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The equity risk premium is much lower than you think it is: empirical estimates from a new approach

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Abstract

We offer ex ante estimates of the equity risk premium based on forecasted accounting numbers. Although our approach is isomorphic to dividend growth models, it generates various diagnostics that help to narrow the range of reasonable assumed growth rates. Our results, based on IBES consensus earnings forecasts over the 1985-1998 period, contrast sharply with those of prior research. Our estimates of risk premium are considerably lower than (about 3 percent) the estimates commonly cited (about 8 percent), and are also more stationary over time. This result has important implications both for academe (e.g., the equity premium puzzle) as well as practice (e.g., discount rates for valuation and over-valued stock markets).

The equity risk premium is much lower than you think it is: empirical estimates from a new approach

The equity risk premium, representing the excess of the expected return on the stock market over the risk-free rate, lies at the core of financial economics. For example, in the traditional Capital Asset Pricing Model, the risk premium is the additional return required to compensate investors for one unit of (beta) risk. Despite numerous attempts to estimate the value of this premium, there is some debate as to which of the many empirical estimates represents the true premium required by equity investors. The most commonly cited estimates are those provided by Ibbotson Associates (1998) in their annual review of historic rates of return observed since 1926 on various portfolios of stocks and bonds. Their data indicate that the risk premium lies in the region of 7 to 9 percent (depending on the maturity of the risk-free rate used). Others, notably Siegel (1992), suggest that there is some variation in this ex post estimate, depending on the particular period examined.

In addition to this ex post approach that is based on observed returns, financial economists have also considered ex ante approaches that estimate the risk premium using forecasted dividends. Expected dividends are often based on earnings forecasts, typically obtained from sell-side stock analysts employed by brokerage houses (e.g, Brigham, Shome and Vinson, 1985, Harris and Marston, 1992, and Moyer and Patel, 1997). In addition to providing buy/sell recommendations, sell-side analysts also provide earnings forecasts, which usually cover the next two years and often include a forecasted growth in earnings that is expected to hold over the next five years (hereafter labeled g'). Services such as First Call, IBES, and Zachs collect these forecasts and make them available to researchers. In the ex ante dividend growth approach, the expected rate of return on the stock market (k^*) is estimated using the Gordon (1962) dividend growth model, described in equation (1). The Gordon model is a special case of the Williams (1938) dividend present value model, detailed in equation (2), when dividends are constrained to grow in perpetuity at a constant rate (hereafter labeled g).

$$p_0 = \frac{d_1}{k^* - g} \Rightarrow \quad k^* = \frac{d_1}{p_0} + g \tag{1}$$

$$p_0 = \frac{d_1}{(1+k^*)} + \frac{d_2}{(1+k^*)^2} + \frac{d_3}{(1+k^*)^3} + \ldots \tag{2}$$

where
p_0 = current price, in year 0,
d_t = dividends expected at the end of year t,
k^*= expected rate of return on the market, derived from the dividend growth model, and
g = expected dividend growth, in perpetuity.

The earnings growth forecast by analysts over the next five years (g') is substituted for dividend growth rate in perpetuity, g, and d_1 is derived from next year's forecasted earnings, using an assumed payout ratio.[1] Estimates of the risk premium using this approach are often similar in magnitude to the ex post estimates in Ibbotson derived from historical data. For example, Moyer and Patel (1997) estimate the risk premium each year over their 11-year sample period (1985-1995) and generate a mean estimate of 9.38 (6.96) percent relative to the short-term (long-term) risk-free rate. Although the mean ex ante estimates are similar to the historical estimates, there is considerable variation across the individual years. Moyer and Patel's estimates of the risk premium relative to the short-term risk-free rate vary between 6.94 and 11.93 percent.

Despite the apparent agreement between the ex post and ex ante estimates, some concerns have been expressed regarding the magnitude of the estimates (see Cochrane, 1997, for a review). Arguments have been offered (e.g, Malkiel, 1996) for why the observed difference between equity returns and risk-free rates is too high an estimate for the true risk premium, and why such a large difference is unlikely to persist in the future. Also, Mehra and Prescott (1985) and a number of subsequent papers have debated the so-called "equity premium puzzle." The essence of this puzzle is that aggregate consumption patterns do not seem to vary enough to

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1 Alternatively, d_1 is estimated by multiplying current dividends by an assumed growth rate (equal either to the forecast growth in earnings, g', or the expected growth in earnings next year, based on e_1/e_0).

justify the high risk premium estimates mentioned in the empirical literature (see Kocherlakota, 1996, for a recent summary).

Evidence from the investment community is generally consistent with the view that the risk premium is much lower than eight percent. Survey evidence (e.g., Benore, 1983) points to rates that are below five percent. Analysis of the discount rates used in the discounted cash flow valuations provided in analyst research reports also suggests that the equity risk premium is below five percent. Some even go so far as to recommend that the premium be dropped to zero.

Notwithstanding these concerns, the academic literature has generally adopted the Ibbotson estimates as being the most reliable, since the weight of the ex post evidence is substantial. Provided the risk premium has remained reasonably stationary over the last seventy years, the observed distribution of the excess of stock returns over risk-free rates enables one to reject the hypothesis that the risk premium is three percent or below, at normal levels of statistical significance.[2] In fact, much of the recent equity premium puzzle literature has searched for explanations that would raise the theoretical estimates toward those provided by Ibbotson (e.g. Abel, 1999)

Why might the historical data imply a risk premium that is too high? Two possible reasons are as follows. First, the period examined is unusually "lucky". While extending the sample period to earlier years is a potential solution to this problem, that approach could contaminate the estimates if the risk premium has experienced structural shifts over the long time periods examined. Second, the data exhibit survivor bias: some stock markets collapsed and those markets that survived, like the US exchanges, exhibit better performance than expected (see Brown, Goetzman, and Ross, 1995).

Although we have no new insights regarding these and other possible reasons for why extant stock markets have done much better than expected, our contribution lies in building

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[2] Cochrane (1977) explains how a mean equity premium of 8 percent derived from 50 years of historical data has a standard deviation of 2.4 percent. Therefore, the true premium lies between 3 and 13 percent with a 95 percent probability.

support for the argument that the prices observed over the past 14 years for the six markets we examine imply a much lower equity risk premium than the Ibbotson estimate. Conversely, assuming that the equity premium is as high as 8 percent results in projected numbers that are simply not consistent with past experience.

Before proceeding to a summary of the paper, we wish to point out why the dividend growth model is potentially misleading and why the results of prior literature using this approach should not be interpreted as supporting the estimates derived from historical returns. Note that g is a hypothetical rate, since it represents the constant rate at which dividends could grow in perpetuity, if such a dividend policy were chosen. It is not anchored in any observable series, such as past or forecast dividend growth rates, or earnings growth rates. Take, for example, two firms that are similar in every way, except that one firm has a higher expected forward dividend yield (d_1/p_0) than the other, say 7 percent and 1 percent. It is not easy to determine whether any selected value of g is too high (effectively depleting the capital in some future period) or too low, (the capital stock would grow "too fast") if dividends were required to grow at that rate.

Equation (1) provides a guide for the appropriate value of g, since it indicates that g represents the excess of the discount rate (k^*) over the forward dividend yield (d_1/p_0). If k^* equals 10 percent, for example, the value of g for the two firms must be 3 percent and 9 percent. These two values of g are substantially different from each other, even though the two firms are not. More important, neither rate seems to be related in any way to any observable series; in particular, both rates are unrelated to the near-term forecast earnings growth rate, which is the proxy used most often for g. This inability to calibrate whether an assumed value of g is appropriate or not is the primary reason why we caution against relying on the dividend growth approach.

Even if by coincidence the hypothetical dividend growth rate in perpetuity, g, equaled expected earnings growth rates in perpetuity, the five-year earnings growth rate forecast by analysts (g') is too high an estimate for the earnings growth rate in perpetuity. Comparing realizations of future earnings with forecasts indicates that forecasted earnings growth rates are consistently optimistic, in all six countries examined. Also, the magnitudes of the forecast growth rates seem too high, relative to those for aggregate statistics. For example, the forecasted 5-year earnings growth rate for the US over our sample period is in the neighborhood of 12 percent.[3] This rate exceeds the estimated growth in GDP (e.g., since 1970, forecasts of expected real growth in GDP have averaged 2.71 percent, and realized real growth has averaged 2.81 percent).[4] And nominal growth in S&P earnings has been only 6.6% since the 1920s (WSJ, 6/16/97, "As stocks trample price measures, analysts stretch to justify buying"). Using too high a growth rate in the ex ante dividend growth model results in too high an expected rate of return on the market, which in turn biases upward the risk premium estimates.[5]

Our approach, labeled the abnormal earnings model, is similar in some ways to the dividend growth model, since it's an ex ante approach and it uses analyst forecast data, but it differs in application and results. It is developed from an accounting-based valuation model that has recently been employed to address questions relating to market myopia (Abarbanell/Bernard, 1995) and market inefficiency (Frankel/Lee, 1996).[6] Put simply, the present value of future dividends, which equals the current stock price in equation (2), can be restated as the sum of the current accounting (or book) value of equity plus a function of future accounting earnings.

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3 The estimates of 5-year earnings growth (*g'*) over our sample period for Canada, France,and the UK are also approximately 12 percent. For Germany and Japan, they vary substantially across different years, and are often less than 12 percent.

4 Although growth in aggregate earnings and growth in GDP are related, they are not identical. Growth in aggregate earnings is generated from earnings per share estimates (i.e., only earnings growth accruing to currently outstanding shares is considered, not earnings growth due to issuing new equity) and will be lower than growth in GDP to the extent that the number of shares is expected to increase over time. On the other hand, growth in aggregate earnings could include growth in earnings from overseas subsidiaries, only a portion of which would be reflected in GDP.

5 Some papers employing the dividend growth model have attempted to compensate for the optimism inherent in analysts' forecasts. Two such adjustments are as follows: a) assume a lower dividend growth rate after year +5, or b) extend the initial (higher) growth rate period over a few years beyond year +5 (e.g. to year +7 in Gordon and Gordon, 1995) and then drop the growth rate to zero thereafter. Such adjustments effectively lower the estimated risk premium and bring it closer to our estimates. However, the fundamental problem with identifying appropriate dividend growth rates in perpetuity, illustrated by the two-firm example in the previous paragraph, still remains: the choice of the stepped down growth rate in a) or the period of high growth in b) is ad hoc.

6 The approach appears to have been discovered independently by a number of economists and accountants over the years. Preinreich (1938) is the first cite, to our knowledge. Edwards and Bell (1961) and Peasnell (1982) are some of the later cites. A number of researchers, Jim Ohlson in particular, have in recent years published a large body of analytical and empirical work that utilizes this insight.

Section I contains more details of the accounting valuation model. Being an ex ante approach, it avoids many of the problems associated with ex post approaches. Concerns about unrepresentative sample periods (extended bull or bear markets) are no longer relevant. Also, since it relies only on contemporaneous forecasts, this approach is not affected by time-varying risk premia. Relative to other ex ante approaches such as the dividend growth model, the abnormal earnings approach puts to better use other information that is currently available and is able to narrow considerably the range of allowable growth rates by generating diagnostics that can be checked for reasonableness. Both benefits are explained in more detail in section I.

Section II contains a description of our sample of IBES forecasts and our methodology, and we report in section III our estimates of the expected market rate of return based on consensus analyst forecasts made as of April of each year between 1985 and 1996 (inclusive). Those estimates are considerably lower than the ex ante estimates of expected market returns based on the dividend growth approach (as well as ex post market returns) over the same period. As a result, our estimates for the equity risk premium, which are in the neighborhood of three percent, are correspondingly lower than risk premium estimates from other approaches. For reasons explained in section III, we use the 10-year Government bond yields as the risk-free rate proxy when computing the risk premium.

Not only are our risk premium estimates lower than those from other approaches, they exhibit less variation over time. Intuitively, the risk premium is a function of the inherent riskiness of the market portfolio and the risk aversion of market participants. We see no reason why these two parameters would vary wildly from period to period, and therefore expect the risk premium estimates to remain relatively stationary over time. The stationarity exhibited by our estimates increases our confidence in the reliability of these estimates.

The results of extensive sensitivity analyses conducted to determine the robustness of our estimates are reported in section IV. Examination of various diagnostics such as implied values of profitability, price-to-book ratios, and price-earnings ratios for future years validates our assumptions and suggests that our estimates are unlikely to be downward biased. Our diagnostics also suggest that the growth estimates underlying prior risk premium estimates are too high. Finally, analysis of other samples confirms our results.

If the risk premium is indeed as low as our estimates suggest, there are important implications. Some of those implications are as follows (see section V for a discussion). First, expected or required rates of return (for capital budgeting, regulated industries, and other investment decisions) that were based on the higher estimates of risk premium provided in the literature are likely to be too high, and might have caused erroneous decisions. Second, in addition to adjusting downwards the required rates of return for risky investments, the spread between the required rates of returns for high and low beta firms is also substantially reduced. Consequently, less effort need be invested in accurately determining the beta associated with investments, given the lower sensitivity of expected rates of return to differences in beta risk. Relatedly, explanations of market anomalies based on unobserved changes in risk need to allow for considerably greater beta changes to explain observed abnormal returns. Third, the magnitude of the risk premium puzzle that needs to be explained is reduced substantially. While such a reduction might still be insufficient to reconcile reasonable risk aversion parameters with the estimates derived from aggregate consumption patterns, reducing the magnitude of the difference to be explained might invigorate the search for alternative explanations. Finally, concerns about current stock prices being too high, relative to underlying fundamentals, are probably overstated. Our results imply that the substantial stock price increases that occurred in recent years are explained completely by improved earnings forecasts and lower risk-free rates.

I. Model Description

The accounting-based valuation model can be stated as follows (see derivation in Appendix).

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \frac{ae_4}{(1+k)^4} + \frac{ae_5}{(1+k)^5} + \ldots \qquad (3)$$

where

bv_t = book (or accounting) value of equity at the end of year t,

$ae_t = e_t - k(bv_{t-1})$, is abnormal earnings, or accounting earnings less a charge for the cost of equity,

e_t = earnings forecast for year t, and

k=expected rate of return on the market portfolio, derived from the abnormal earnings approach.

Equation (3) indicates that the current stock price equals the current book value of equity plus the present value of future expected abnormal earnings. Abnormal earnings, a proxy for economic profits or rents, adjusts accounting earnings by deducting a charge for the use of equity capital. Note that the algebra in the appendix requires that this charge be based on a beginning-of-period equity investment that is measured in book values, not market values. To separate the two sets of empirical estimates reported in section III, we use the labels k and k^* for the expected market rate of return estimated from the abnormal earnings and dividend growth approaches.

Since the IBES database provides analysts' earnings forecasts only for years 1 through 5, to incorporate abnormal earnings beyond that date we assume that abnormal earnings grow at a constant rate (g'') after year 5. Equation (3) is thus adapted as follows.

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \frac{ae_4}{(1+k)^4} + \frac{ae_5}{(1+k)^5} + \left[\frac{ae_5(1+g'')}{(k-g'')(1+k)^5}\right] \qquad (4)$$

The last, bracketed term captures the present value of abnormal earnings after year 5, and the terms before are derived from accounting statements (bv_0) and analyst forecasts (e_1 to e_5).[7] It is worth repeating that there are three separate growth rates in this paper, and they refer to different streams, correspond to different periods, and arise from different sources. The first rate, g, refers to dividends, corresponds to growth in perpetuity, and is a rate assumed by the researcher. The second rate, g', refers to accounting earnings, corresponds to the first five years, and is provided by financial analysts. The third rate, g'', refers to abnormal earnings, corresponds to the period beyond year +5, and is assumed by the researcher.

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[7] Computation of abnormal earnings for years 2 through 5 require book values of equity for years 1 through 4. These book values are derived by imposing an assumed dividend payout rate (py) and using the relation that $bv_t = bv_{t-1} + e_t(1-py)$.

Financial economists have often expressed concerns about accounting earnings deviating from "true" earnings (and book values of equity deviating from market values), in the sense that accounting numbers are noisy measures and easily manipulated. However, the relation above is not impaired by differences between accounting and economic numbers, nor is it affected by the latitude available within accounting rules to report different accounting numbers.[8] In fact, as shown in the appendix, the stream of accounting variables underlying equation (3) can be mapped exactly to the dividend stream in equation (2) using relation (A3).

If the accounting stream in equation (3) can be replaced by the corresponding dividend stream in equation (2), what then are the benefits of estimating the market discount rate using equation (4) relative to using the dividend growth approach described by equation (1)? As indicated in the introduction, the fundamental problem with implementing the dividend growth approach is the arbitrary choice of the assumed rate at which dividends grow in perpetuity. The primary advantage of using the accounting stream is the ability to check the underlying growth assumption. The 5-year earnings growth rates forecast by IBES analysts in our sample are around 12 percent. Assuming this growth rate for dividends in perpetuity, we obtain risk premia similar to those estimated in prior research: in excess of 8 percent, relative to the 10-year risk free rate. In contrast, the dividend growth rate in perpetuity that corresponds to the assumptions we use to generate the lower risk premium estimates is only about 7.5 percent.[9] The debate then

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8 A simple example might help to illustrate this important feature of the abnormal earnings approach. Suppose a manager chooses not to depreciate in year 1, in order to increase reported accounting earnings. While this would increase e_1 and ae_1, it would also increase bv_1, the book value of equity at the end of year 1. Inflating this number, which is the investment base for year 2, in turn reduces the value of ae_2 because of the higher charge for the cost of equity in year 2 (k times bv_1). This effect reduces all future period abnormal earnings as long as the book value is inflated. There is also a reduction in earnings at some future date because of the depreciation not taken in year 1. It turns out that the increase in ae_1 caused by not depreciating in year 1 is exactly offset by the reduction in the present value of future years' abnormal earnings. In other words, distortions created by accounting numbers do not vitiate the relation in equation (3), provided the accounting rules are applied consistently (see appendix for more details).

9 The dividend stream that corresponds exactly to the abnormal earnings stream in equation (4) does not grow at a smooth rate. It can be replaced, however, by an equivalent smooth (growing at a fixed rate) dividend stream that has the same present value. That growth rate is the excess of the estimated market rate of return (k) over the forward dividend yield (see equation (1)).

boils down to which of the two rates for dividend growth in perpetuity is more reasonable: 12 percent or 7.5 percent?

The abnormal earnings approach helps to resolve this debate. It uses current market and accounting information and forecasted accounting numbers to project future streams for a number of value-relevant indicators for different growth assumptions. Examination of the trends forecast for these indicators suggests that dividend growth rates in perpetuity of around 12 percent are simply too high: the levels of future profitability and growth in profitability implied by those trends are inconsistent with intuition and past experience.

The second benefit of using the abnormal earnings approach is that it uses considerable information that is currently available, unlike the dividend growth approach which is based only on expected dividends in year +1 and an assumed growth rate. In the abnormal earnings approach, the proportion of total value that is based on “soft” numbers, those derived from growth rates assumed by the researcher, is substantially reduced. The first six terms on the right-hand side of equation (4) are derived from “hard” numbers, obtained either from current accounting statements (bv_0) or from analyst forecasts (ae_1 to ae_5). Only the last term, representing the present value of abnormal earnings beyond year 5, is determined by a growth rate assumed by the researcher (g''). Reducing the proportion of total value derived from soft numbers reduces the importance of the growth rate assumed by the researcher. It also makes it easier to see that the prior estimates of k are probably too high. To generate risk premium estimates as high as conventional estimates, the assumed value of growth in abnormal earnings beyond year +5 (g'') would need to exceed 20 percent in perpetuity!

Like other ex ante approaches, our approach assumes that the stock market efficiently incorporates analyst forecasts into prices, and that analysts make unbiased forecasts. There is, however, a large body of research that has documented instances of mispricing relating to information available in analyst forecasts, and also evidence of various biases exhibited by analysts. Fortunately, the extent of mispricing documented is relatively small. Also, the evidence on mispricing suggests that some firms are underpriced and others are overpriced.

Therefore, some of that mispricing should cancel out at the market level, and be of less concern for our market-level study. Turning to the issue of analysts making efficient forecasts, although some of the biases exhibited by analysts would similarly cancel out in the aggregate, there is evidence of a systematic optimism bias in analysts' earnings forecasts. Assuming that stock prices adjust for any such optimism in forecasted earnings, our estimates of the risk premium, which are based on unadjusted earnings forecasts, will need to be adjusted downward even further.

II. Data and methodology

IBES collects individual analyst earnings forecasts and makes them available electronically to subscribers. The forecasts are for different horizons (1-quarter ahead, 1-year ahead, 2-years ahead and so on). At the annual level, most analysts make forecasts for 1 and 2 years ahead and also provide an expected growth rate for earnings that they expect for the next "cycle". Although the duration of such cycles is not explicitly specified, it is informally interpreted to represent the next 5 years. Consequently, we use the forecasted 5-year growth rate to generate earnings forecasts for years +3, +4, and +5. Some analysts also provide specific forecasts for 3, 4, and 5 years out, for a subset of firms. That subsample is investigated to confirm that the earnings forecasts for years +3 to +5 inferred from 5-year growth rates are unbiased proxies for the actual forecasts for those years.

IBES provides archival data to researchers in two forms: a detailed dataset that has the individual forecasts and a summary dataset that has the mean/median value of all available individual forecasts as of a particular date each month for each horizon. Except for some sensitivity analyses that are based on the detailed dataset, this paper's results are drawn from the summary dataset. In addition to the earnings forecasts, IBES also provides data for actual earnings per share, dividends per share, share prices, and the number of outstanding shares.[10]

[10] The actual earnings per share numbers reported by IBES do not match exactly with any of the earnings per share data items on Compustat (before or after taxes/before or after extraordinary items/primary or fully

Mean analyst forecasts are collected from the summary IBES database as of April each year, up to and including April 1996.[11] Ideally, the forecasts and prices should be gathered as soon as possible after the year-end immediately after the book value of equity is known. Rather than collect forecasts at different points in the year, depending on the fiscal year-end of each firm, we opted to collect data as of the same month each year for all firms to ensure that the risk-free rate is the same across each annual sample. Since most firms have December year-ends and the book value of equity for the prior year is likely to have just been made available to the public for those firms by April, we used forecasts as of April each year.[12]

While optimistic analyst forecasts might bias upwards our risk premium estimates, there are two methodological simplifications that create a small bias in the opposite direction. First, although the valuation relation is based on dividends being paid at the end of each year, the actual cash flows occur during the year, typically in four quarterly payments. Actual prices are higher than they would have been if cash flows occurred only at the end of the year, and this depresses the estimated k. Second, April is past the "beginning" of the year, corresponding to the date that last year's dividend is paid. As a result, the price as of April is higher than it would have been at the beginning of the year. The bias in the estimated risk premium when this effect is ignored is slightly greater for firms with fiscal year-ends other than December. Neither effect is material, however, and overall we still expect our risk premium estimates to be biased upward.

Equity book values are collected from Compustat's Industrial Annual, Research, and Full Coverage Annual Files, for years up to and including 1995. Only firms with IBES forecasts for

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diluted). IBES employees indicated to us that their actual numbers are after-tax earnings before extraordinary items and discontinued operations, but the effects of certain write-offs and accounting changes are also excluded. The choice between primary and fully-diluted basis for earnings per share is determined by IBES to correspond with the basis used by a majority of contributing analysts for that firm-year. In the few cases when earnings per share is derived on a fully-diluted basis, we use the dilution factor provided by IBES to convert those earnings to primary earnings per share.

11 We are awaiting the release of Compustat files for 1997 to update the sample to include forecasts as of April 1997 and 1998.

12 Book values of equity can be obtained from balance sheets, which are required to be filed with the SEC within 90 days after the fiscal year-end. For firms that do not meet this deadline, the book value of equity can be inferred by adding fourth quarter earnings and subtracting fourth quarter dividends from the equity value as of the end of the third quarter. There are very few firms that do not announce fourth quarter earnings and dividends within 90 days after the fiscal year-end.

1 and 2 years ahead and a 5-year growth forecast as well as non-missing data for actual book value, earnings, dividends, and price per share and number of shares are included in the sample. Earlier years in the IBES database provided too few firms to represent the overall market. From 1985 onwards, the number of firms with complete data increases substantially, and there are at least 1,500 firms in each year thereafter. As a result, our sample period begins in April, 1985, and extends over the 12 years ending in April, 1996.

All firm-years with complete data in each year of the sample period are aggregated to generate market-level earnings, book values, dividends, and prices. The number of firms with available data (reported in column 1 of Table I) increases steadily from 1,571 in April, 1985 to 3,196 firms in April, 1996. Comparison with the total number of firms and market capitalization of all firms on NYSE, AMEX, and NASDAQ each April indicates that although our sample represents only about 30 percent of all such firms, it represents 90 percent or more of the total market capitalization. In other words, although there are many publicly-traded firms excluded from our sample, most excluded firms are of low capitalization. Overall, we believe our sample is fairly representative of the value-weighted market, and refer to it as "the market" hereafter. Actual data for year 0 (the full fiscal year preceding each April when forecasts were collected) is provided in columns 2 through 6. Table I also includes market-level forecast earnings (in columns 7 through 11) for years +1 and +2, as well as estimated numbers for +3, +4, and +5. These last three estimates are generated by applying the 5-year forecasted growth rate, g', on forecasted earnings for year +2.[13]

Table I reveals an interesting finding relating to dividend payouts. The ratio of market dividends to market earnings is around 50 percent. This seems unusually high compared to anecdotal estimates of average dividend payouts. We offer two potential explanations for this difference. First, our ratio based on aggregates is similar to a value-weighted average dividend

[13] Very few firm-years had negative values for 2-year-ahead forecasts, even though quite a few firms reported losses in the current year. The few observations with negative year +2 earnings forecasts are deleted from our samples, to avoid applying a positive growth rate on negative year +2 forecasts when extrapolating to estimate forecasts for years +3 to +5.

payout and is thus more representative of large firms, which tend to have higher dividend payouts than small firms. Second, market earnings include many loss firms. Typically, dividend-payout averages are generated from payouts computed at the firm level, and loss firms are excluded from consideration because the payout ratio is meaningless when the denominator is negative. There are a substantial number of loss firms in each year, and this number is unusually high in the early 1990's when accounting earnings were depressed because of write-offs and accounting changes. Dividends remained relatively unaffected, however, and including loss firms raises the ratio of aggregate dividends to aggregate earnings, relative to the average dividend payout of all profitable firms.

III. Results

The next step is to infer the expected rate of return on the market portfolio, k, from equation (4). This is the discount rate that equates the market value each April with the function of current book value and future forecasted abnormal earnings as of those dates.[14] As mentioned earlier, abnormal earnings (ae_t) represent accounting earnings (e_t) less a charge for the cost of equity. This charge equals the discount rate, k, times the beginning book value of equity (bv_{t-1}). Future book values of equity are estimated using the average market-level dividend payout ratio of 50% observed in table I. That is, book value for year +1 is assumed to equal current book value (as of the end of year 0) plus 50% of earnings forecast for year +1. Book values for years +2 through +5 are estimated in a similar manner. Earnings forecasts along with the future book value estimates enable computation of abnormal earnings for years +1 through +5.

For years beyond year +5, abnormal earnings are assumed to grow at a constant rate, g''. To assess the range of reasonable values for g'', it is important to describe some features of abnormal earnings. Expected abnormal earnings would equal zero if book values of equity

[14] We search manually for the solution to this polynomial, with the first iteration in the neighborhood of the risk-free rate. Since equation (4) is a polynomial of the fifth order in k, there are five roots for k. As noted by Botosan (1997), only one root is reasonable, the others are negative and/or imaginary.

reflected market values.[15] If book values measured input costs fairly but did not include the portion of market values that represented economic rents not yet earned, abnormal earnings would reflect those rents and be expected to be positive. However, the magnitude of such rents at the aggregate market level is likely to be small, and is likely to decrease over the horizon period for the usual reasons (competition, antitrust actions, and so on).

Notwithstanding economic arguments for the expected level of abnormal earnings, a strong force that generates systematic and substantial positive abnormal earnings is accounting conservatism: book values are less than market values because assets (liabilities) tend to be understated (overstated) on average. For example, many investments are written off too rapidly relative to their value (e.g., research and development). Given the likely sources of abnormal earnings, growth in abnormal earnings can reasonably be expected to be quite low, much lower than expected growth rates for earnings and dividends. As a result, much of the earlier literature utilizing the abnormal earnings approach has assumed a zero growth in abnormal earnings past the "horizon" date.[16]

Although we too are convinced that abnormal earnings in aggregate are unlikely to exhibit long-term growth rates as high as those exhibited by earnings or dividends, we believe an assumption of zero nominal growth in abnormal earnings is too pessimistic. Growth in abnormal earnings is best understood by examining the behavior of the accounting rate of return (ratio of accounting earnings to beginning-of-year book value of equity) under conservative accounting relative to the cost of equity. Simulations and theoretical analyses (e.g., Zhang, 1997) of the steady-state behavior of the accounting rate of return under conservative accounting suggest two important determinants of growth in abnormal earnings: growth in investment and the degree of accounting conservatism. In essence, the accounting rate of return approaches the cost of equity but remains above it in the long-term. As a result, even though the excess of the rate of return

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15 In an efficient market, market values are expected to adjust each period so that no abnormal returns are expected in the future. Similarly, if book values are marked to market values each period, the resulting abnormal earnings would be expected to be zero in future periods.

16 In fact, many papers have assumed that abnormal earnings *decline* to zero, past the horizon date.

over the cost of equity declines slightly over time, the dollar magnitude of nominal abnormal earnings for the overall market is likely to increase because of nominal growth in investment.

Based on this understanding, we assume that abnormal earnings beyond year +5 are expected to exhibit zero growth in real terms. That is, we assign to g'' a value equal to the expected inflation rate. We estimate the expected inflation rate by subtracting three percent from the contemporaneous nominal 10-year risk-free rate, where the three percent adjustment is assumed to represent the real risk-free rate.[17] Since we recognize that this assumption is only an educated guess, we consider in section IV.D other values of g'' also. Fortunately, our estimated risk premium is relatively robust to variation in the assumed growth rate, g''. This lack of sensitivity is due to the relatively small proportion of current market value that is captured by the growth term in equation (4), relative to equation (1).

Table II provides the results of estimating the market discount rate from equation (4), and the associated risk premium. Aggregate market and book values for all firms in our annual samples are reported in the first two columns. The next five columns (3 through 7) contain present values of abnormal earnings for years +1 through +5, based on the estimated discount rate, k, and the next column (column 8) contains the present value of the terminal value, representing abnormal earnings growing at the rate g'' after year +5. Recall that this growth rate was assumed to equal the risk-free rate less three percent.

There is some debate as to which maturity is appropriate when selecting the risk-free rate. The risk premium literature has used both shorter (30-day or 1-year) and longer (30-year) maturities for the risk-free rate. On the one hand, longer maturities exceed the true risk-free rate because they incorporate the uncertainty associated with intermediate variation in risk-free rates. On the other hand, short-term rates are likely to be below the true risk-free rate, since some portion of the observed upward sloping term structure could reflect increases in expected future short-term rates. Since the flows (dividends or abnormal earnings) being discounted extend

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[17] The observed yields on recently issued inflation-indexed government bonds supports this assumption.

beyond one-year, it would not be appropriate to use the current short-term rate to discount flows that have been forecast based on rising interest rates.

Rather than report two sets of results based on the long and short term risk-free rates, we report results based on an intermediate term rate: the 10-year Government T-bond yield (reported in column 9 of Table II). Since the term structure for risk-free rates was consistently upward sloping during our sample period, our intermediate-term rate was always lower than the long-term rate and always higher than the short-term rate. The mean risk-free rates (as of April each year) over the sample period for the 1 and 30-year maturities are 143 basis points less and 25 basis points more than the mean 10-year rate of 7.89 percent. The mean risk premium relative to the 1-year or 30-year risk-free rates can easily be inferred from the mean risk premium we estimate relative to the 10-year rate: just add 143 basis points to or subtract 25 basis points from our estimate. Given the substantial difference between the short-term and long-term risk-free rates, it is important that the maturity of the risk-free rate used be controlled for when making comparisons across studies.

Column 10 in Table II provides the estimated market discount rate, k, and the related risk premium is noted in column 12. The estimated market discount rates vary between a high of 14.42 percent in 1985 and a low of 9.64 percent in 1993. The corresponding risk-free rates vary with the estimated k's, between a high of 11.64 percent in 1985 and 6.05 percent in 1993. The difference between the two rates, representing the estimated risk premium, has a mean value of 3.46 percent and remains within a fairly tight band: between a low of 2.78 percent in 1985 and a high of 3.90 percent in 1986.

To provide a reference point, we also report in column 11 the estimated market discount rates based on the dividend growth model, k^*, described by equation (1).[18] Consistent with the prior literature, we assume that the 5-year growth in earnings forecast can be substituted for the

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18 Another reference point is the ex post return observed over the sample. That return is in excess of 13 percent, implying a risk premium of over 5 percent. Given that only 12 years of data are used to estimate it, this mean return is less reliable than other ex post estimates observed over longer periods.

growth in dividends from year 1 onward. We obtain estimates for k^* that are almost identical to those reported by Moyer and Patel (1997).[19] The values of k^* in column 11 are much larger than the corresponding values of k in column 10, and exhibit more time-series variability. The difference between k^* and k, and therefore the difference between the respective risk premia, has a mean of 3.88 percent and ranges between 1.68 percent (in 1985) and 5.05 percent (in 1993).

Whereas the assumed dividend growth rate in perpetuity (g) is critically important in the dividend growth model, the assumed growth rate in abnormal earnings beyond year +5 (g'') is relatively less important in the abnormal earnings approach. Examination of the value profile reported in columns 2 through 8 of table II indicates the relative magnitude of the terminal value, which is the only term influenced by g''. In 1985, for example, only 27 percent of the current market value is represented by the terminal value, approximately 68 percent is captured by current book value and five percent is captured by the terms representing abnormal earnings for years +1 to +5. In essence, changes in the assumed growth rate for abnormal earnings beyond year +5 have a smaller effect on risk premium estimates, relative to the effect of similar changes in the assumed growth rate in perpetuity for dividends (this issue is discussed further in section IV.D).

The lower level of time-series variability of the risk premium estimates derived from k, compared to that for k^*, is consistent with the view that the abnormal earnings approach provides more reliable estimates. We recognize that the stationarity of our estimates is partially induced by the link between risk-free rates and assumed growth rates ($g''=r_f$ -3%)). The terminal value term in equation (4), which is determined by g'', would vary with the risk-free rate, and create a positive correlation between the estimated market discount rate (k) and the risk-free rate, which in turn would dampen the variation in the estimated risk premium (k less r_f). There are, however, two reasons to believe that this effect is only partially responsible for the observed stationarity in

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19 Similar results are expected because the underlying data is taken from the same source. Any differences between the samples and procedures used in this paper and those in Moyer and Patel (1997) are small; for example, they use the S&P 500 index whereas we use all firms with available data.

our risk premium estimates. First, there are considerable shifts over the 12-year sample period in the proportion of total value residing in the terminal value term (from 27 percent in 1985 to 50 percent in 1996), and yet the risk premium remains relatively unaffected. Second, as discussed in section IV.D, the risk premium estimates for a synthetic market portfolio constructed to have no growth in abnormal earnings (g''=0) also exhibit very little time-series variability. For this portfolio, there is no mechanical link between g'' and the risk-free rate.

In sum, our estimates of the equity risk premium using the abnormal earnings approach are considerably lower and more stationary than those estimated in the past using other approaches. Prior estimates of risk premia using historical data and ex ante dividend growth approaches are at least twice as large as those we derive using the abnormal earnings approach. The contrast between our results and the traditional estimates of risk premium is even more stark in light of the well-known optimism in analyst forecasts; adjusting for that bias would decrease further our estimates of the risk premium.

IV . Sensitivity Analyses

This section summarizes our attempts to gauge the robustness of our conclusion about the risk premium being much lower than prior estimates. We consider first two relations regarding the price-to-book and price-earnings ratios that allow us to check whether our projections under the two models are reasonable. Next, we examine the sensitivity of our risk premium estimates to variation in the rate at which abnormal earnings are expected to grow after year +5 (g''). To isolate the effect of g'', we examine synthetic market portfolios that are constructed to have no abnormal earnings past year +5. We also examine whether growth rates as high as those assumed in prior dividend growth models result in future values of price-to-book and price-to-earnings ratios that are reasonable. Finally, we summarize the results we obtained by repeating our analysis on two other samples: Value Line forecasts for domestic firms, and IBES forecasts for international firms.

A. Price-to-book ratios and the level of future profitability

The first relation we use is that between the price-to-book ratio and future levels of profitability (see derivation in the appendix), where future profitability levels are measured by the excess of the market's accounting rate of return (roe_t) over the required rate of return, k.

$$\frac{p_0}{bv_0} = 1 + \frac{roe_1\text{-}k}{(1+k)} + \frac{roe_2\text{-}k}{(1+k)^2}\left(\frac{bv_1}{bv_0}\right) + \frac{roe_3\text{-}k}{(1+k)^3}\left(\frac{bv_2}{bv_0}\right) + \ldots \qquad (A15)$$

where

$roe_t = \dfrac{e_t}{bv_{t-1}}$, is the accounting rate of return in year t, computed on book value of equity in t-1.

This relation indicates that the price-to-book ratio, or the P/B ratio, is explained by the product of future expected abnormal profitability (roe_t-k) and growth in equity book values (bv_t/bv_0). Firms expected to earn an accounting rate of return on equity equal to the cost of capital should trade currently at book values (p_0/bv_0=1). Similarly, the P/B ratio expected in year +5, which is determined by the assumed growth in abnormal earnings after year +5 (g''), should be related to accounting profitability beyond year +5.[20] One way to investigate the validity of the assumed growth rates is to examine if the profiles of future P/B ratios and profitability levels are reasonable and related to each other as predicted by equation (A15).

Table III provides data on current and expected future values of P/B ratios and profitability. Current market and book values are reported in columns 1 and 2, and the implied market and book values in year +5 are reported in columns 3 and 4. These values are used to generate current and year +5 P/B ratios, reported in columns 5 and 6. Columns 7 through 12 contain the forecasted accounting rate of return on equity for years +1 to +6, which can be compared with the estimated market discount rate, k, reported in column 13, to obtain forecasted profitability.

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[20] The relation between the price to book ratio in year 5 and the level of profitability after year 5 can be seen by rewriting equation A15 as if year 0 is year 5. The relation between g'', the growth in abnormal earnings beyond year 5, and the price to book ratio in year 5 can be seen by examining the relation between g'' and the *difference* between price and equity book value (as described in the discussion before equation (A17) in the appendix).

The current P/B ratio has been greater than one in every year in the sample period, and has increased steadily over time, from 1.5 in 1985 to 2.7 in 1996. As indicated by equation (A15), P/B ratios in excess of one imply that forecast *roe* in future years should exceed the estimated equity cost of capital, *k*. This prediction is confirmed by the results in Table III: all forecasted *roe* values for years +1 through +6 exceed the corresponding values of *k*. The increase over the sample period in P/B ratios is mirrored by corresponding increases both in forecast profitability (roe_t - *k*) in years +1 through +5 as well as forecast profitability in the post-horizon period (after year +5), as measured by the implied price to book ratio in year +5.[21] Finally, the tendency for P/B ratios to revert gradually over the horizon towards 1 (indicated by the year +5 values in column 6 being smaller than the year 0 values in column 5) is a desirable attribute (see discussion in section II on the expected long-term trends for abnormal earnings). Overall, we find the results in table III comforting, since there are no discrepancies among current and future P/B ratios and profitability estimates.

B. Price-earnings ratios and forecast growth in profitability

The second relation we use to check our assumption regarding post-horizon growth in abnormal earnings is the price-earnings ratio, described by equation (A16) below (see derivation in appendix). Price-earnings ratios are a function of the present value of future changes in abnormal earnings, multiplied by a capitalization factor (=*1/k*).

$$\frac{p_0}{e_1} = \frac{1}{k}\left[1 + \frac{\Delta ae_2}{e_1(1+k)} + \frac{\Delta ae_3}{e_1(1+k)^2} + \ldots\right] \qquad (A16)$$

where
$Dae_t = ae_t - ae_{t-1}$,is the change in expected abnormal earnings over the prior year.

The price-earnings ratio on the left-hand side deviates slightly from the traditional representation in the sense that it is a “forward” price-earnings ratio, based on expected earnings

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[21] The growth in book value terms in equation (A15), which add a multiplicative effect, have been ignored in the discussion because of the built-in correlation with the level of profitability. As the *roe* increases, the growth in book value also increases because the dividend payout has been held constant at 50 percent for all years (by assumption).

for the upcoming year, rather than a "trailing" price-earnings ratio, which is based on earnings over the year just concluded. The relation between future earnings growth and forward price-earnings ratios is clean, unlike that for trailing price-earnings ratios. [22] Therefore, we use only the forward price-earnings ratio here and refer to it simply as the P/E ratio.

The results reported in table IV provide P/E ratios and growth in abnormal earnings derived from analyst forecasts, at the market level. The first 4 columns provide market values for year 0 and year +5 and the corresponding upcoming expected earnings. These items are used to generate the current and year +5 P/E ratios reported in columns 5 and 6, which can be compared to the values of *1/k* reported in column (18).[23] According to equation (A16), absent growth in abnormal earnings, the P/E ratio should be equal to *1/k*, and for positive (negative) expected growth in abnormal earnings, the P/E ratio should be greater (less) than those values of *1/k*. Forecast growth rates in abnormal earnings for years +2 through +6 are reported in columns 7 through 11.[24] According to equation (A16), the growth in abnormal earnings should correctly be scaled by upcoming earnings expected for year +1 (e_1) and then discounted. However, we report undiscounted percent growth numbers to allow comparisons across years.

P/E ratios in year 0 (column 5) are greater than the corresponding values of *1/k* in all sample years. Consistent with price-earnings ratios always exceeding *1/k*, abnormal earnings are forecast to exhibit positive growth for all cells in columns 7 to 11. The growth in abnormal earnings in year +2 is fairly large in magnitude and varies considerably over the sample period. After that, abnormal earnings growth declines steadily from +3 through +5 and stabilizes for year +6 and beyond at the assumed values for g'', which represents the expected nominal inflation rate

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22 Since the numerator of the P/E ratio is an ex-dividend price (p_0), the payment of a large dividend (d_0)would substantially reduce the ex-dividend price without affecting trailing earnings (e_0), thereby destroying the relation between prices and earnings. This complication does not arise when expected earnings for the upcoming period (e_1) are used instead of e_0.

23 If the numbers in Table IV appear to be not as high as those reported in the popular press, note that forward P/E ratios are smaller than corresponding trailing P/E ratios. There are two reasons for this general tendency. First, next year's earnings are greater than current earnings because of earnings growth. Second, current earnings contain one-time or transitory components that are on average negative, whereas forecast earnings focus on core or continuing earnings.

24 We did not compute abnormal earnings growth between year 0 and year +1, since we did not collect equity book values from year –1, which are required to compute abnormal earnings for year 0.

(r_f-3%). P/E ratios in year +5 are also greater than the corresponding values of *1/k* in all sample years. That is to be expected since the corresponding values of *g"* are all positive. Note that the tendency for growth in abnormal earnings to trend down from year 0 to year +5 is mirrored by the corresponding tendency for P/E ratios to trend towards *1/k* in all sample years.

For purposes of comparison with other work, we also report in columns 12 through 17 of table IV the growth in forecast earnings (as opposed to growth in abnormal earnings) for years +1 through +6. Note that percent growth in abnormal earnings need not be related in a systematic manner to percent growth in earnings, and that there is no explicit link in equation (A16) between price-earnings ratios and earnings growth. However, the prior literature makes intuitive links between price-earnings ratios and future earnings growth, and earnings growth is easier to comprehend and relate to than growth in abnormal earnings. Forecasted growth in earnings declines over the horizon, similar to the pattern exhibited by growth in abnormal earnings. Note the similarity in the pattern of earnings growth for all years in the sample period: the magnitudes of earnings growth estimates appear to settle at around 12 percent by year +5, before dropping sharply to values around 7 percent in the post-horizon period (year +6). This issue is discussed further in section IV.D below.

The results in table IV confirm the predictions derived from equation (A16) as well as the intuitive links drawn in the literature. As with the results for P/B ratios, the trends for P/E ratios and growth in abnormal earnings exhibit no apparent discrepancies that might suggest that the assumed abnormal earnings growth rates past year +5 are unreasonable.

C. Bias in analyst forecasts

Our next analysis investigates the potential for bias in analyst estimates used relating to the earnings forecast for years +1 through +5. If, for example, analysts tend to be conservative and under-forecast earnings, our estimates of the risk premium would be biased downward. We examine that possibility and some other possible reasons why IBES forecasts might result in biased estimates of the risk premium.

First, we consider the possibility that the imputed earnings numbers for years +3 to +5, based on applying the 5-year growth forecasts on year +2 earnings forecasts, create unintended biases. To investigate this possibility, we construct a sample of firms that had complete forecasts for all future years between +1 and +5; i.e., we did not need to impute forecasts for years +3, +4 and +5. Although the sample size was reduced considerably and only six of 12 years had a minimum number of firms to represent reasonably the overall market, the results obtained are very similar to those reported in table II for the corresponding years. Therefore, no bias appears to be caused by imputing earnings forecasts for years +3 to +5.

Second, we examine whether analysts that provide forecasts for years +1, +2 and a 5-year growth forecast are systematically different from analysts that only provide forecasts for years +1 and +2. Since the consensus data we use pools all available forecasts for each of the three horizons mentioned (+1, +2, and 5-year growth), the 5-year growth consensus could be representing a group of analysts that are systematically more or less optimistic than those that do not make 5-year growth forecasts but make forecasts for +1 and +2. Our analysis based on the detailed IBES files with individual analyst forecasts (results not reported) indicates that this is not the case. Forecasts for year +1 and +2 made by analysts that also provide 5-year growth forecasts are similar to forecasts for year +1 and +2 made by analysts that do not provide 5-year growth forecasts.

Finally, we investigate the extent of bias exhibited by consensus forecasts for different horizons by comparing forecasts with actual earnings (as reported by IBES) for those years. There is considerable prior evidence suggesting that analyst make optimistic forecasts. That bias is confirmed in our IBES sample. We compute the forecast error for each firm in our sample, representing the median consensus forecast as of April less actual earnings, for different forecast horizons (year +1, +2, … +5) for each year between 1985 and 1995. Table V contains the median forecast errors (across all firms in the sample for each year), scaled by share price. Forecasted earnings exceed actual earnings every year and for all horizons. Further, the extent of optimism increases with the horizon examined.

Since the forecast errors are scaled by price, comparing the magnitudes of the median forecast errors with the inverse of the trailing P/E ratios (or E/P ratios) is similar to a comparison of forecast errors with earnings levels. That comparison suggests that the bias is fairly substantial. While the trailing E/P ratios for our sample vary between 5 and 9 percent, the forecast errors in Table V vary between values that are in the neighborhood of 0.5 percent for year +1 to around 3 percent in year +5. Comparing the magnitudes of year +5 forecast errors with the implied E/P ratios indicates that forecasted earnings exceed actual earnings by as much as 50 percent at that horizon.[25]

D. Impact of variation in the assumed growth rate in abnormal earnings beyond year +5 (g")

We begin by considering two alternative cases for g'': 3 percent less and 3 percent more than the base case analyzed so far, where g'' was assumed to equal the expected inflation rate in the base case. Increasing (decreasing) the assumed value of g'' increases (decreases) the estimated required rate of return, k, but the effect is smaller than that in the dividend growth model because five of the six terms in equation (4) are unaffected by changes in g''. As mentioned in section III, our assumed growth rate of $g''=r_f$-3% is higher than any rate assumed in the prior abnormal earnings literature. Adding another three percent to the growth rate, which would require abnormal earnings to grow at a 3 percent real rate (similar to the GDP), probably causes the assumed growth rate to exceed market expectations, for the reasons mentioned in section III. Dropping 3 percent from the base case, as in the lower growth scenario, would be equivalent to assuming a very low nominal growth rate in abnormal earnings, and would be only slightly more optimistic than the assumptions in much of the prior abnormal earnings literature.

For the higher growth rate scenario ($g''=r_f$), the average risk premium over the 12-year sample period increases to a mean of 4.70 percent across the 12 years in our sample, from a

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[25] In addition to increasing with forecast horizon, the optimism bias is greater for certain years where earnings were depressed temporarily. The higher than average dividend payouts observed in Table 1 for 1987 and 1992 indicate temporarily depressed earnings in those years, and the forecast errors are also higher than average for those years. For example, the two largest median year +2 forecast errors are 1.86 and 1.81 percent, and they correspond to 2-year out forecasts made in 1985 and 1990.

mean of 3.46 percent in the base case. That is, even for this very high growth rate in abnormal earnings, the increase in the estimated risk premium is modest, and leaves it substantially below the traditional estimates of the risk premium. While increasing the growth rate increases the terminal value, it also reduces the present value of that terminal value because of the higher market discount rate it engenders. For the lower growth rate assumption, $g''=r_f$-6%, the estimated market risk premium falls to a mean rate of 2.34 percent. In sum, our risk premium estimates remain relatively insensitive to large changes in the assumed growth rate for abnormal earnings beyond year 5.

We consider next a synthetic market portfolio constructed to have no expected future abnormal earnings, to avoid the need for an assumed abnormal earnings growth rate beyond year +5.[26] As described in equation (A15), portfolios with P/B ratios equal to 1 should exhibit no abnormal earnings; i.e. the return on equity should on average equal the cost of equity. To construct portfolios with P/B equal to 1, we split all firms with available data each year into two groups: those with P/B above 1 and those with P/B less than 1. Equity market and book values for each group are aggregated to determine the overall P/B ratio for each group. The two groups are then assigned weights (that sum to 1), depending on the distance of their P/B ratios from 1, so that the weighted-average P/B for the synthetic market portfolio is 1. All current and forecast data for sample firms each year are then multiplied by the corresponding weights, to obtain the data required to estimate k from equation (4). The last term in equation (4), representing the terminal value of abnormal earnings beyond year +5, is set to zero.

As indicated by the level of the P/B ratio and the general increase in market P/B noted in Table III, the proportion of firms with P/B greater than 1 is always much greater than one-half, and this proportion increases over the sample period. The weight assigned to this group is only 10 percent in 1985 and decreases to 1 percent in 1996. As a result, less profitable firms, indicated by P/B ratios below 1, dominate the synthetic market portfolio. While this synthetic

[26] We thank Steve Penman for suggesting this analysis.

market is not similar to the actual market in many respects, it is similar along one important dimension: the weighted-average betas for the synthetic portfolios were close to 1 for each year of the sample.[27]

A complete analysis of this synthetic market portfolio was conducted, similar to that reported in Tables I through IV for the actual market portfolio. Some key results are as follows. First, the risk premium estimates from the synthetic market are similar to, but slightly lower than, those reported in Table II. They have a mean of 2.24 percent, compared to the mean risk premium of 3.46 percent in Table II. This result is consistent with our feeling that the inputs to our Table II analysis (analyst forecasts for years +1 to +5 and the assumed growth rate beyond year +5) probably result in risk premium estimates that are biased upward. Second, the risk premium estimates exhibit more volatility, and range from a minimum of 1.08 percent in 1992 to a maximum of 3.34 percent in 1990. The volatility observed for the estimates in Table II is likely to be biased downward because of the smoothing induced by pegging the abnormal earnings growth rate to the risk-free rate (see time-series stationarity discussion in section III). There is no reason to expect such a bias in the volatility exhibited by the synthetic market risk premium estimates. Although the synthetic market risk premium estimates exhibit greater volatility than the estimates in Table II, they are still considerably less volatile than the estimates from the dividend growth model (see $k^*\text{-}r_f$ from Table II).[28]

Our final investigation of the potential bias caused by assumed abnormal earnings growth rates relates to the case where analysts' five-year earnings growth rates (g') are used to extrapolate dividends in perpetuity; i.e. g' is substituted for g. Recall that this is a common assumption in prior research employing the dividend growth approach. The results described in section IV.C indicate that the 5-year earnings growth rates of about 12 percent forecast by IBES analysts are higher than the actual earnings growth rates observed *in every year of the sample*

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27 The mean beta for the 12 synthetic portfolios is 1, the standard deviation is 0.05, and the range varies from a minimum of 0.9 in 1985 to a maximum of 1.08 in 1994.

28 The standard deviation for $k\text{-}r_f$ from Table II is 0.33, for $k\text{-}r_f$ for the synthetic market is 0.69, and for $k^*\text{-}r_f$ from Table II is .1.29.

period. Although these results suggest very strongly that the five-year earnings growth rates should not be used in perpetuity, it is still possible that the forecasts were efficient ex ante, but just turned out to be optimistic ex post. If so, the ex ante dividend growth approach that has typically extrapolated that growth rate to perpetuity would be reasonable, and the k^* values we estimate would be unbiased. To examine this possibility, we repeated the entire analysis based on the assumption that the forecasted five-year growth rates hold for dividends in perpetuity; i.e. the assumptions underlying k^*. Our objective is to determine if there are clues that indicate that these growth assumptions are too high, even in an ex ante sense.

The projected dividend streams underlying the dividend growth model for each year were converted to future earnings, book values, and abnormal earnings, for years +1 through +15, and the various diagnostic indicators reported for Tables III and IV were computed for those future years. The results are summarized in Table VI. Rather than report results for all years, we only report values from the beginning (year +1), the middle (year +8), and the end (year +15) of the projections. For the reason indicated in footnote 22, we are unable to compute abnormal earnings growth for year +1,and report instead the year+2 numbers for that diagnostic.

The behavior of future P/E ratios (reported in columns 1, 2, and 3) appears to be reasonable. The P/E ratios decline gradually over the horizon towards the value of $1/k^*$, corresponding to the discount rates implied by the assumed growth rates (see k^* values reported in column 13). However, the patterns observed for abnormal earnings suggest that the assumed growth rates are too high. Generally, the earnings numbers are less than the normal earnings required by the discount rate k^*, (=bv_{t-1} x k^*,) in the early years of the horizon, which causes negative abnormal earnings. The abnormal earnings become more negative initially, resulting in the negative growth numbers reported in column 4 for year +2. Subsequently, the abnormal earning become less negative and then slightly positive by the middle of the horizon, which results in the very large positive growth numbers reported in column 5 (for year +8). Thereafter, the abnormal earnings continue to grow at the unusually high rates reported in column 6 (for year +15).

The trends for the P/B ratio (reported in columns 7 to 9) and the corresponding profitability numbers, *roe*, (reported in columns 10 to 12) confirm that the assumed growth rates are too high. Unlike the results in Table III, where P/B ratios decline over the horizon, here this ratio continues to increase, for every year in the sample. This unexpected trend is mirrored by the continued increase in profitability over the horizon. While the values of *roe* in year +1 are generally below the expected return, k^*, they exceed the cost of capital by year +8 and continue to increase thereafter. Such an increasing trend in profitability at the market level is without precedent and runs counter to intuition.

It is, however, the trend one would expect if the assumed growth rate in earnings is unreasonably high. These high growth rates result in overstated discount rates (cost of equity capital). The profitability in the early part of the horizon is below this overstated discount rate, which results in negative abnormal earnings. Over time, however, the very high growth rates result in profitability numbers that begin to exceed even the overstated discount rate.

The unusually high levels of profitability and growth in profitability by year +15, in combination with projected P/B ratios that continue to increase over the horizon confirm that the market's expectations for growth could not be as high as the 12 percent growth rates underlying our dividend growth model estimates. While the market's expectations could potentially have been that high during a portion of the sample period, it is highly unlikely that the market would continue to overestimate growth rates in every year of the sample period.

Overall, our analysis suggests strongly that dividend growth rates in perpetuity (g) in the neighborhood of 12 percent, obtained from analysts' five-year earnings growth forecasts (g'), are simply too high. Our analysis also suggests that our assumed growth rates for abnormal earnings beyond year +5 ($g''=r_f - 3\%$), while still slightly optimistic, are much more reasonable. Since our estimates of the risk premium are relatively insensitive to variation in assumed values of g'', because of the large proportion of total value that is captured by other terms in equation (4), we are reasonably confident that the true risk premium is not much higher than our estimates.

E. Value Line forecasts

Another source of earnings forecasts that are often used by researchers is Value Line. Value Line forecasts are unique in that they provide a terminal end-of-horizon price in addition to earnings and dividends forecasts for the years in between. To the extent that Value Line does not expect stocks to be systematically overpriced or underpriced at any point in time, one can infer the implied discount rate that equates current market value to the present value of future dividends and forecasted terminal value (by adapting equation (2) to collapse all dividends beyond year +5 into the fifth year price). Unfortunately, Value Line forecasts are not available to us in machine-readable form and need to be collected by hand. Rather than collect Value Line forecast data for all firms for all years, we focus on a smaller subset. We selected a representative set of the largest firms (all firms in the Dow Jones Index) as of April for one year each from the beginning and end of our sample period (1985 and 1995).

Our Value Line results differ considerably from those observed for the IBES sample: the implied market discount rates for the Value Line sample are 20% for 1985 and 8.5% for 1995, relative to 14.42% and 11.02% in Table II for the IBES sample.[29] Our analysis indicates that the 1985 discount rate is too high and the 1995 discount rate is too low (based on various implications that can be derived from those rates). We believe that the 1985 Value Line estimate of 20% is too high for many of the reasons that were uncovered for the high values of k* discussed in section IV.D. The implied long-term growth in dividends is 12.5 percent, which is probably too high for the reasons mentioned earlier. The P/B ratio increases steadily from a level of 1.5 in year 0 (April, 1985) to 3.0 by year +15, and the corresponding *roe* values increase from 16.4 percent to 20.21 percent by year +15. Neither trend is reasonable. Also, because of

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[29] The results for the Value Line sample do not vary greatly when the dividend growth approach, described in equation (1), is replaced by the abnormal earnings approach, described in equation (4), to estimate the market discount rate. The 1985 rate remains unchanged at 20 percent and the 1995 rate increases from 8.5 percent to 9.8 percent. This implies that the earnings, dividends, book value and price forecasts from Value Line are internally consistent. Botosan (1997) estimates firm-specific discount rates as of 1990 using the abnormal earnings approach on Value Line estimates and finds results similar to those observed for our 1985 Value Line sample: the mean and median discount rate for her sample of 122 manufacturing firms was 20.1 percent and 19 percent.

the high discount rates, all abnormal earnings for years +1 through +5 are negative. Negative abnormal earnings are not expected since the price-to-book value is greater than 1 (=1.4) for this sample.[30]

We believe the 1995 Value Line market discount rate estimate of 8.5 percent is too low because it implies a negative abnormal earnings growth rate for years after year +6. As described in section III, a negative growth in abnormal earnings is not normally expected in the presence of conservative accounting (the 1995 P/B ratio of 4.1 for our Value Line sample suggests a large degree of accounting conservatism). The constant dividend growth rate implied for the long-term (after year 5) is only 5 percent, which appears to be lower than any rate suggested in the literature. Also, the market discount rate of 8.5 percent represents a risk premium of only 1.36 percent, relative to the risk-free rate of 7.14 percent. Not only is this a low risk premium in absolute terms, it is much lower than the risk premium of 8.36 percent estimated for 1985 (20 percent less the risk-free rate of 11.64 percent). This much variation in implied risk premia over a ten-year period is not easily explained.

Overall, we are considerably less comfortable with our estimates of risk premium from Value Line forecasts, relative to our estimates from IBES data. The risk premium estimates vary too much, and the estimated discount rates appear unreasonably high (low) in 1985 (1995). An alternative explanation that provides a rationale for the Value Line forecasts is that they believed these stocks were mispriced.. Specifically, if stocks were systematically underpriced in 1985 and systematically overpriced in 1995, then replacing actual prices with higher (lower) “correct” prices in 1985 (1995) would alter the estimated risk premia towards more consistent values. Unfortunately, we are not able to confirm this explanation. Examination of the details in the text

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30 As described in equation (4), when market values exceed book values, the present value of all abnormal earnings must be positive (and equal to the excess of market value over book). If the first five abnormal earnings are negative, it is not possible to generate a positive value for the terminal abnormal earnings using a steady growth in abnormal earnings (g”) on a negative year +5 abnormal earnings; a more complex pattern of abnormal earnings growth is required in the post-horizon period.

and other indicators provided by Value Line for these firms did not suggest that Value Line expected systematic mispricing in either of the two years for the sample firms.

F. International IBES forecasts

Claus and Thomas (1999) repeat the same analysis conducted here for stock market aggregates for five other countries: Canada, France, Germany, Japan, and the UK. As explained in the Appendix, the abnormal earnings approach does not require that accounting procedures be similar across countries. As long as the earnings forecasts are based on “expected clean-surplus” and have the same basis as reported earnings in the respective countries, the abnormal earnings approach is valid. The Claus and Thomas risk premium estimates are quite similar to those reported here: the mean risk premia lie between 2 and 3 percent for all countries, except for Japan, for which the estimated premium is even lower (0.25 percent).. These results are remarkable because the relevant characteristics of the different countries (earnings growth estimates, price-to-book ratios, levels of and growth in profitability, etc.) exhibit considerable variation, and yet the risk premium estimates are fairly similar. As with the US results, these estimates exhibit much less time-series variability than the corresponding risk premia estimates obtained using the dividend growth model (k^*).

In the absence of market imperfections and segmented markets, one would expect the risk premia to be similar across countries. Of course, differences in asset risk across the different country aggregates would cause differences in the individual country risk premia. However, estimation of country index betas (slope of the regression of monthly country index returns on the Morgan Stanley world index) suggests that the differences in beta risk across country indices are relatively small.[31] Observing similar estimates of the risk premium in different countries increases the likelihood that our estimates are reasonable. In combination with the time-series stationarity exhibited by the international and domestic estimates, the similarity in risk premia

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31 The country betas we estimated are as follows: Canada=0.82; France=1.01; Germany=0.89; Japan=1.32; US=0.79; UK=0.99.

across countries is a result that is intuitively appealing and increases the likelihood that our domestic risk premium estimates are correct.

V. Conclusion

There is general acceptance among academic financial economists that the equity risk premium is 7 percent or more relative to the long-term risk-free rate, and another 200 basis points higher relative to the short-term risk-free rate. This view is based for the most part on the past performance of the US stock market. We claim that these estimates are too high, at least for the 1985-1998 period, and that the risk premium implied by market prices is only half as much, or less. Our claim is based on ex ante estimates of the risk premium, derived by estimating the discount rate that equates current prices to forecasts of future dividends (earnings) obtained from IBES. Dividend growth rates need to be about 12 percent in perpetuity to support risk premium estimates of about 7 percent. Not only are such growth rates substantially in excess of any reasonable forecasts of aggregate growth (e.g. GDP), as pointed out by Malkiel (1996), the projected streams for various indicators, such as price-to-book ratios and profitability levels, are internally contradictory or inconsistent with intuition and past experience.

While it is true that IBES analysts predict near-term (five-year ahead) earnings growth forecasts that are typically in the neighborhood of 12 percent, we show that these forecasts are systematically optimistic relative to actual earnings observed in our sample period. As a result, prior studies that have projected analyst 5-year growth rates to perpetuity generate risk premium estimates that are also too high.

There is no question that the historical performance of the US stock market suggests that our estimates are too low. If we are right, it is probably the case that the historical performance is either a statistical aberration (an unusual stretch of good luck) or caused by some form of survivor bias. In support of our view, we find that our estimates are remarkably stationary over the 12 years we study, and also remarkably similar to the ex ante estimates obtained for four other countries by Claus and Thomas (1999). Despite substantial differences (time-series and cross-country) in the forecast growth rates and other value-relevant items, the risk premium appears to be relatively constant.

Given the stationarity of our estimates of the risk premium, we are tempted to conclude that the recent extended bull stock market is not irrational: it is completely supported by improved consensus analyst forecasts of future earnings and a general decline in risk-free rates. Whether or not those earnings forecasts will turn out to be more optimistic than in prior years is an empirical question, but there is no evidence in our results which suggests that the market is accepting a lower risk premium in 1996, relative to the risk premium it demanded in 1985. [32]

Although the historical estimate of the risk premium is widely accepted among academics, there is less evidence of similar unanimity among practitioners. For example, there is some evidence of influential investors expressing in survey evidence that they demand a risk premium that is in the neighborhood of our estimates (see Benore, 1983, for example). Perhaps, other evidence can be brought to bear on the expectations of market participants.

There is little doubt about the importance of obtaining unbiased estimates of the equity risk premium. It has important implications for a variety of financial decisions as well as academic studies. We believe we have generated sufficient evidence to cause a reevaluation of this question.

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[32] There have been some suggestions that the publicity given to the evidence that stock portfolios held for longer periods almost always beat bond portfolios held over the same periods could have created a reduction in perceived risk of equity, and lowered the cost of equity in recent years.

Table I

Actual and forecast numbers for market earnings, book value, dividends, and prices

The market consists of firms on the IBES Summary files with forecasts for years +1, +2, and a 5-year earnings growth estimate (g') as of April each year, and actual earnings per share, dividends per share, number of shares outstanding and share prices as of the end of the prior fiscal year (year 0). Book values of equity for year 0 are obtained from Compustat. Forecasted earnings per share for years +3, +4 and +5 are determined by applying g', the forecasted 5-yeat growth rate, to year +2 forecasted earnings. Per share numbers are multiplied by the number of shares to get numbers at the firm level, and these are added across firms to get numbers at the market level each year. All amounts, except for dividend payout, are in millions of dollars.

Forecast as of	# of	Actual Values for Year 0					Forecast Earnings for Years +1 to +5				
April	firms	Earnings	Dividends	Payout	Book value	Market value	Year +1	Year +2	Year +3	Year +4	Year +5
column #	1	2	3	4	5	6	7	8	9	10	11
1985	1,559	154,858	71,134	46%	1,191,869	1,747,133	180,945	205,294	228,208	254,181	283,706
1986	1,613	155,201	73,857	48%	1,214,454	2,284,245	178,024	203,677	226,018	251,313	280,035
1987	1,774	146,277	81,250	56%	1,323,899	2,640,743	186,319	220,178	244,174	271,432	302,529
1988	1,735	167,676	86,237	51%	1,430,672	2,615,857	222,497	246,347	273,204	303,642	338,262
1989	1,809	229,070	97,814	43%	1,541,231	2,858,585	261,278	284,616	315,204	349,721	388,776
1990	1,889	228,216	107,316	47%	1,636,069	3,143,879	257,657	295,321	328,803	366,798	410,028
1991	1,939	218,699	108,786	50%	1,775,199	3,660,296	241,760	294,262	328,513	367,521	412,073
1992	2,106	202,275	113,962	56%	1,911,383	4,001,756	252,109	308,567	344,742	386,098	433,552
1993	2,386	247,988	127,440	51%	2,140,668	4,918,359	295,862	356,086	397,969	445,840	501,081
1994	2,784	290,081	129,186	45%	2,168,446	5,282,046	339,694	402,689	450,559	505,315	568,179
1995	2,965	365,079	147,575	40%	2,670,725	6,289,760	444,593	518,600	579,954	650,120	730,648
1996	3,360	446,663	175,623	39%	3,182,952	8,207,274	512,921	588,001	659,732	742,244	837,577
1997	3,797	547,395	201,017	37%	3,679,110	10,198,036	614,932	709,087	800,129	905,787	1,029,061
1998	3,673	526,080	178,896	34%	3,412,303	12,908,495	577,297	682,524	775,707	884,529	1,012,294

Table II
Implied expected rate of return on the market (k) and equity risk premium (k-r_f)

The market is an aggregate of firms on the IBES Summary files with forecasts for years +1, +2, and a 5-year earnings growth estimate (g') as of April each year, and actual earnings, dividends, number of shares outstanding and prices as of the end of the prior full fiscal year (year 0). Book values of equity for year 0 (bv_0)are obtained from Compustat. Forecasted earnings for years +3, +4 and +5 are determined by applying g', the forecasted 5-year growth rate, to year +2 forecasted earnings. k is the expected market rate of return, or the implied discount rate, that satisfies the valuation relation in equation (4) below. Abnormal earnings (ae_t) equal reported earnings less a charge for the cost of equity (=beginning book value of equity * k). Assuming that 50% of earnings are retained allows the estimation of future book values from current book values and forecast earnings. The terminal value represents all abnormal earnings beyond year 5. Those abnormal earnings are assumed to grow at a constant rate g'', which is assumed to equal the real risk free rate, and is set equal to the current 10-year risk-free rate less 3%. The expected rate of return on the market is also estimated using equation (1), and is labelled k^*. Equation (1) is derived from the dividend growth model, and dividend growth in perpetuity, g, is assumed to equal the 5-year earnings growth rate, g'. All amounts, except for rates of return, are in millions of dollars.

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \frac{ae_4}{(1+k)^4} + \frac{ae_5}{(1+k)^5} + \left[\frac{ae_5(1+g'')}{(k-g'')(1+k)^5}\right] \qquad (4)$$

$$k^* = \frac{d_1}{p_0} + g \qquad (1)$$

Forecasts as of April	Market value	Book value	PVabnormal earnings +1	Pvabnormal earnings +2	PVabnormal earnings +3	PVabnormal earnings +4	PVabnormal earnings +5	PVterminal value	10-year r_f	k from (4)	k^* from (1)	k-r_f
column #	1	2	3	4	5	6	7	8	9	10	11	12
1985	1,747,133	1,191,869	8,353	15,970	19,411	22,559	25,469	464,136	11.43%	14.38%	16.14%	2.95%
1986	2,284,245	1,214,454	36,874	45,744	48,984	52,201	55,435	828,345	7.30%	11.28%	14.90%	3.98%
1987	2,640,743	1,323,899	35,189	50,699	54,192	57,743	61,412	1,057,289	8.02%	11.12%	15.08%	3.10%
1988	2,615,857	1,430,672	43,398	46,911	50,259	53,564	56,877	933,609	8.71%	12.15%	15.52%	3.44%
1989	2,858,585	1,541,231	57,447	56,207	58,532	60,838	63,156	1,020,687	9.18%	12.75%	14.85%	3.57%
1990	3,143,879	1,636,069	49,791	61,586	65,603	69,534	73,430	1,187,789	8.79%	12.33%	15.41%	3.54%
1991	3,660,296	1,775,199	41,063	68,719	75,020	81,270	87,540	1,529,982	8.04%	11.05%	15.16%	3.01%
1992	4,001,756	1,911,383	45,289	76,241	83,650	91,132	98,787	1,694,789	7.48%	10.57%	15.55%	3.09%
1993	4,918,359	2,140,668	82,037	113,113	121,980	131,171	141,010	2,183,434	5.97%	9.62%	15.12%	3.65%
1994	5,282,046	2,168,446	101,980	129,363	136,974	144,921	153,317	2,452,364	6.97%	10.47%	15.02%	3.50%
1995	6,289,760	2,670,725	135,110	161,831	169,683	177,951	186,749	2,788,101	7.06%	11.03%	14.96%	3.97%
1996	8,207,274	3,182,952	178,155	202,987	216,527	230,881	246,277	3,952,265	6.51%	9.96%	14.96%	3.45%
1997	10,198,036	3,679,110	220,311	252,050	270,195	289,684	310,885	5,184,242	6.89%	10.12%	13.88%	3.23%
1998	12,908,495	3,412,303	276,647	325,652	352,789	382,642	415,799	7,745,477	5.64%	8.15%	13.21%	2.51%
Mean									7.71%	11.07%	14.98%	3.36%

Table III

Price-to-book ratios (p_t/bv_t), forecast accounting return on equity (roe_t) and expected rates of return (k)

To examine the validity of assumptions underlying k, which is the expected market rate of return or the implied discount rate that satisfies the valuation relation in equation (4), current price-to-book ratios are compared with estimated future returns on equity (roe_t) to examine fit with equation (A15) below. The market is an aggregate of firms on the IBES Summary files with forecasts for years +1, +2, and a 5-year earnings growth estimate as of April each year, and actual earnings, dividends, number of shares outstanding and prices as of the end of the prior full fiscal year (year 0). Book values of equity for year 0 (bv_0)are obtained from Compustat. Return on equity (roe_t) equals forecast earnings scaled by beginning book value of equity (bv_{t-1}). See tables 1 &2 for procedures used to generate future earnings and book values. Market and book value amounts are in millions of dollars.

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \frac{ae_4}{(1+k)^4} + \frac{ae_5}{(1+k)^5} + \left[\frac{ae_5(1+g")}{(k-g")(1+k)^5}\right] \qquad (4)$$

$$\frac{p_0}{bv_0} = 1 + \frac{roe_1\text{-}k}{(1+k)} + \frac{roe_2\text{-}k}{(1+k)^2}\left(\frac{bv_1}{bv_0}\right) + \ldots \qquad (A15)$$

	Year 0 equity values		Year 5 equity values		Price/book ratio		Forecast accounting return on equity						k
Forecasts as of April	Market value (p_0)	Book value (bv_0)	Market value (p_5)	Book value (bv_5)	in year 0 (p_0/bv_0)	in year 5 (p_5/bv_5)	in yr 1 (roe_1)	in yr 2 (roe_2)	in yr 3 (roe_3)	in yr 4 (roe_4)	in yr 5 (roe_5)	in yr 6 (roe_6)	from eq. (4)
column #	1	2	3	4	5	6	7	8	9	10	11	12	13
1985	1,747,133	1,191,869	2,676,683	1,768,036	1.5	1.5	15%	16%	16%	17%	17%	17%	14.38%
1986	2,284,245	1,214,454	3,197,490	1,783,987	1.9	1.8	15%	16%	16%	17%	17%	17%	11.28%
1987	2,640,743	1,323,899	3,727,459	1,936,215	2.0	1.9	14%	16%	16%	16%	17%	17%	11.12%
1988	2,615,857	1,430,672	3,779,033	2,122,648	1.8	1.8	16%	16%	16%	17%	17%	17%	12.15%
1989	2,858,585	1,541,231	4,200,867	2,341,029	1.9	1.8	17%	17%	17%	18%	18%	18%	12.75%
1990	3,143,879	1,636,069	4,589,685	2,465,373	1.9	1.9	16%	17%	17%	18%	18%	18%	12.33%
1991	3,660,296	1,775,199	5,181,184	2,597,264	2.1	2.0	14%	16%	16%	17%	17%	17%	11.05%
1992	4,001,756	1,911,383	5,574,848	2,773,918	2.1	2.0	13%	15%	16%	16%	17%	17%	10.57%
1993	4,918,359	2,140,668	6,595,210	3,139,088	2.3	2.1	14%	16%	16%	17%	17%	17%	9.62%
1994	5,282,046	2,168,446	7,336,322	3,301,664	2.4	2.2	16%	17%	18%	18%	19%	18%	10.47%
1995	6,289,760	2,670,725	8,837,148	4,132,682	2.4	2.1	17%	18%	18%	19%	19%	19%	11.03%
1996	8,207,274	3,182,952	11,206,787	4,853,189	2.6	2.3	16%	17%	18%	18%	19%	18%	9.96%
1997	10,198,036	3,679,110	14,103,523	5,708,609	2.8	2.5	17%	18%	18%	19%	20%	19%	10.12%
1998	12,908,495	3,412,303	16,838,377	5,378,478	3.8	3.1	17%	18%	19%	20%	21%	20%	8.15%
Mean					2.2	2.1	15%	17%	17%	18%	18%	18%	11.07%

Table IV
Forward price-to-earnings ratios (p_t/e_{t+1}) and growth in forecast abnormal earnings and earnings.

To examine the validity of assumptions underlying k, which is the expected market rate of return or the implied discount rate that satisfies the valuation relation in equation (4), current and forecast forward price-to-earnings ratios are compared with growth in forecast abnormal earnings to examine fit with equation (A16) below. The market is an aggregate of firms on the IBES Summary files with forecasts for years +1, +2, and a 5-year earnings growth estimate as of April each year, and actual earnings, dividends, number of shares outstanding and prices as of the end of the prior full fiscal year (year 0). Book values of equity for year 0 (bv_0)are obtained from Compustat. Abnormal earnings (ae_t) equal reported earnings less a charge for the cost of equity (=beginning book value of equity * k). See tables 1 &2 for procedures used to generate future earnings and book values. Market equity values and earnings amounts are in millions of dollars.

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \frac{ae_4}{(1+k)^4} + \frac{ae_5}{(1+k)^5} + \left[\frac{ae_5(1+g'')}{(k-g'')(1+k)^5}\right] \qquad (4)$$

$$\frac{p_0}{e_1} = \frac{1}{k}\left[1 + \frac{\Delta ae_2}{e_1(1+k)} + \frac{\Delta ae_3}{e_1(1+k)^2} + \ldots\right] \qquad (A16)$$

	Year 0 values		Year 5 values		Forward P/E ratio		growth in forecast abnormal earnings					growth in forecast earnings						*1/k*
Forecasts as of April	Market value (p_0)	Earnings (e_1)	Market value (p_5)	Earnings (e_6)	in year 0 (p_0/e_1)	in year 5 (p_5/e_6)	+2	+3	+4	+5	+6	+1	+2	+3	+4	+5	+6	from eq.(4)
column #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1985	1,747,133	180,945	2,676,683	308,308	9.7	8.7	119%	39%	33%	29%	8%	17%	13%	11%	11%	12%	9%	7
1986	2,284,245	178,024	3,197,490	299,896	12.8	10.7	38%	19%	19%	18%	4%	15%	14%	11%	11%	11%	7%	9
1987	2,640,743	186,319	3,727,459	324,573	14.2	11.5	60%	19%	18%	18%	5%	27%	18%	11%	11%	11%	7%	9
1988	2,615,857	222,497	3,779,033	364,573	11.8	10.4	21%	20%	20%	19%	6%	33%	11%	11%	11%	11%	8%	8
1989	2,858,585	261,278	4,200,867	420,673	10.9	10.0	10%	17%	17%	17%	6%	14%	9%	11%	11%	11%	8%	8
1990	3,143,879	257,657	4,589,685	442,911	12.2	10.4	39%	20%	19%	19%	6%	13%	15%	11%	12%	12%	8%	8
1991	3,660,296	241,760	5,181,184	442,291	15.1	11.7	86%	21%	20%	20%	5%	11%	22%	12%	12%	12%	7%	9
1992	4,001,756	252,109	5,574,848	463,780	15.9	12.0	86%	21%	20%	20%	4%	25%	22%	12%	12%	12%	7%	9
1993	4,918,359	295,862	6,595,210	531,812	16.6	12.4	51%	18%	18%	18%	3%	19%	20%	12%	12%	12%	6%	10
1994	5,282,046	339,694	7,336,322	607,937	15.5	12.1	40%	17%	17%	17%	4%	17%	19%	12%	12%	12%	7%	10
1995	6,289,760	444,593	8,837,148	783,736	14.1	11.3	33%	16%	16%	17%	4%	22%	17%	12%	12%	12%	7%	9
1996	8,207,274	512,921	11,206,787	893,185	16.0	12.5	25%	17%	17%	17%	4%	15%	15%	12%	13%	13%	7%	10
1997	10,198,036	614,932	14,103,523	1,100,714	16.6	12.8	26%	18%	18%	18%	4%	12%	15%	13%	13%	14%	7%	10
1998	12,908,495	577,297	16,838,377	1,069,786	22.4	15.7	27%	17%	17%	18%	3%	10%	18%	14%	14%	14%	6%	12
Mean					14.6	11.6	47%	20%	19%	19%	5%	18%	16%	12%	12%	12%	7%	9

Table V

Optimism bias in IBES forecasts: median forecast errors for forecasts made between 1985 and 1995

The following table represents the median of all forecast errors scaled by share price for each year examined. The forecast error is calculated for each firm as of April each year, and equals the median consensus forecasted earnings per share minus the actual earnings per share, scaled by price. The year when the forecasts were made is listed in the first row, while the first column lists the horizon of that forecast. For each year and horizon combination, we report the median forecast error and the number of firms in the sample. To interpret the Table, consider the values of 0.78 percent and 1,680 reported for the +1/1985 combination., in the top left-hand corner of the table. This means that the median value of the difference between the forecasted and actual earnings for 1986 was 0.78 percent of price, and that sample consisted of 1,680 firms with available forecast errors. The results confirm that analyst forecasts are systematically positively biased and that this bias increases with the forecast horizon; however, the extent of any such bias has been steadily declining over time.

		Year forecast was made													
		1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	mean
Forecast Year +1	Median	0.78%	0.65%	0.37%	0.07%	0.44%	0.58%	0.39%	0.17%	0.15%	0.03%	0.04%	0.00%	0.00%	0.28%
	Obs.	1,680	1,707	1,878	1,815	1,868	1,932	1,959	2,176	2,492	2,710	2,895	3,261	3,462	
Forecast Year +2	Median	2.05%	1.40%	0.79%	0.99%	1.74%	1.88%	1.21%	0.87%	0.58%	0.34%	0.32%	0.27%	.	1.04%
	Obs.	1,545	1,572	1,732	1,701	1,757	1,815	1,896	2,084	2,287	2,594	2,694	2,852	.	
Forecast Year +3	Median	2.84%	0.99%	1.44%	2.22%	2.78%	2.39%	1.50%	0.95%	0.63%	0.54%	0.45%	.	.	1.52%
	Obs.	1,406	1,449	1,596	1,576	1,634	1,744	1,826	1,936	2,159	2,396	2,346	.	.	
Forecast Year +4	Median	2.63%	2.04%	2.80%	3.19%	3.17%	2.83%	1.54%	0.91%	0.77%	0.60%	.	.	.	2.05%
	Obs.	1,285	1,344	1,492	1,474	1,586	1,696	1,724	1,825	2,024	2,132	.	.	.	
Forecast Year +5	Median	3.54%	3.44%	3.86%	3.59%	3.43%	2.91%	1.36%	0.94%	0.74%	.	.	.	.	2.65%
	Obs.	1,201	1,260	1,411	1,432	1,528	1,621	1,618	1,704	1,815	.	.	.	.	

Table VI

Projections of market fundamentals for the case where dividends grow in perpetuity at analysts' 5-year earnings growth forecasts

To examine the validity of assumptions underlying k^*, which is the expected market rate of return or the implied discount rate that satisfies the valuation relation in equation (1), where g is assumed to equal the 5-year earnings growth rates forecast by analysts, projections are made of market fundamentals for 15 years out. The four fundamentals examined are P/E ratios, growth in abnormal earnings (Dae_t), P/B ratios and accounting rate of return (roe_t). The relation between P/B ratios and roe_t is given by equation (A15), and the relation between P/E ratios and Dae_t is given by equation (A16). The market is an aggregate of firms on the IBES Summary files with forecasts for years +1, +2, and a 5-year earnings growth estimate as of April each year, and actual earnings, dividends, number of shares outstanding and prices as of the end of the prior full fiscal year (year 0). Book values of equity for year 0 (bv_0)are obtained from Compustat. Abnormal earnings (ae_t) equal earnings (e_t) less a charge for the cost of equity ($=bv_{t-1}$ x k^*).

$$k^* = \frac{d_1}{p_0} + g \qquad (1)$$

$$\frac{p_t}{bv_t} = 1 + \frac{roe_{t+1} - k^*}{1+k^*} + \frac{roe_{t+2} - k^*}{\left(1+k^*\right)^2}\left(\frac{bv_{t+1}}{bv_t}\right) + \ldots \qquad \text{(A15)}$$

$$\frac{p_t}{e_{t+1}} = \frac{1}{k^*}\left[1 + \frac{\Delta ae_{t+2}}{e_{t+1}(1+k^*)} + \frac{\Delta ae_{t+3}}{e_{t+1}\left(1+k^*\right)^2} + \ldots\right] \qquad \text{(A16)}$$

Forecasts	forward P/E ratios (p_t/e_{t+1})			% growth in abnormal earnings			P/B ratios (p_t/b_t)			return on equity (e_t/bv_{t-1})			exp. rate of
as of April	Year +1	Year +8	Year +15	Year +2	Year +8	Year +15	Year +1	Year +8	Year +15	Year +1	Year +8	Year +15	return (k^*)
column	1	2	3	4	5	6	7	8	9	10	11	12	13
1985	9.66	9.49	9.28	-12.5%	5902.5%	21.6%	1.5	1.7	1.8	15.2%	18.0%	19.7%	16.1%
1986	12.83	12.62	12.36	-28.8%	42.3%	17.4%	1.9	2.3	2.5	14.7%	17.7%	19.7%	14.9%
1987	14.17	13.88	13.52	-28.7%	42.4%	17.4%	2.1	2.6	3.0	14.1%	18.6%	21.8%	15.1%
1988	11.76	11.54	11.27	-41.2%	34.4%	16.6%	1.9	2.2	2.4	15.6%	18.9%	21.1%	15.5%
1989	10.94	10.78	10.57	44.1%	18.5%	13.8%	1.9	2.0	2.0	17.0%	18.2%	18.8%	14.9%
1990	12.20	12.01	11.76	-44.3%	33.6%	16.8%	2.0	2.3	2.4	15.7%	18.7%	20.5%	15.4%
1991	15.14	14.89	14.58	-18.9%	69.7%	19.3%	2.2	2.7	3.1	13.6%	17.9%	20.9%	15.2%
1992	15.87	15.56	15.17	-18.9%	69.2%	19.4%	2.2	3.0	3.5	13.2%	18.7%	23.0%	15.5%
1993	16.62	16.22	15.74	-24.7%	48.7%	18.5%	2.4	3.1	3.5	13.8%	18.6%	21.9%	15.1%
1994	15.55	15.25	14.90	-73.4%	29.5%	16.7%	2.5	2.9	3.1	15.7%	18.8%	20.7%	15.0%
1995	14.15	13.88	13.56	190.1%	23.1%	15.6%	2.4	2.6	2.7	16.6%	18.7%	19.8%	15.0%
1996	16.00	15.65	15.24	-133.8%	27.4%	16.6%	2.6	2.9	3.1	16.1%	18.7%	20.0%	15.0%
1997	16.58	16.15	15.67	-125.5%	28.1%	17.2%	2.8	3.1	3.2	16.7%	19.2%	20.5%	15.4%
1998	22.36	21.78	21.15	-124.3%	28.7%	17.9%	3.9	4.3	4.4	16.9%	19.6%	21.0%	15.5%

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Appendix

This appendix provides additional details of the valuation relations used in the paper. Our aim is to help readers unfamiliar with the literature walk through the derivations from first principles. (See Lee, 1996, for a related effort.) We have culled these relations over time from many different papers, teaching notes, and other materials. The link between abnormal earnings and economic value (stock price) has been derived independently by a number of economists and accountants over the years. Preinreich (1938) is the first cite, to our knowledge. Edwards and Bell (1961) and Peasnell (1982) are some of the later cites. Other relations, such as those describing the price-to-book and price-to-earnings ratios, appear to have been developed more recently, and have been described extensively in work conducted by Steve Penman.

Based on the classic dividend valuation approach, current stock price can be set equal to the present value of future expected dividends.

$$p_0 = \frac{d_1}{(1+k_1)} + \frac{d_2}{(1+k_1)(1+k_2)} + \frac{d_3}{(1+k_1)(1+k_2)(1+k_3)} + \ldots \qquad (A1)$$

where
p_0 = current price,
d_t = dividends expected at the end of year t, and
k_t = discount rate expected for year t.[1]

Next, replace dividends by earnings and changes in equity book value using the "expected clean surplus" assumption; i.e., the change in book value of equity over year t is expected to equal earnings in year t less dividends in year t. Dividends equal cash dividends plus stock repurchases less stock issues.

$$bv_t = bv_{t-1} + e_t - d_t \qquad (A2)$$

where
bv_t = book value of equity at the end of year t, and

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1 Although in practice discount rates are usually assumed to remain constant over time for purposes of simplification, occasionally time-varying rates are used to capture effects such as those caused by expected recapitalizations (undoing a leveraged buy out) and a steeply-sloped term structure of interest rates.

e_t = earnings forecast for year t.

It is worth explaining the "clean-surplus" relation described by equation (A2): the change in book value during the year should equal earnings less net dividends (net of equity issues and repurchases). That is, except for transactions between the firm and stockholders (represented by net dividends), all other items affecting equity book values should pass through the income statement. Transactions relating to various reserve accounts that bypass the income statement would violate the clean-surplus assumption. Given that a few transactions do in fact bypass the income statement, how restrictive is the clean surplus relation?[2] We believe that the clean surplus relation is unlikely to be restrictive since it is sufficient that all such transactions that bypass the income statement have a net expected value of zero *for the future*. That is, the fact that such transactions have occurred in the past or are likely to occur in the future is not a problem as long as the analyst estimates do not in aggregate (across all firms) include in earnings a positive or negative amount for items that are likely to bypass the income statement.

Rearranging A2 allows dividends to be expressed as earnings less changes in book value

$$d_t = e_t - (bv_t - bv_{t-1}) \qquad (A3)$$

Substituting d_t values from A3 into A1 gives

$$p_0 = \frac{e_1 - (bv_1 - bv_0)}{(1+k_1)} + \frac{e_2 - (bv_2 - bv_1)}{(1+k_1)(1+k_2)} + \frac{e_3 - (bv_3 - bv_2)}{(1+k_1)(1+k_2)(1+k_3)} + \ldots \qquad (A4)$$

Adding and subtracting $k_1{*}bv_0$, $k_2{*}bv_1$, and so on, to each of the terms in the numerator, gives

$$p_0 = \frac{e_1-k_1bv_0+k_1bv_0+bv_0-bv_1}{(1+k_1)} + \frac{e_2-k_2bv_1+k_2bv_1+bv_1-bv_2}{(1+k_1)(1+k_2)} + \frac{e_3-k_3bv_2+k_3bv_2+bv_2-bv_3}{(1+k_1)(1+k_2)(1+k_3)} + \ldots (A5)$$

which can be expressed as

$$p_0 = \frac{e_1-k_1bv_0+bv_0(1+k_1)-bv_1}{(1+k_1)} + \frac{e_2-k_2bv_1+bv_1(1+k_2)-bv_2}{(1+k_1)(1+k_2)} + \frac{e_3-k_3bv_2+bv_2(1+k_3)-bv_3}{(1+k_1)(1+k_2)(1+k_3)} + \ldots (A6)$$

or

$$p_0 = \frac{e_1-k_1bv_0}{(1+k_1)} + \frac{bv_0(1+k_1)}{(1+k_1)} - \frac{bv_1}{(1+k_1)} + \frac{e_2-k_2bv_1}{(1+k_1)(1+k_2)} + \frac{bv_1(1+k_2)}{(1+k_1)(1+k_2)} - \frac{bv_2}{(1+k_1)(1+k_2)} +$$
$$\frac{e_3-k_3bv_2}{(1+k_1)(1+k_2)(1+k_3)} + \frac{bv_2(1+k_3)}{(1+k_1)(1+k_2)(1+k_3)} - \frac{bv_3}{(1+k_1)(1+k_2)(1+k_3)} + \ldots \qquad (A7)$$

[2] See Frankel and Lee (1997b) for additional discussion of the extent to which the clean surplus relation is violated under current accounting rules.

Simplifying, we get

$$p_0 = bv_0 + \frac{e_1\text{-}k_1bv_0}{(1+k_1)} + \frac{e_2\text{-}k_2bv_1}{(1+k_1)(1+k_2)} + \frac{e_3\text{-}k_3bv_2}{(1+k_1)(1+k_2)(1+k_3)} + \ldots \qquad (A8)$$

The numerators seek to proxy for economic earnings, or accounting earnings less a charge for the cost of equity. However, the numerators are only approximately equal to economic earnings because the charge for equity is based on book values of equity employed, rather than market values. This proxy for economic earnings is commonly called abnormal earnings, and is defined as

$$ae_t = e_t - k_tbv_{t-1} \qquad (A9)$$

Substituting ae_1, ae_2, and so on for the numerators in equation *(A8)* gives

$$p_0 = bv_0 + \frac{ae_1}{(1+k_1)} + \frac{ae_2}{(1+k_1)(1+k_2)} + \frac{ae_3}{(1+k_1)(1+k_2)(1+k_3)} + \ldots \qquad (A10)$$

In other words, the excess of current market values over current book values of equity are captured by the present value of future expected abnormal earnings.

The ratio of current market values to book values (or the price-to-book ratio) can be derived from equation *(A8)*, by multiplying and dividing each of the numerators by bv_0, bv_1, and so on. This gives

$$p_0 = bv_0 + \frac{bv_0(e_1/bv_0\text{-}k_1)}{(1+k_1)} + \frac{bv_1(e_2/bv_1\text{-}k_2)}{(1+k_1)(1+k_2)} + \frac{bv_2(e_3/bv_2\text{-}k_3)}{(1+k_1)(1+k_2)(1+k_3)} + \ldots \qquad (A11)$$

or

$$p_0 = bv_0 + \frac{roe_1\text{-}k_1}{(1+k_1)}bv_0 + \frac{roe_2\text{-}k_2}{(1+k_1)(1+k_2)}bv_1 + \frac{roe_3\text{-}k_3}{(1+k_1)(1+k_2)(1+k_3)}bv_2 + \ldots \qquad (A12)$$

where

$roe_t = \frac{e_t}{bv_{t-1}}$, is the accounting rate of return in year t on the book value of equity.

Dividing both sides of equation *(A12)* by bv_0 gives

$$\frac{p_0}{bv_0} = 1 + \frac{roe_1\text{-}k_1}{(1+k_1)} + \frac{roe_2\text{-}k_2}{(1+k_1)(1+k_2)}\left(\frac{bv_1}{bv_0}\right) + \frac{roe_3\text{-}k_3}{(1+k_1)(1+k_2)(1+k_3)}\left(\frac{bv_2}{bv_0}\right) + \ldots \qquad (A13)$$

Equation (A13) shows that the price to book ratio is a function of the excess of the accounting return on equity (*roe_t*) over the cost of equity capital (*k_t*) in each year times the growth in the book value of equity, from today to year t (*bv_t/bv_0*).

Often, discount rates are expected to remain constant over time (based on a flat term structure for risk-free rates and risk stationarity). In this case, equations *(A10)* and *(A13)* can be rewritten with $k=k_1=k_2=k_3$ and so on.

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \dots \qquad (A14)$$

$$\frac{p_0}{bv_0} = 1 + \frac{roe_1-k}{(1+k)} + \frac{roe_2-k}{(1+k)^2}\left(\frac{bv_1}{bv_0}\right) + \frac{roe_3-k}{(1+k)^3}\left(\frac{bv_2}{bv_0}\right) + \dots \qquad (A15)$$

For the special case of constant discount rates, it can be shown that the Price-Earnings ratio can be expressed as the following function of growth in abnormal earnings.

$$\frac{p_0}{e_1} = \frac{1}{k}\left[1 + \frac{\Delta ae_2}{e_1(1+k)} + \frac{\Delta ae_3}{e_1(1+k)^2} + \dots\right] \qquad (A16)$$

where

$Dae_t = ae_t - ae_{t-1}$,is the change in expected abnormal earnings over the prior year.

For purposes of implementation, the right-hand side of equation (A14) can be separated into two parts: a) the first part is based on data from years 0 through 5, years for which specific estimates are available and b) a terminal value that captures the abnormal earnings for all years past year 5 by assuming constant growth in perpetuity beyond year 5 (g''). Note that this terminal value, which captures the present value as of year 5 of all future abnormal earnings, is considerably less than the market value as of that date. In fact, this terminal value represents the difference between the market and book values of equity as of year 5.

$$p_0 = bv_0 + \frac{ae_1}{(1+k)} + \frac{ae_2}{(1+k)^2} + \frac{ae_3}{(1+k)^3} + \frac{ae_4}{(1+k)^4} + \frac{ae_5}{(1+k)^5} + \left[\frac{ae_5(1+g'')}{(k-g'')(1+k)^5}\right] \quad ¼(A17)$$

where

g''= growth in abnormal earnings after year 5.