Business Cycles

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Standard RBC model (1)

- Good discussion and summary in King and Rebelo (1999).
- The model is a stochastic version of the neoclassical growth model.
- Main points of Kydland and Prescott (1982):
  1. Fully microfounded dynamic stochastic economy with rational expectations.
  2. They oppose productivity shocks to traditional Keynesian approach which stresses demand side / monetary shocks as a source business cycles.
  3. Calibration methodology instead of traditional reduced form macroeconometric models.

Standard RBC model (2)

- Households (measure 1) choose sequences $\{k_{t+1}, n_t\}_{t=0}^{\infty}$ to maximize

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \right]$$

s.t. $\quad w_t n_t + r_t k_t + (1-\delta) k_t \geq c_t + k_{t+1} \quad$ and $\quad 0 \leq l_t \leq 1$

- Firms hire productive factors to maximize profits

$$\max_{\tilde{k}_t, \tilde{n}_t} \{ z_t F(\tilde{k}_t, \tilde{n}_t) - W_t \tilde{n}_t - R_t \tilde{k}_t \}$$

Standard RBC model (3)

- A key element is rational expectations: households and firms make decisions taking into account all the current public information to forecast the future.
- z_t is technological shock following a known stochastic process (usually first-order (AR(1)) persistent in logs).

$$\log z_{t+1} = \rho \log z_t + \epsilon_t$$

  where ϵ_t is a independent identically distributed (iid) shock.
- Aggregate behavior is consistent with individual behavior

$$K_t = k_t = \tilde{k}_t \quad \text{and} \quad N_t = n_t = \tilde{n}_t$$

Recursive formulation (1)

- Households:

$$V(k,z) = \max_{k',n} \{u(Wn + Rk + (1-\delta)k - k', 1-n) + \beta E_{z'}[V(k',z')|z]\}$$

- Optimality conditions:

$$\begin{aligned} k': &\quad 0 = -u_1 + \beta E[V_1(k',z')|z] \\ n: &\quad 0 = u_1 W - u_2 \\ \text{Env:} &\quad V_1(k,z) = u_c(R + 1 - \delta) \end{aligned}$$

Recursive formulation (2)

- Euler equation: $u_1 = \beta E[u_1'(R' + 1 - \delta)|z]$
- Intratemporal equation / labor supply: $u_2/u_1 = W$
- Prices evolve stochastically due to z.
- $R = zF_1(K, N)$ and $W = zF_2(K, N)$
- We can characterize optimal rules for production, consumption, capital accumulation and labor effort.
- There is a stochastic steady state distribution in which economy fluctuates in response to shocks (Brock and Mirman 1972).

Taking the model to the data (1)

Two interrelated steps of Calibration:

1. Give appropriate functional forms for $u(c, n)$, $F(k, n)$ and the stochastic process of shock.
2. Find "reasonable" parameters to compare series coming from the simulated economy with actual data.

Calibration: Overview (1)

- Selection of independently estimated parameters based on microevidence.
- Unknown parameters chosen to target some statistics of interest (mean, variance, etc).
- Model evaluation by comparing other empirical moments to model-generated moments.
- Special attention has been paid to:
  - Long-run mean or variances of cyclical components.
  - Persistence (autocorrelation) of cyclical components.
  - Correlation between cyclical components of variables.
  - Lead - lagged correlation between cyclical components.

Calibration: Overview (2)

- Because solving RBC models need computer-intensive methods, the choice made by Kydland and Prescott (1982) was the only one feasible in 1982.
- However, there has been substantial improvement in techniques to estimate the structural parameters of these models using a formal Method of Moments, Maximum Likelihood or Bayesian methods.
- Today these kind of models are routinely estimated or calibrated using new numerical and computational techniques.
- There are empirical applications in Macroeconomics, Industrial Organization, Labor Economics, Public Finance and virtually all economic fields.

Baseline Calibration of RBC Model (1)

- Production side mainly determines steady-state.
- Assume Cobb-Douglas technology because the share of labor share income in output is roughly constant.

$$Y = zK^{\alpha}N^{1-\alpha} \quad \text{or} \quad Y = K^{\alpha}(zN)^{1-\alpha}$$

- Pick parameters to target some stylized facts:
- Constant labor participation in output $\rightarrow \alpha = 1/3$
- Target average interest rate because it is stable. Since in deterministic steady state it is the discount factor set $\beta = 0.065/4$, the average quarterly return of Standard & Poor Index 1984-1986.

Baseline Calibration of RBC Model (2)

- Depreciation $\approx 0.1/4$ per quarter.
- Set $\bar{z}$=1 as a normalization (detrended data)
- Using Cobb-Douglas and that $r + \delta = zF_1(K, N)$ we obtain implied K/N steady-state ratio

$$K/N = \left(\frac{\alpha z}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$

- This also implies a steady-state wage

$$w = (1-\alpha)(K/N)^{\alpha}$$

Baseline Calibration of RBC Model (3)

- To get stationary hours worked in the long run, we need the utility function

$$u(c,n) = \frac{(cv(1-n))^{1-\sigma}}{1-\sigma} \quad \text{if } \sigma \neq 1$$
$$u(c,n) = \log c + \log v(1-n) \quad \text{if } \sigma = 1$$

with with $v(1-n)$ twice-continuously differentiable, increasing and concave ($v' > 0$, $v'' < 0$)

- The coefficient of risk aversion $\sigma \in [1,2]$. Following King and Rebelo (1999) we choose $\sigma = 1$.

$$u(c,n) = \log c + \theta \log(1-n)$$

Baseline Calibration of RBC Model (4)

- Set θ so that the average number of hours worked is 20% of available time in the US. (Think about $24 \times 7 = 168$ weekly hours and about $7 \times 5 = 35$ hours worked per week.)
- Intratemporal condition implies that

$$w = \frac{\theta}{1-n} c \to \theta = (w/c)(1-n)$$

  where $c = f(k/n) - \delta(k/n)$ and $1 - n = 0.8$
- One strategy is to log-linearize and solve using decision rules as functions of states variables k and z

Log-linearized solution (1)

- Compute approximation around the stationary steady state
- For intratemporal condition

$$w = z(1-\alpha)\left(\frac{k}{n}\right)^{\alpha} = \frac{\theta c}{1-n}$$

$$d\log z + \alpha(d\log k - d\log n) = d\log c + dn/(1-n^{\star})$$

$$\hat{z} + \alpha(\hat{k} - \hat{n}) = \hat{c} + \hat{n}(n^{\star}/(1-n^{\star})) \quad (1)$$

- Euler equation

$$\frac{1}{c} = \beta E\left(\frac{1}{c'} z \left(\frac{n}{k}\right)^{1-\alpha}\right)$$

$$-\hat{c} = E\left(-\hat{c}' + \hat{z}' + (1-\alpha)(\hat{n}' - \hat{k}')\right)$$

$$\hat{c} = E\hat{c}' + (1-\alpha)(E\hat{k}' - E\hat{n}') \quad (2)$$

Log-linearized solution (2)

- Budget constraint

$$\begin{aligned} zk^{\alpha}n^{1-\alpha} &= c + k' - (1-\delta)k \\ \hat{z} + \alpha\hat{k} + (1-\alpha)\hat{n} &= \frac{dc + dk' - (1-\delta)dk}{y^{\star}} \\ &= \hat{c}(c^{\star}/y^{\star}) + \hat{k}'(k^{\star}/y^{\star}) - \hat{k}((1-\delta)k^{\star}/y^{\star}) \end{aligned} \tag{3}$$

- Doing some algebra on equation (1), we obtain

$$\hat{c} = \hat{z} + \alpha\hat{k} - \left(\alpha + \frac{n^{\star}}{1-n^{\star}}\right)\hat{n} \tag{4}$$

$$E\hat{c}' = \rho\hat{z} + \alpha E\hat{k}' - \left(\alpha + \frac{n^{\star}}{1-n^{\star}}\right)E\hat{n}' \tag{5}$$

Log-linearized solution (3)

- Replacing (4) and (5) into the Euler equation (2), we eliminate $\hat{c}$

$$(1-\rho)\hat{z} + \alpha\hat{k} - \left(\alpha + \frac{n^\star}{1-n^\star}\right)\hat{n} = E\hat{k}' - \left(\frac{1}{1-n^\star}\right)E\hat{n}' \qquad (6)$$

- Replacing (4) in the budget constraint (3), we express $\hat{n}$ in terms of $\hat{k}'$, $\hat{k}$ and $\hat{z}$.

$$\begin{aligned}
&\left(1-\alpha+\frac{c^\star}{y^\star}\left(\alpha+\frac{n^\star}{1-n^\star}\right)\right)\hat{n} \\
&= \frac{k^\star}{y^\star}\hat{k}' + \left(\alpha\frac{c^\star}{y^\star} - (1-\delta)\frac{k^\star}{y^\star} - \alpha\right)\hat{k} + \left(\frac{c^\star}{y^\star} - 1\right)\hat{z} \\
&\hat{n} = A_0\hat{k}' + A_1\hat{k} + A_2\hat{z} \qquad (7)
\end{aligned}$$

Log-linearized solution (4)

- Replacing the latter expression (7) into the Euler equation (6) yields

$$
\begin{aligned}
&(1-\rho)\hat{z} + \alpha\hat{k} - \left(\alpha + \frac{n^\star}{1-n^\star}\right)(A_0\hat{k}' + A_1\hat{k} + A_2\hat{z}) \\
&= E\hat{k}' - \left(\frac{1}{1-n^\star}\right)(A_0 E\hat{k}'' + A_1 E\hat{k}' + A_2\rho\hat{z}) \qquad (8)
\end{aligned}
$$

- Guess and verify approach: The conjectured solution is

$$\hat{k}' = B_0\hat{k} + B_1\hat{z}$$

- Hence, the expectations take the forms

$$
\begin{aligned}
E\hat{k}' &= B_0\hat{k} + B_1\hat{z} && (9)\\
E\hat{k}'' &= B_0 E\hat{k}' + B_1 E\rho\hat{z} = B_0(B_0\hat{k} + B_1\hat{z}) + B_1 E\rho\hat{z} && (10)\\
&= B_0^2\hat{k} + B_1(B_0+\rho)\hat{z} && (11)
\end{aligned}
$$

Log-linearized solution (5)

- Substituting (9) and (11) into (8) we get

$$
\begin{aligned}
&(1-\rho)\hat{z}+\alpha\hat{k}-\left(\alpha+\frac{n^\star}{1-n^\star}\right)(A_0\hat{k}'+A_1\hat{k}+A_2\hat{z})\\
&=B_0\hat{k}+B_1\hat{z}-\left(\frac{1}{1-n^\star}\right)\Big[A_0(B_0^2\hat{k}+B_1(B_0+\rho)\hat{z})\\
&+A_1(B_0\hat{k}+B_1\hat{z})+A_2\rho\hat{z}\Big]
\end{aligned}
$$

- Reorganizing we verify that the conjecture was right

$$
\begin{aligned}
&\hat{k}'\left(\alpha+\frac{n^\star}{1-n^\star}\right)=\hat{k}\left(\alpha-\left(\alpha+\frac{n^\star}{1-n^\star}\right)A_1-B_0-\frac{A_0B_0^2+A_1B_0}{1-n^\star}\right)\\
&+\hat{z}\left(1-\rho-\left(\alpha+\frac{n^\star}{1-n^\star}\right)A_2-B_1-\frac{A_0B_1(B_0+\rho)+A_1B_1+A_2\rho}{1-n^\star}\right)
\end{aligned}
$$

Log-linearized solution (6)

- The coefficients accompanying $\hat{k}$ and $\hat{z}$ must be equated to the conjectured

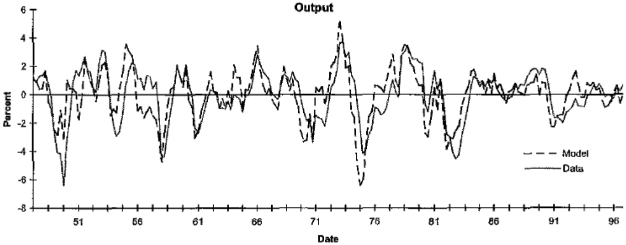
$$\frac{A_0}{1-n^\star}B_0^2 + \left(A_0\left(\alpha + \frac{n^\star}{1-n^\star}\right) + \frac{A_1}{1-n^\star} + 1\right)B_0$$
$$+\left(\alpha - A_1\left(\alpha + \frac{n^\star}{1-n^\star}\right)\right) = 0$$

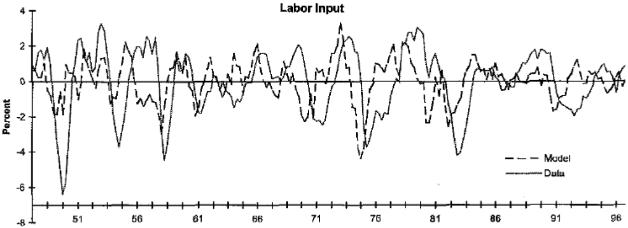
which generates a quadratic equation in B_0. Take the root that $-1 < B_0 < 1$.

$$B_1\left(A_0\left(\alpha + \frac{n^\star}{1-n^\star}\right) + \frac{A_0(B_0+\rho)+A_1}{1-n^\star} + 1\right)$$
$$-\left(1-\rho-\left(\alpha + \frac{n^\star}{1-n^\star}\right)A_2 - \frac{\rho A_2}{1-n^\star}\right) = 0$$

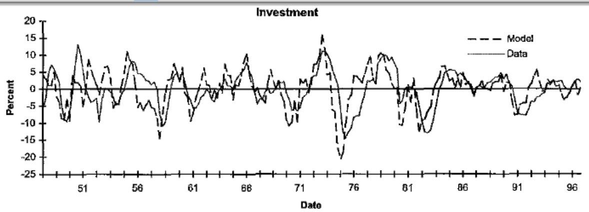
- Substitute the stable solution for B_0 into the latter equation to get B_1.
- Obtain $\hat{n}$ by replacing solution for $\hat{k}$.

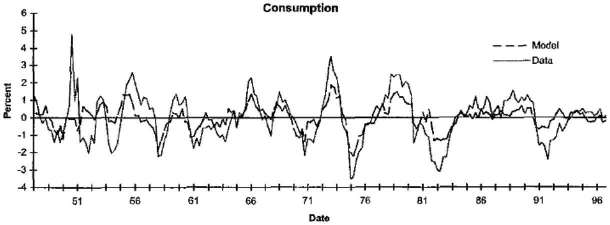
Simulated model: output, hours worked





Simulated model: investment, consumption





Simulated model: cyclical moments

Table 3
Business cycle statistics for basic RBC model[a,b]

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

[a] All variables have been logged (with the exception of the real interest rate) and detrended with the HP filter.
[b] The moments in this table are population moments computed from the solution of the model. Prescott (1986) produced multiple simulations, each with the same number of observations available in the data, and reported the average HP-filtered moments across these simulations.

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