Economic Growth Theory

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Endogenous growth: spillovers

- Romer (1986) model based on two ideas:
- Learning-by-doing: Capital accumulation generates learning that causes efficiency gains (cost reductions)
- Capital accumulation generated knowledge is public non-rival good, which is freely exploited.
- Capital accumulation of one firm increases productivity of all other firms
- Firm i technology: $Y_i = F(K_i, AL_i)$
- Linear technological change: $A = K$.
- Consumer side is standard: assume CRRA preferences.

Endogenous growth: spillovers (2)

- Competitive markets: $r + \delta = f'(k, K)$ and $w = f(k, K) - f'(k, K)k$ with $f(k_i, K) = F(K_i/L_i, K)$.
- All agents behave in the same way, i.e. $L_i = L$ and $K_i = k_i L = K$.
- Average product of capital in firm i is

$$Y_i/K_i = y_i/k_i = f(1, K/k_i) \equiv \widetilde{f}(K/k_i)$$

- In equilibrium $y_i/k_i = \widetilde{f}(L)$, hence average productivity does not decrease in k and neither does the marginal productivity of capital

$$\frac{dy_i}{dk_i} = \frac{d}{k_i} k_i \widetilde{f}(k_i/K) = \widetilde{f}(k_i/K) - \widetilde{f}'(k_i/K)K/k_i$$
$$= \widetilde{f}(L) - L\widetilde{f}'(L) = B(L)$$

Endogenous growth: spillovers (3)

- Using Euler equation and conjecturing a balanced growth path solution

$$(1+g_c) = [\beta(\widetilde{f}(L) - L\widetilde{f}'(L) - \delta)]^{1/\sigma}$$

- Using budget constraint and assuming constant growth rate for capital

$$c/K = -K'/K + 1 - \delta + Y/K = c/K = \widetilde{f}(L) - g_K - \delta$$

- So $g_K = g_C$. Since $Y = \widetilde{f}(L)K$, $g = g_K = g_C = g_Y$
- Notice that private productivity of capital $<$ social productivity of capital.
- Room for government intervention.

Endogenous growth: spillovers (4)

- Social planner's problem

$$V(k) = \max_{k'}\{u(\widetilde{f}(L)k' + (1-\delta)k - k') + \beta V(k')\}$$

- Euler: $u_c = \beta(u_c'(\widetilde{f}(L) + 1 - \delta))$
- Using CRRA: $1 + g = (\beta(\widetilde{f}(L) + 1 - \delta))^{1/\sigma}$
- Key: planner's consider both private return of capital and the positive externality it generates on other firms.

Endogenous growth: spillovers (5)

- Optimal policy: subsidize capital to increase accumulation using revenue from lump-sum taxation.
- Scale effect: countries with large L benefit the most from spillover.
- Caveat: What's the right definition of L in this case...?

Research and Competition

- Attempt to explain technological change,i.e. how productivity grows.
- If innovation is costly and imitation is free, no firm would pay Research and Development (R&D) because equilibrium profits are zero.
- Innovator would make negative profits while imitators get 0.
- Usual solution is *ex post* monopoly power for the innovator via property right law or patent system.
- If it is feasible to prevent other firms from using the innovation, the new good (or idea) is said to be *excludable*.
- If ideas or goods can simultaneously be used with no congestion, i.e. the marginal return of the use does not decrease in the number of users, they are called *nonrival*.

R&D model: preliminaries

- Firms are compensated for their R&D via *ex post* monopoly.
- We can think of this as patents, that is (temporary) monopoly over the created idea (partially excludable good).
- Scale effects: due to nonrival use of an idea, the bigger the market, the bigger the profits. Corollary: no perfect competition.
- How are new ideas generated? What's the relation between old and new ideas?
- Ideas are embedded into intermediate perishable new goods (for the sake of simplicity).
- Endogenous Technological Change (Romer 1987; Romer 1990).

Lab-equipment model (Romer 1987)

- Let x_n be the the n-th idea or capital good. There are N goods at some time t.
- Technology: $Y_t = L_t^{1-\alpha} \sum_{n=1}^{N_t} x_t(n)^\alpha$
- Full depreciation of intermediate goods or capital.
- Aggregate resource constraint $C_t + \eta X_t + Z_t \leq Y_t$ with $X_t \equiv \sum_{n=1}^{N_t} x_t(n)$.
- η units of consumption good are needed to create 1 unit of input $x(n)$.
- New inputs $x(n)$ are generated by paying a fixed cost, i.e $Z_t = \lambda(N_{t+1} - N_t)$.
- Final good Y is sold in a competitive market.

Lab-equipment model (Romer 1987) (2)

- Hence, profit maximization leads to

$$w_t = (1-\alpha)L_t^{-\alpha}\sum_{n=1}^{N_t} x_t(n)^\alpha = (1-\alpha)Y_t/L_t$$
$$p_t(n) = \alpha L^{1-\alpha} x_t(n)^{\alpha-1}$$

- where w_t is the wage and $p_t(n)$ is the price of input n.
- The cost of supplying 1 unit of $x(n)$ is η units of consumption.
- Capital good producers can prevent other firms from producing its good.

Lab-equipment model (Romer 1987) (3)

- Static optimization problem. Every period they produce $x(n)$ to maximize their profits

$$\pi(n) = (p(n) - \eta)x(n) = \alpha L^{1-\alpha} x(n)^{\alpha} - \eta x(n)$$

- From FOC the optimal production is $x(n)^{\star} = x^{\star} = \alpha^{\frac{2}{1-\alpha}} \eta^{-\frac{1}{1-\alpha}} L$ and the price is $p(i) = \eta/\alpha$.
- Optimal profit flow is $\pi(i)^{\star} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \eta^{-\frac{1}{1-\alpha}} L$

Lab-equipment model (Romer 1987) (4)

- In equilibrium, at any time t the present value of the stream of profits must equal the fixed cost of invention of a capital good λ. Then,

$$\lambda = \sum_{\tau=t}^{\infty} \frac{\pi_t}{(1+r)^t} = \pi(i)^{\star} \sum_{\tau=t}^{\infty} (1+r)^{-t}$$

- Since this relation must hold for all τ and the profit stream is constant, r must be constant
- It may occur that λ is higher than the present value of profits for any $\{r_\tau\}_{\tau=t}^{\infty} \rightarrow$ no innovation.

Lab-equipment model (Romer 1987) (5)

- If λ is sufficiently low, then $r^{\star} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\eta^{-\frac{1}{1-\alpha}}(L/\lambda)$
- Profits increase in L because firms can make more profits in larger economies.
- Households own labor and buy assets a. They maximize lifetime utility with CRRA preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$
$$\text{s.t} \quad c_t + a_{t+1} \leq w_t l + (1+r_t)a_t$$

- As usual, this leads to the standard Euler equation

$$g_c = c'/c = (\beta(1+r_t))^{\frac{1}{\sigma}}$$

Lab-equipment model (Romer 1987) (6)

- In equilibrium $r^\star = r_t$ so that g_c is constant.
- Replacing $x(n) = x^\star$ in output we get

$$Y_t = N_t L \alpha^{\frac{2\alpha}{1-\alpha}} \eta^{-\frac{1}{1-\alpha}} = NL\psi$$

- which implies $g_N = g_Y$
- Finally using the budget constraint

$$N_t L\psi = C_t + N_t x^\star + \lambda(N_{t+1} - N_t)$$
$$L\psi = \frac{C_t}{N_t} + x^\star + c(g_N - 1)$$

- Hence, constant g_N implies C and N grow at the same rate. Then, $g = g_N = g_Y = g_C$

Lab-equipment model (Romer 1987) (7)

- Is the decentralized outcome socially efficient?
- Intuition: Necessary *ex post* monopoly generates too low input quantity.
- Once an input is created, it can be used for the whole economy by only paying the marginal cost of production.
- Optimal benevolent planner's problem

$$\max_{\{c_t\}_{t=0}^{\infty},\left\{\{x_t(n)\}_{n=1}^{N_t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$\text{s.t.} \quad Y_t \geq Lc_t + X_t + Z_t$$

Lab-equipment model (Romer 1987) (8)

- Recursive formulation is

$$V(N) = \max_{N', \{x(n)\}_{n=1}^{N}} \left\{ u\left(L^{-\alpha} \sum_{n=1}^{N} x(n)^{\alpha} - (\eta/L) \sum_{n=1}^{N} x(n) - (\lambda/L)(N' - N) \right) + \beta V(N') \right\}$$

- First-order condition wrt $x(n)$ yields

$$0 = L^{-\alpha} \alpha x(n)^{\alpha-1} - \eta/L \quad \Rightarrow x(n) = \alpha^{\frac{1}{1-\alpha}} \eta^{-\frac{1}{1-\alpha}} L \equiv \tilde{x}$$

Lab-equipment model (Romer 1987) (9)

- It's optimal to produce same quantity of inputs for all n, t. Hence, the problem simplifies to

$$V(N) = \max_{N'} \left\{ u\left(L^{-\alpha} N \tilde{x}^{\alpha} - (\eta/L) N \tilde{x} - (\lambda/L)(N' - N)\right) + \beta V(N') \right\}$$

- First-order condition wrt to N' and envelope conditions are

$$0 = -u_c(\lambda/L) + \beta V'(N')$$
$$V'(N) = u_c(L^{-\alpha} \tilde{x}^{\alpha} - \eta \tilde{x}/L + \lambda/L)$$

Lab-equipment model (Romer 1987) (10)

- Replacing the value of $\tilde{x}$, we obtain the following Euler equation

$$u_c = u_c' \beta (1 + r_s) \quad \rightarrow \quad g_c = c'/c = (\beta(1 + r_s))^{\frac{1}{\sigma}}$$

- Where $r_s = (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} \eta^{-\frac{1}{1-\alpha}} (L/\lambda)$
- Comparing market interest rate r and social planner's r_s we realize that

$$r_s = \alpha^{\frac{1}{1-\alpha}} r$$

- Hence, social return of R&D investment is higher than private since $\alpha < 1$.
- Optimal intervention: subsidize input production financed with lump-sum tax.

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