Economic Growth Theory

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Introducing labor effort (1)

- This section follows discussion in Barro and Sala-i-Martin (2004, ch 9)
- Households not only care about consumption, but also about leisure
- Standard setup follows conventional labor supply ideas: each household has one unit of time per period and has to choose between work (bad) and leisure (good).

$$\max_{\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, 1-h_t) \right\}$$

$$\text{subject to} \quad c_t + k_{t+1} \leq f(k_t, h_t) + (1-\delta)k_t$$

$$\text{and} \quad 0 \leq h_t \leq 1, c_t \geq 0, k_{t+1} \geq 0$$

Introducing labor effort (2)

- If utility is strictly increasing in consumption and leisure, we can write the problem as

$$\max_{\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u(f(k_t, h_t) + (1-\delta)k_t - k_{t+1}, 1 - n_t) \right\}$$

subject to $\quad 0 \leq h_t \leq 1, c_t \geq 0, k_{t+1} \geq 0$

- Is n_t a control or state variable?
- Moreover, the solution of the household's problem can be written by means of a Bellman equation

$$V(k) = \max_{k' > 0, h \in [0,1]} \{u(f(k,h) + (1-\delta)k - k', 1 - h) + \beta V(k')\}$$

Introducing labor effort (3)

- Assuming differentiability of the Value Function, we have the following first-order conditions

$$0 = \frac{\partial V}{\partial k'} = -u_1(c, 1-h) + \beta V_1(k')$$
$$0 = \frac{\partial V}{\partial h} = u_1(c, 1-h) f_2(k, h) - u_2(c, 1-h)$$

- First condition gives standard Euler equation
- The second condition and market clearing gives a implicit labor supply equation

$$f_2(k, h) = w = \frac{u_2(c, 1-h)}{u_1(c, 1-h)}$$

Introducing labor effort (4)

- Can labor effort choice be consistent with models with balanced growth path?
- Answer: only if the utility function satisfies some (asymptotic) functional forms
- In the labor supply condition consumption and wage grow at the same time. Why?
- First, let conjecture there is balanced growth path, i.e all growth rates are constant

$$Y = wH + RK \quad \rightarrow \quad (Y/H) = w + R(K/H) = (C/H) + (I/H)$$

- We can write down the latter as $y = w + Rk = c + i$. Therefore, we have that

$$g_y = y'/y = (w/y)(w'/w) + R(k/y)(k'/k)$$
$$g_y = (w/y)g_w + R(k/y)g_k$$

Introducing labor effort (5)

- From the resource constraint we know that

$$g_y = (c/y)g_c + (i/y)g_i$$

- From the law-of-motion of capital we know that $g_k = g_i$ because

$$g_k = k'/k = 1 - \delta + i/k$$

- This implies that $g_c = g_i = g_k = g_y = g$. Why?
- Suppose $g_c > g_y$. Eventually $c = y$ and then $g_c = 0$, violating balanced growth path.
- Suppose $g_c < g_y$. Then $c/y \to 0$, and $(i_e + i_s)/y \to 1$. Eventually investment needs to slowdown, violating balanced path (and transversality condition)

Introducing labor effort (6)

- Using the equation of wages we get

$$g = (w/y)g_w + R(k/y)g$$

- The growth rate of wages must be $g_w = g$ for fixed ratios k/y and R (the latter follows from the Euler equation, as we saw before)
- Hence, we have that

$$\frac{w}{c} = \frac{u_2}{cu_1} \quad \rightarrow \quad \log w/c = \log u_2 - \log u_1 - \log c$$

Introducing labor effort (7)

- Differentiating the last equation we get

$$0 = \frac{du_2}{u_2} - \frac{du_1}{u_1} - \frac{dc}{c}$$
$$0 = \frac{1}{u_2}(u_{21}dc - u_{22}dn) - \frac{1}{u_1}(u_{11}dc - u_{12}dn) - \frac{dc}{c}$$

- In the long run hours worked should not diminish nor increase so that $dn = 0$. In addition, assuming that $g_c \approx d \log c \neq 0$, the last expression yields

$$\left(\frac{cu_{12}}{u_2}\right) = 1 + \left(\frac{cu_{11}}{u_1}\right)$$

Introducing labor effort (8)

- In the standard neoclassical model we learned that CRRA preferences generate constant growth rate of consumption. Here we have the same Euler equation. Hence, it must be true that

$$\frac{cu_{11}}{u_1} = -\sigma$$

- Therefore, we get that

$$\frac{u_{12}}{u_2} = \frac{\partial \log u_2}{\partial c} = \frac{1-\sigma}{c}$$

Introducing labor effort (9)

- Integrating both sides by c yields

$$\log u_2 = (1-\sigma)\log c + \log \psi(h)$$

where $\psi(h)$ is an arbitrary function of n. Thus,

$$u_2 = c^{1-\sigma}\psi(h)$$

- Integrating with respect to h, we obtain

$$u(c, 1-h) = c^{1-\sigma}\Psi(h) + \varphi(c) \quad \text{with} \quad \Psi(h) = \int_0^h \psi(x)dx$$

where $\varphi(c)$ is an arbitrary function of c.

Introducing labor effort (10)

- Finally, we need that condition $\frac{cu_{11}}{u_1} = -\sigma$ holds. Hence,

$$u_1 = (1-\sigma)c^{-\sigma}\Psi(h) + \varphi'(c)$$
$$cu_{11} = -\sigma(1-\sigma)c^{-\sigma}\Psi(h) + c\varphi''(c)$$

- Therefore, we need that

$$c\varphi''(c) = -\sigma\varphi'(c) \quad \rightarrow \quad -\frac{c\varphi''(c)}{\varphi'(c)} = \sigma$$

- Hence... $\varphi(c)$ needs to be a CRRA function!

Introducing labor effort (11)

- Thus, the functional form we are after is (considering that it has to be increasing)

$$u(c, 1-h) = \frac{c^{1-\sigma}}{1-\sigma}(\Psi(h)+1)$$

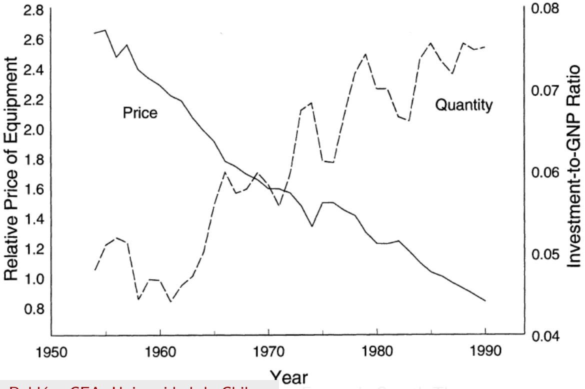
- With this specification, the labor supply equation becomes

$$\Psi'(h)/\Psi(h) = (\sigma - 1)w/c$$

- It may be inconsistent with finite work effort

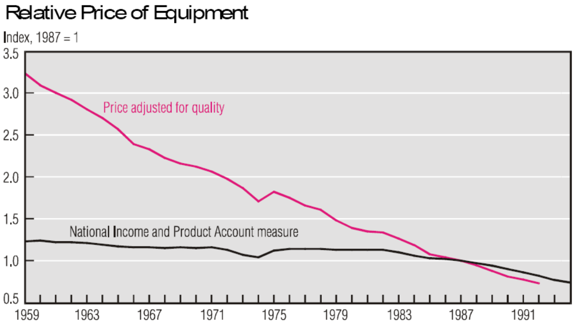
Capital-embodied technological change (GHK model)

- Due to Greenwood, Hercowitz, and Krusell (1997)
- Price per equivalent unit of equipment has sharply declined.
- Equipment use has steadily increased.



Capital-embodied technological change (GHK model)

- Technological change in equipment. Multiple examples: cars, computers, etc.



Capital-embodied technological change (GHK model)

- Physical depreciation: Value reduction of K due to productive use.
- Economic depreciation: Value reduction of K due to the fact that more productive units of capital or cheaper capital are available.

Setup GHK Model (1)

- Households maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

with $U(c,l) = (\theta \log c + (1-\theta)\log(1-l))$ with $0 \leq l_t \leq 1$

- Technology

$$y = zF(k_e, k_s, l) = zk_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s}$$

Setup GHK Model (2)

- Capital laws-of-motion

$$k_e' = (1-\delta_e)k_e + i_e q$$
$$k_s' = (1-\delta_e)k_s + i_s$$

- q is the amount of equipment that can be obtained from 1 unit of consumption.
- $1/q$ is the price of equipment in terms of consumption.
- Government revenue

$$\tau = \tau_k(r_e k_e + r_s k_s) + \tau_l wl$$

Setup GHK Model (3)

- Government returns taxes as a lump-sum transfer to households T.
- Resource constraint

$$y = c + i_e + i_s$$

- Technological progress

$$q_t = \gamma_q^t \qquad z_t = \gamma_z^t$$

GHK model: Competitive equilibrium (1)

- Aggregate state of the world $\lambda = (q, z, K_e, K_s, L)$.
- Recursive formulation: households

$$V(k_e, k_s; \lambda) = \max_{c, l, k_e', k_s'} U(c, l) + \beta V(k_e', k_s; \lambda)$$

$$\text{s.t} \quad c + k_e'/q + k_s' = (1 - \tau_k)(R_e(\lambda)k_e + R_s(\lambda)k_s) + (1 - \tau_l)W(\lambda)l + (1 - \delta_e)k_e/q + (1 - \delta_s)k_s + T(\lambda)$$

- First order conditions

$$0 = -\theta/cq + \beta V_{k_e'}$$

$$0 = -\theta/c + \beta V_{k_s'}$$

$$0 = (\theta/c)(1 - \tau_l)W(l) - (1 - \theta)/(1 - l)$$

GHK model: Competitive equilibrium (2)

- Envelope conditions

$$V_{k_e} = (\theta/c)((1-\tau_k)R_e(\lambda) + (1-\delta_e)/q)$$
$$V_{k_s} = (\theta/c)((1-\tau_k)R_s(\lambda) + (1-\delta_s))$$

- Euler equations

$$c'/c = \beta((1-\tau_k)qR_e(\lambda) + 1 - \delta_e)$$
$$c'/c = \beta((1-\tau_k)R_s(\lambda) + 1 - \delta_s)$$

- k_e and k_s yield the same marginal return in equilibrium

$$(qR_e(\lambda) - R_s(\lambda)) = (\delta_e - \delta_s)/(1-\tau_k)$$

GHK model: Competitive equilibrium (3)

- Prices are determined considering firms'problem

$$\max_{\hat{k}_e, \hat{k}_s, \hat{l}} \left\{ zF(\hat{k}_e, \hat{k}_s, \hat{l}) - R_e(\lambda)\hat{k}_e - R_e(\lambda)\hat{k}_e - W(\lambda)\hat{l} \right\}$$

- In equilibrium individual decision determine aggregate, i.e $L = l = \hat{l}$, $K_e = k_e = \hat{k}_e$ and $K_s = k_s = \hat{k}_s$
- Clearly, marginal return of factors equal their prices in equilibrium

$$W(\lambda) = zF_l(k_e, K_s, L)$$
$$R_e(\lambda) = zF_{k_e}(K_e, K_s, L) \quad R_s(\lambda) = zF_{k_s}(K_e, K_s, L)$$

Solving GHK model (1)

Balanced growth path: all variables grow at constant rate

- Guess-and-verify approach
- Resource constraint imply y, c, i_e, i_s grow at same rate g. Why?

$$y'/y = (c'/c)(c/y) + (i_e'/i_e)(i_e/y) + (i_s'/i_s)(i_s/y)$$

- Suppose $g_c > g_y$. Eventually $c = y$ and then $g_c = 0$, violating balanced path.
- Suppose $g_c < g_y$. Then $c/y \to 0$, and $(i_e + i_s)/y \to 1$. Eventually investment needs to slowdown, violating balanced path.
- Hence, $g_y = g_c = g_e = g_s = g$

Solving GHK model (2)

Balanced growth path: all variables grow at constant rate

- Moreover

$$g = k_e'/k_e = 1 - \delta_e + i_e q/k_e \quad \rightarrow k_e \quad \text{grow at } g\gamma_q$$
$$g = k_s'/k_s = 1 - \delta_s + i_s/k_s \quad \rightarrow k_s \quad \text{grow at } g$$

- Using production function, the output growth is

$$\frac{y'}{y} = \frac{z'}{z}\left(\frac{k_e'}{k_e}\right)^{\alpha_e}\left(\frac{k_s'}{k_s}\right)^{\alpha_s}$$
$$g = \gamma_z (g\gamma_q)^{\alpha_e}(g)^{\alpha_s}$$
$$g = \gamma_z^{\frac{1}{1-\alpha_e-\alpha_s}}\gamma_q^{\frac{\alpha_e}{1-\alpha_e-\alpha_s}}$$

Calibrating GHK model (1)

- Calibration: using independent parameters to assess the model predictive power.
- A priori parameters: $\gamma_q = 1.032$ (Gordon's price index), $\delta_s = 0.056$, $\delta_e = 0.124$ and $\tau_l = 0.4$.
- Values for $\beta, \theta, \alpha_e, \alpha_s, g, \tau_k$ are determined by solving the system of equations conformed by
  - Equipment and Structure Euler equations (2)
  - Investment-to-output ratios (2)
  - Intratemporal labor supply (1)
  - Budget constraint (1)

Calibrating GHK model (2)

...such that the following long-run empirical restrictions hold

- Average annual growth rate per hour worked 1.24% $\rightarrow g = 1.024$
- Ratio hours worked to nonsleeping hours 0.24 $\rightarrow l = 0.24$
- Total capital share of 0.3 $\rightarrow \alpha_e + \alpha_s = 0.3$
- $i_e/y = 0.073$ and $i_s/y = 0.041$
- Return of capital of 7% $\rightarrow \beta/g = 1/1.07$

Results GHK

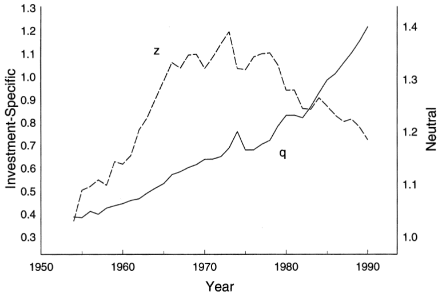


FIGURE 3. TECHNOLOGICAL CHANGE

- Construction of k_e using quality corrected price.
- Neutral technological change shows downturn after 1970.
- Equipment-embedded technological change accounts for 60% of the US growth 1954-1990

Barro, R. J. and X. Sala-i-Martin (2004).
Economic Growth (Second ed.).
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