suppose we want to study the dynamic response of the economy to a shock of at time 0 in y_1, the dynamic impact of the shock $U_0^* = [u_{10}, 0, ..., 0]'$.
- Then, we can write the subsequent responses by means of

$$Y_1 = P_1 U_0^*$$
$$Y_2 = P_1 Y_1 + P_2 U_0^* = (P_1^2 + P_2) U_0^*$$
$$Y_3 = P_1 Y_2 + P_2 Y_1 + P_3 U_0^* = (P_1^3 + P_1 P_2 + P_2 P_1 + P_3) U_0^*$$
$$...$$

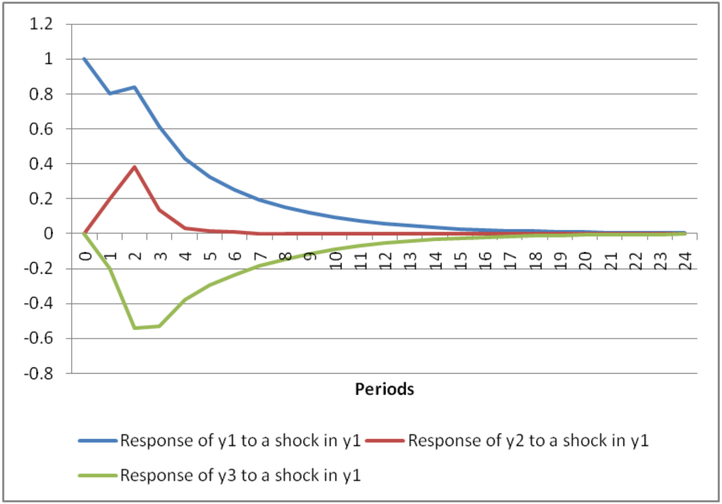
- Although easy to compute using a simple computer program, further period expressions become more and more complicated.

An example of VAR

- Here is an example of VAR(2)

$$\begin{bmatrix} y1_t \\ y2_t \\ y3_t \end{bmatrix} = \begin{bmatrix} 0.8 & -0.2 & 0.3 \\ 0.2 & 0.1 & 0.5 \\ -0.2 & -0.1 & 0.3 \end{bmatrix} \begin{bmatrix} y1_{t-1} \\ y2_{t-1} \\ y3_{t-1} \end{bmatrix} + \begin{bmatrix} 0.3 & -0.2 & 0.1 \\ 0.3 & -0.1 & 0.1 \\ -0.3 & 0.3 & -0.1 \end{bmatrix} \begin{bmatrix} y1_{t-2} \\ y2_{t-2} \\ y3_{t-2} \end{bmatrix} + \begin{bmatrix} u1_t \\ u2_t \\ u3_t \end{bmatrix}$$

Impulse-Response graph



VAR shocks identification (1)

- However, how do we identify a particular shock in this context?
- Contemporaneous errors are usually correlated.
- Hence, if we want to study the response to a shock on $y1$, we are introducing introducing multiple shocks to the dynamic system.
- One solution is to "orthogonalize" the errors using Cholesky decomposition of the variance matrix of residuals Σ.

$$\Sigma = GG'$$

where G is a lower triangular matrix that satisfies the above condition.

VAR shocks identification (1)

- One problem is that the Cholesky decomposition is not unique. The order of the variables matters.
- Hence "...the researcher has to specify the instantaneous *causal* ordering of the variables." Lütkepohl (2005, p.59), which means we need to make some "timing" choices.
- For instance, in a RBC model we assume that capital stock from the previous period does not affect production in the current period.

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