Revenue Management

René Caldentey

Outline

I. A Brief History of RM

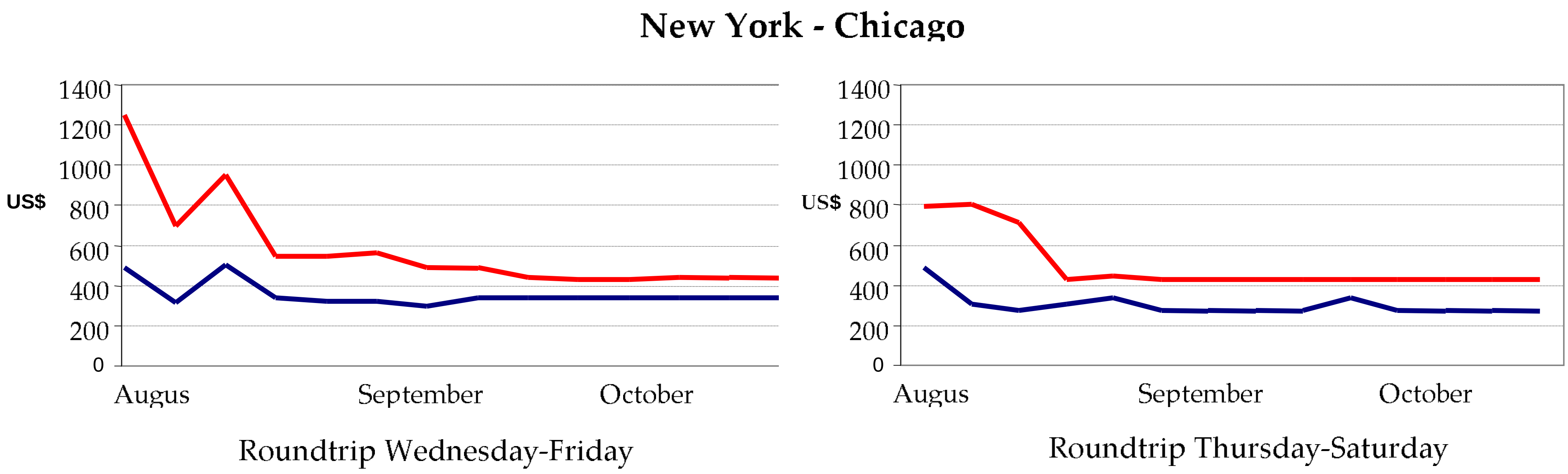
II. RM Framework

III. Components of a RM System

IV. Examples of Models and Methodology

V. Summary and Future Directions

...But Before an Example



A Wednesday-Friday ticket is 15%-20% more expensive than a Thursday-Saturday ticket!!

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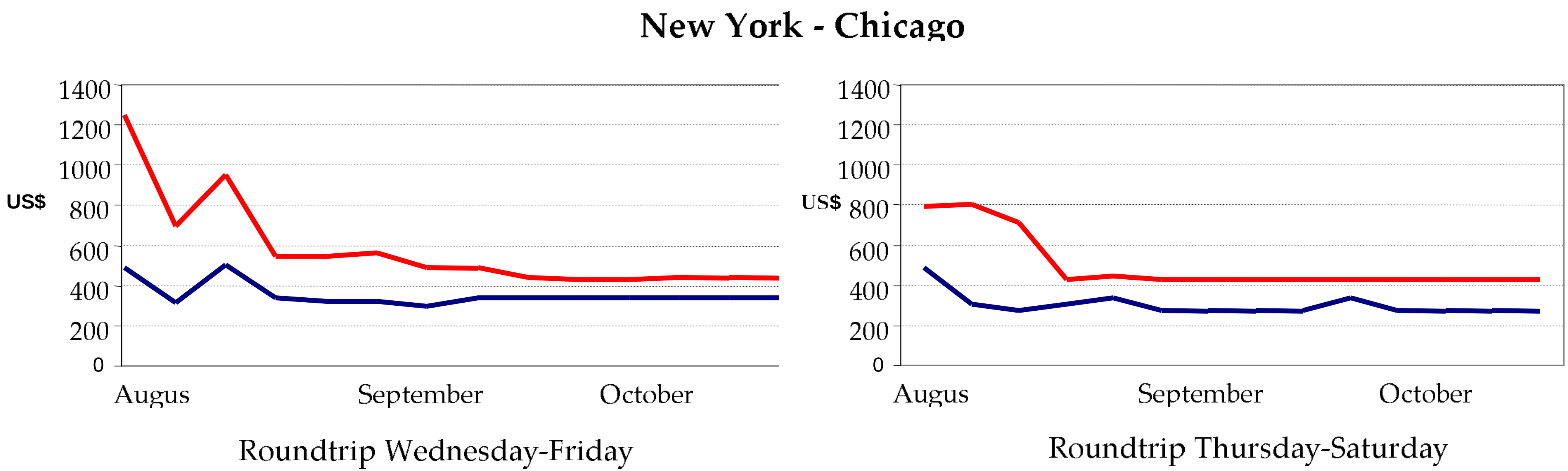

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Revenue Management:

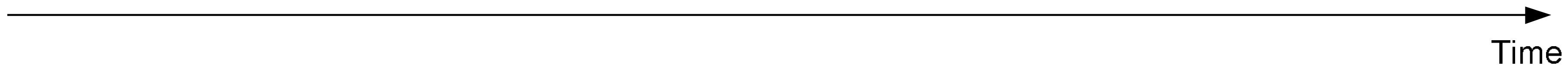
"Selling the Right product to the Right customer at the Right price."

...and a Warning

Coca-Cola: A Mayor Calor, Más Alto el Precio.

A Brief History

Airline Industry



A Brief History

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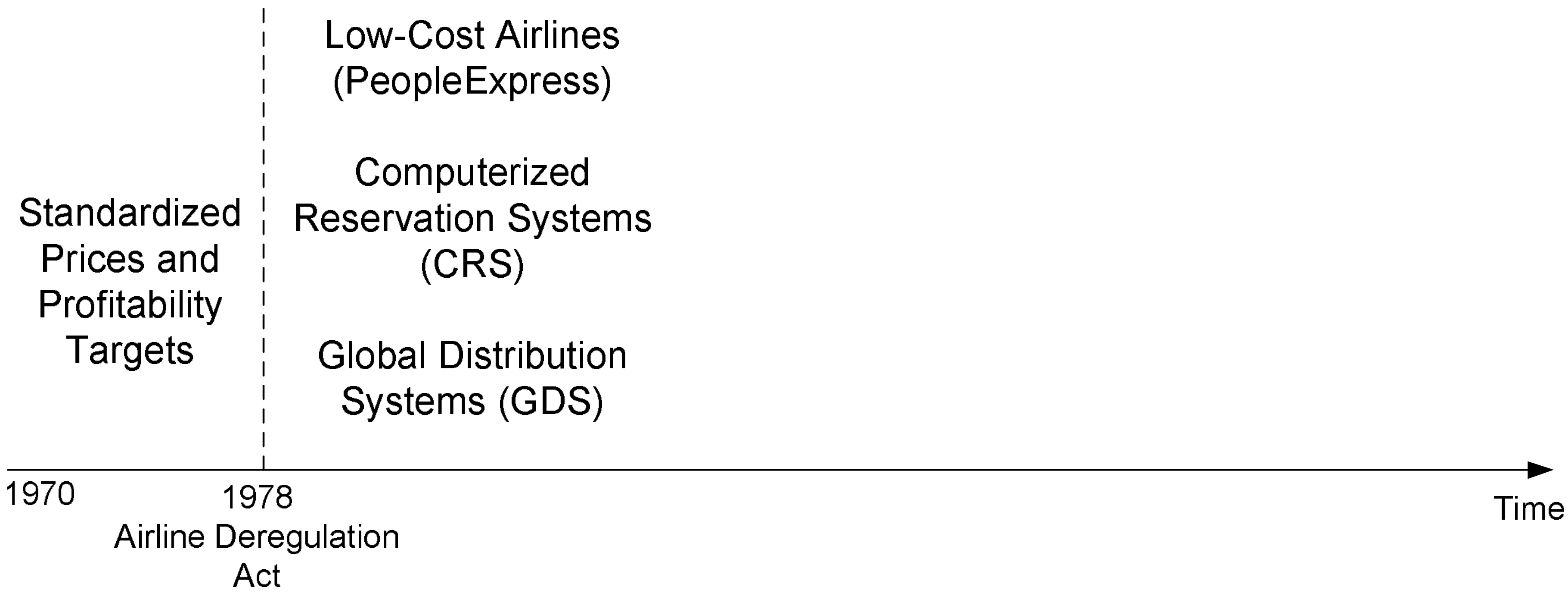

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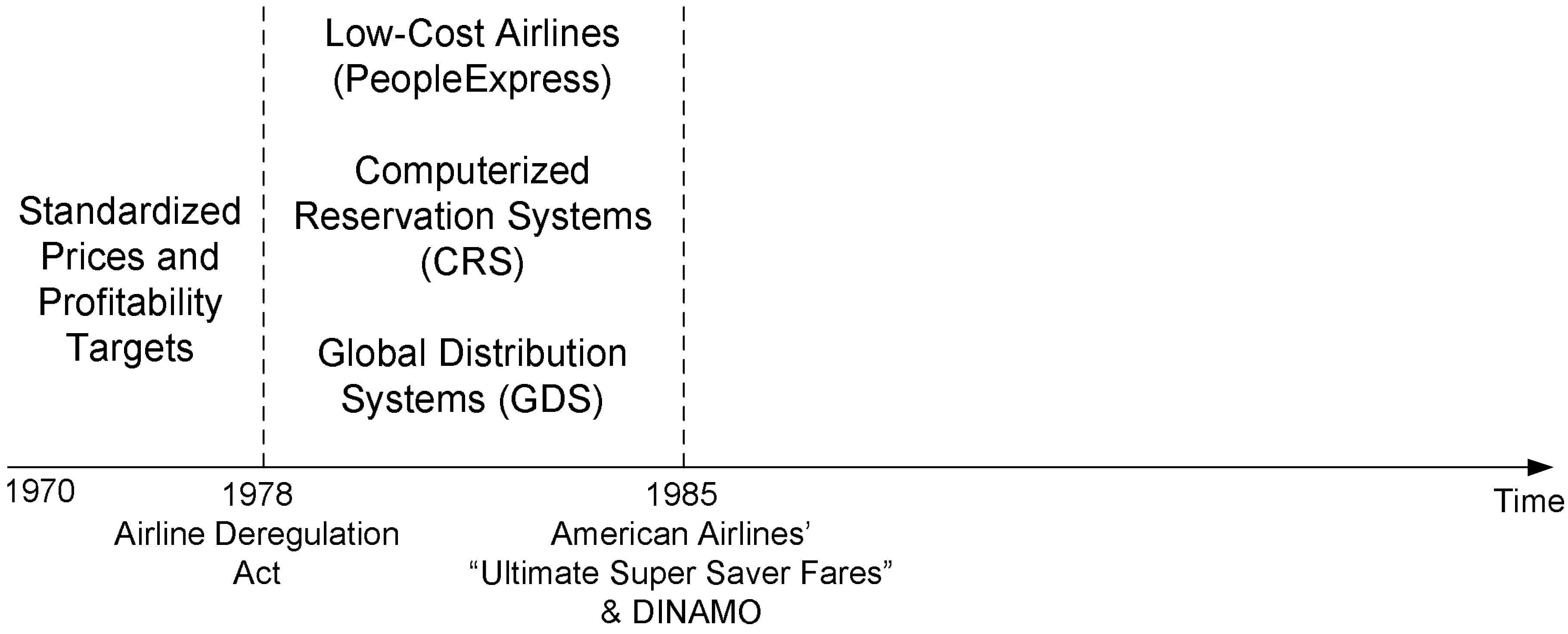

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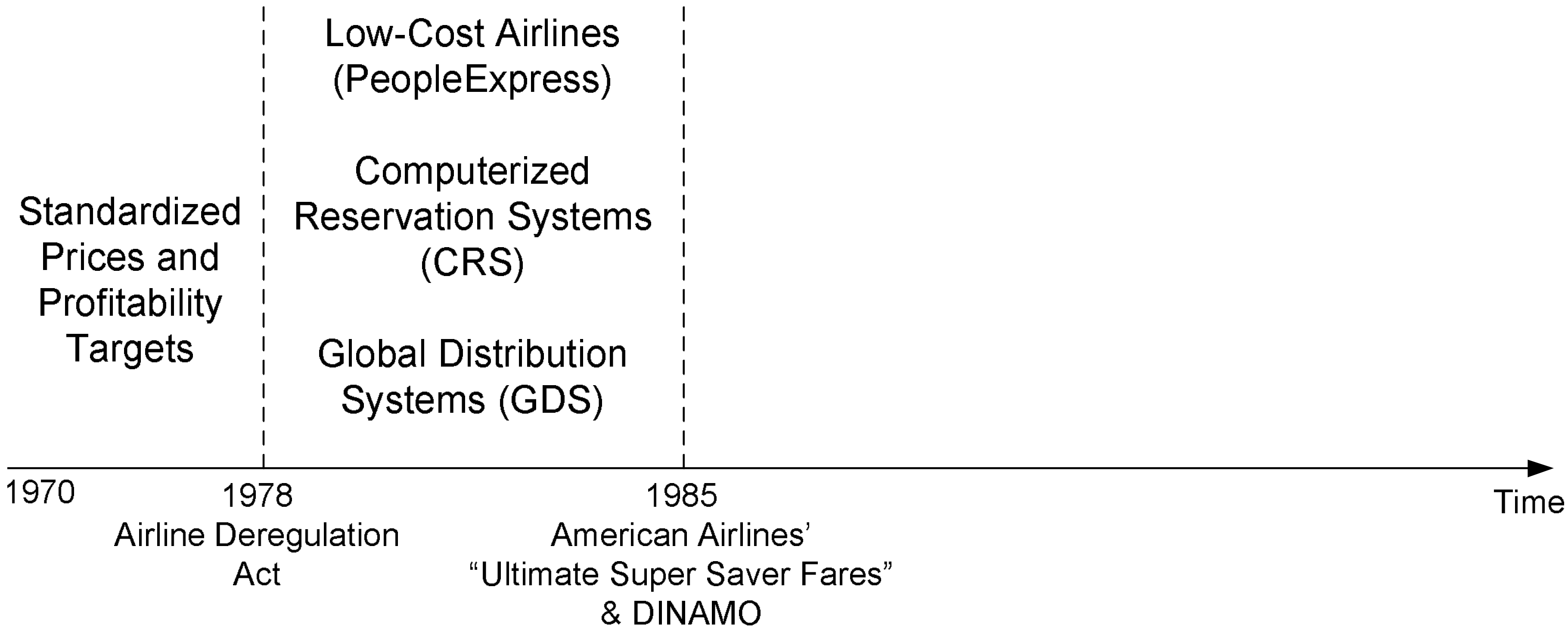

Ultimate Super Saver Fares:

1) Fare Restrictions: Buy 30 days in advance, Saturday overnight, non-refundable.

2) Capacity Control: Restricted number of discount seats sold on each flight.

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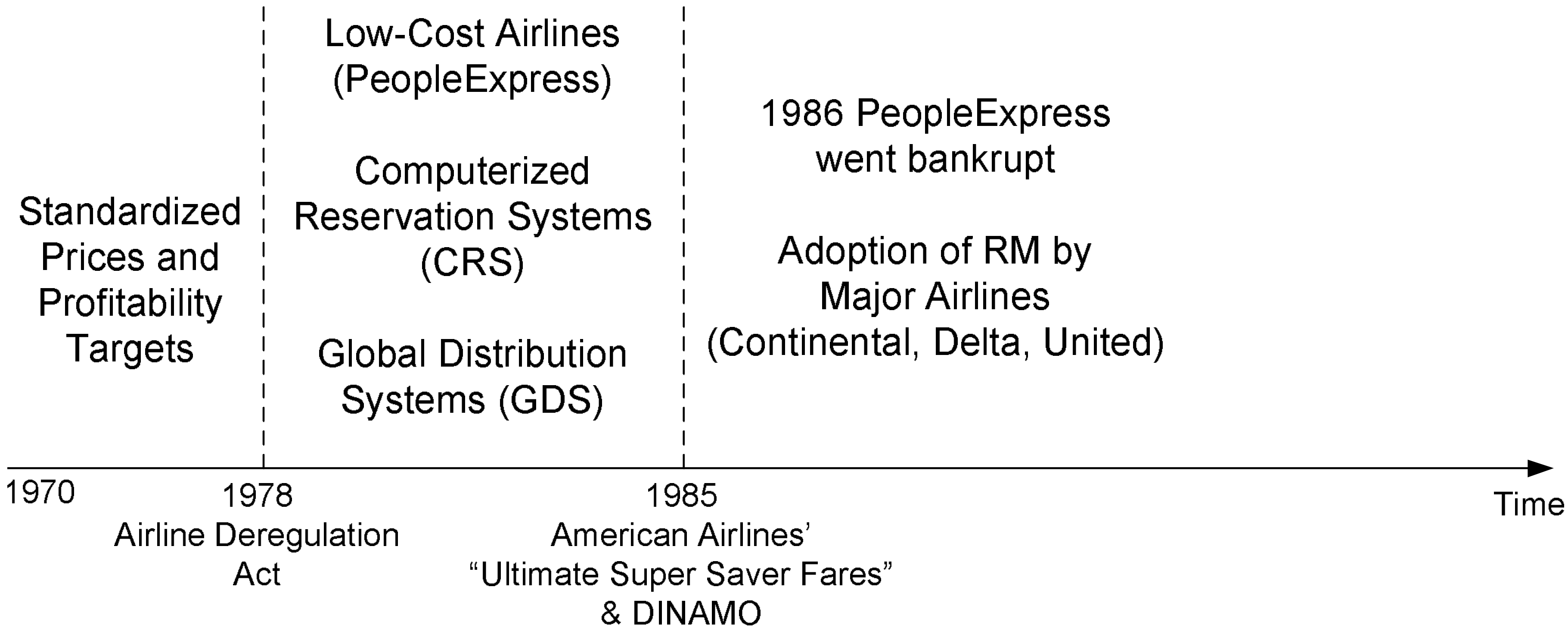

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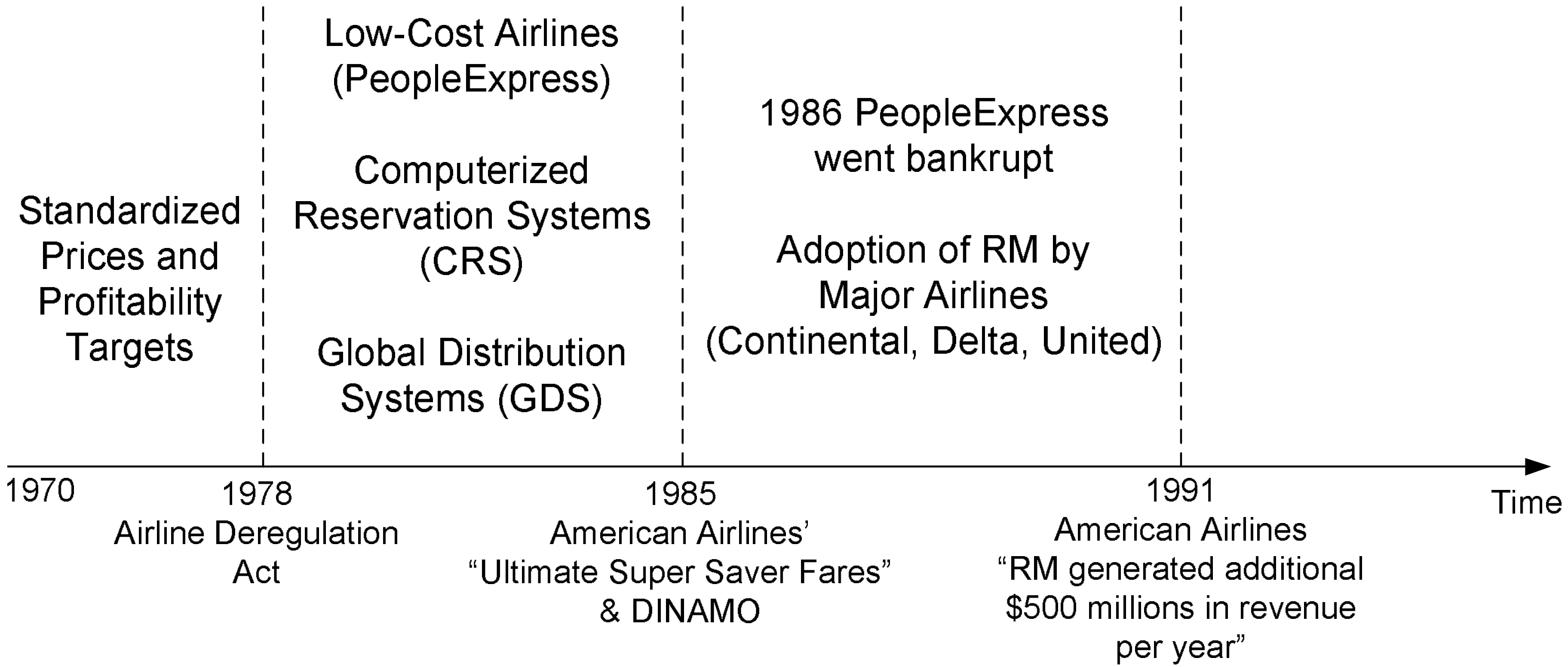

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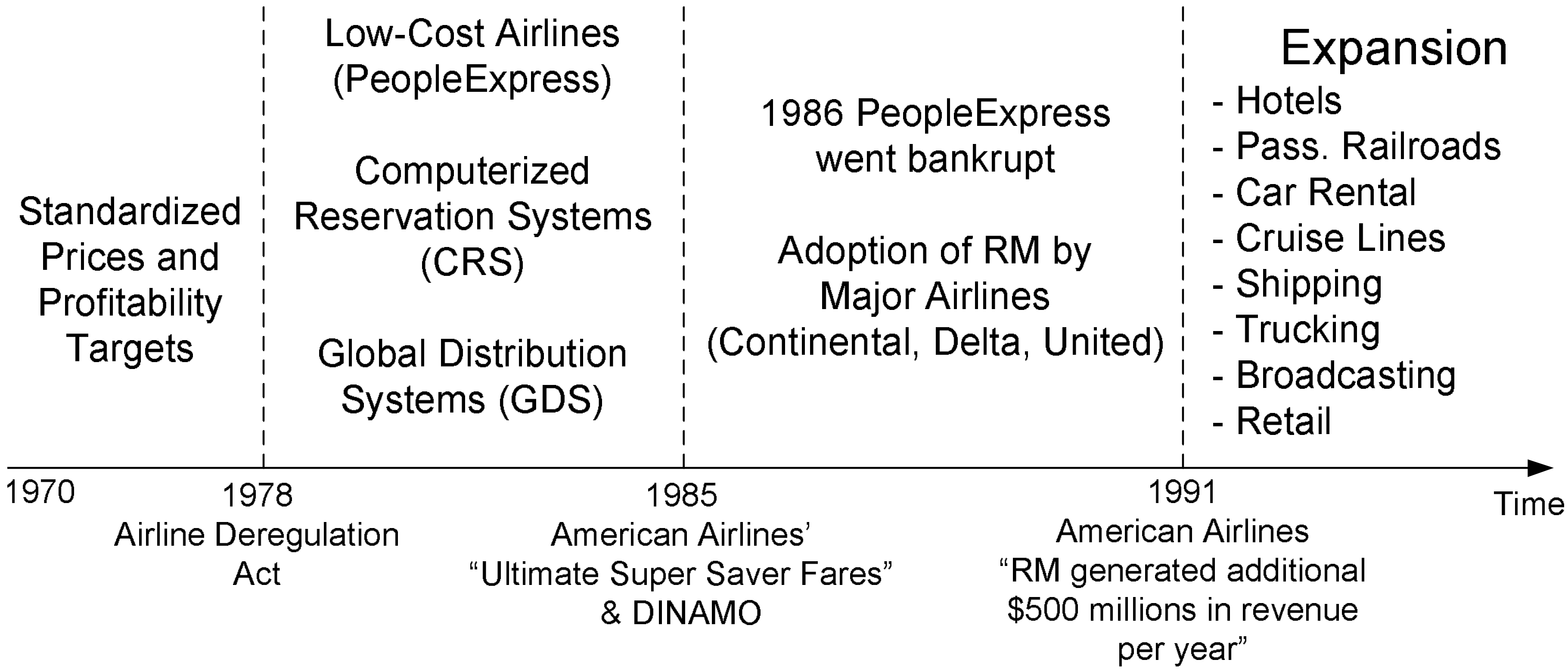

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Revenue Management Decisions

RM addresses three types of demand management decisions

- Structural Decisions
  - Selling formats
  - Product bundling
  - Terms of sale
- Price Decisions
  - Initial prices
  - Markdowns
  - Promotions
- Quantity Decisions
  - Accept/Reject demand
  - Rationing by
    - ✓ channel
    - ✓ location
    - ✓ time

Strategic
Changed relatively infrequently

Tactical
Price or quantity is used depending on commitments, flexibility of channel, time scale, etc.

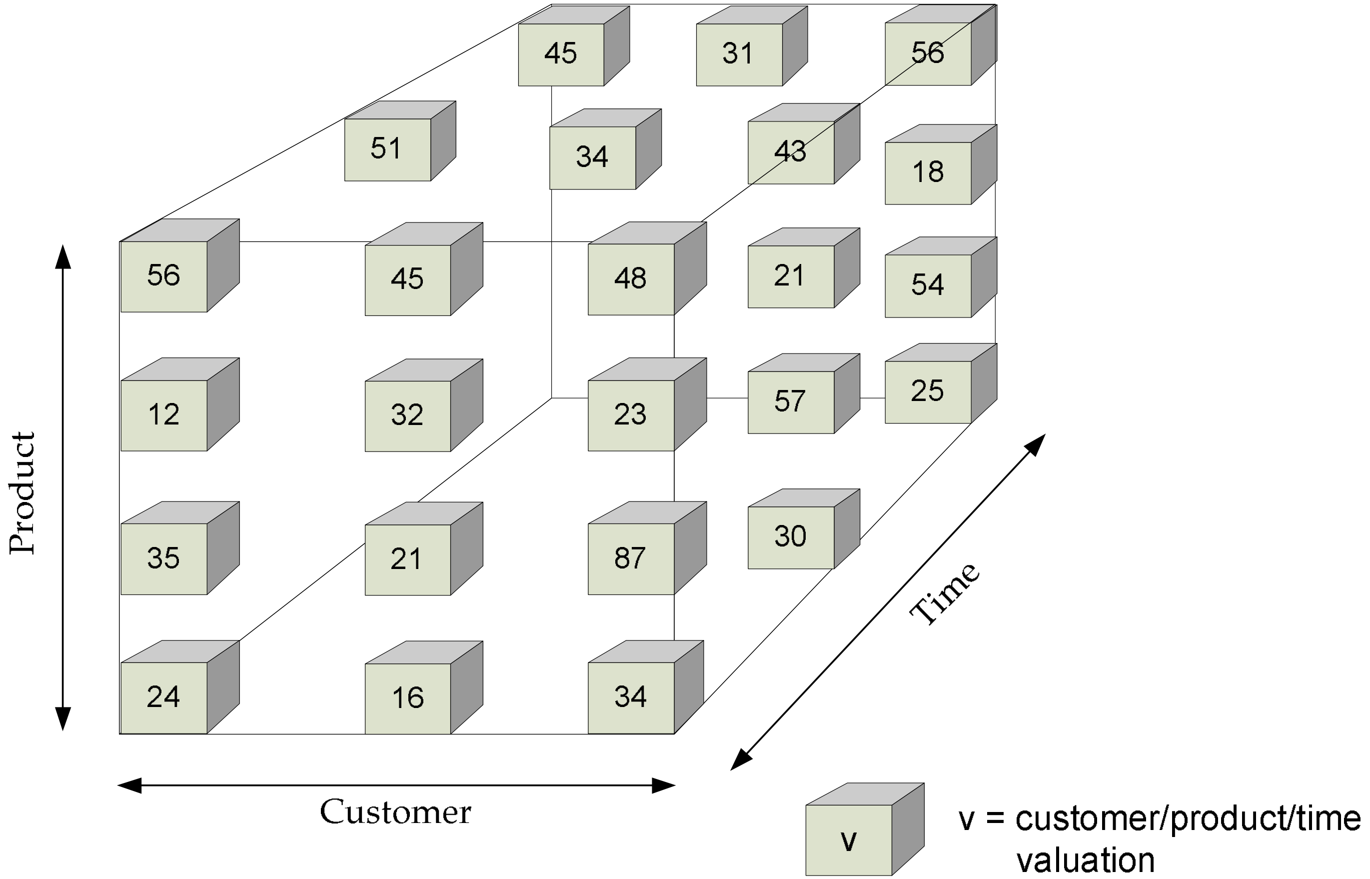
What's New About RM?

- Demand management decisions are old news
  - Practice
  - Economic theory

- The 'new twist' is <u>how</u> the decisions are made
  - Information technology
    - ✓ Databases
    - ✓ Enterprise planning and execution systems
    - ✓ Internet
  - Scientific decision making
    - ✓ Statistics
    - ✓ Economic & behavioral modeling
    - ✓ Optimization

When Does RM Apply?

A conceptual view of a firm's "demand landscape"



RM tries to exploit this landscape and manage the resulting trade-offs

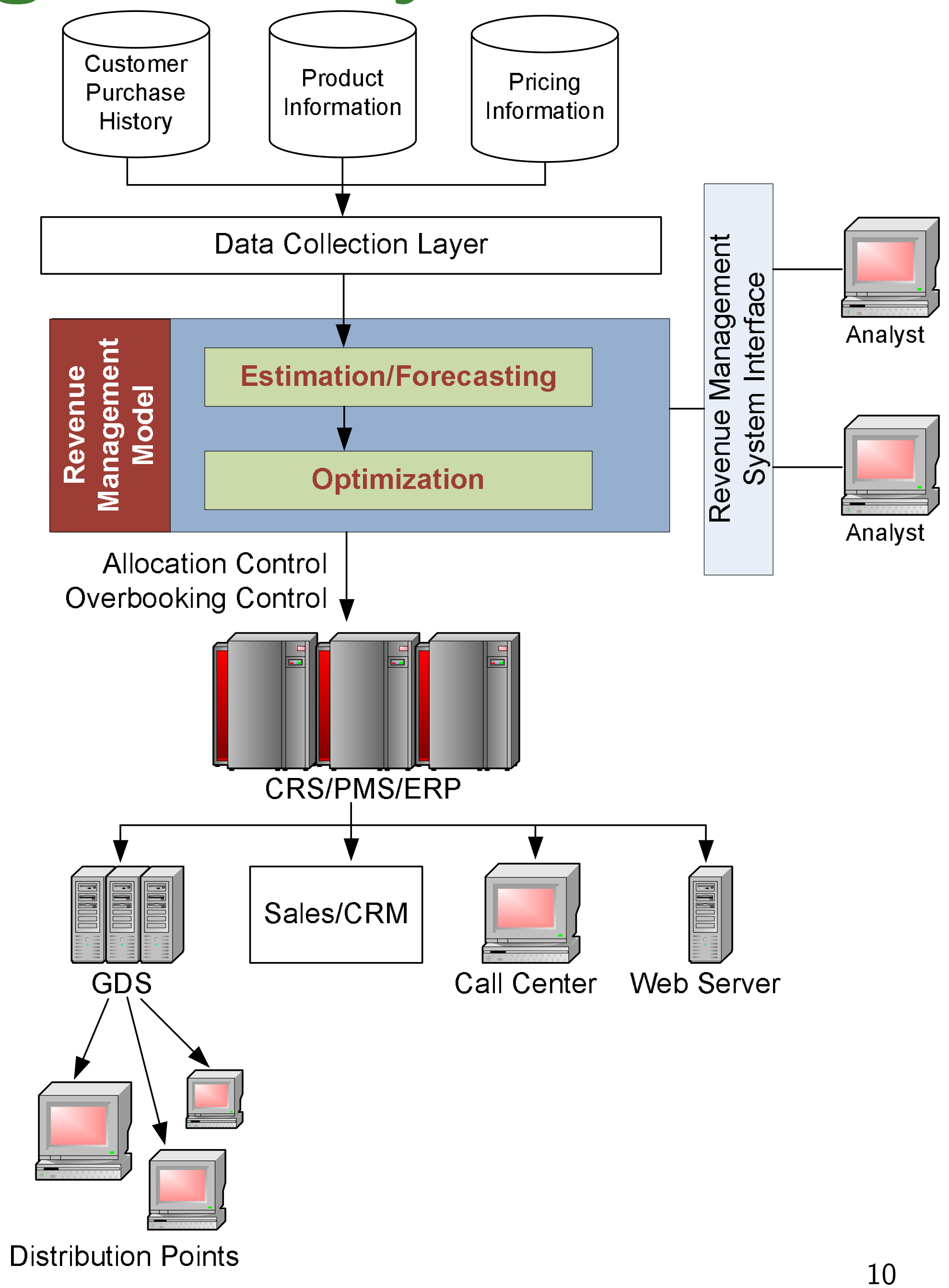
Conditions Favoring RM

1. Customer heterogeneity
2. Demand variability and uncertainty
3. Fixed selling horizon / Perishable products
4. Production inflexibility
5. Price is not a signal for quality
6. Data and IS infrastructure exist
7. Management culture accepting of science and tech.

A Revenue Management System

Two main methodological components

- Forecasting
- Optimization

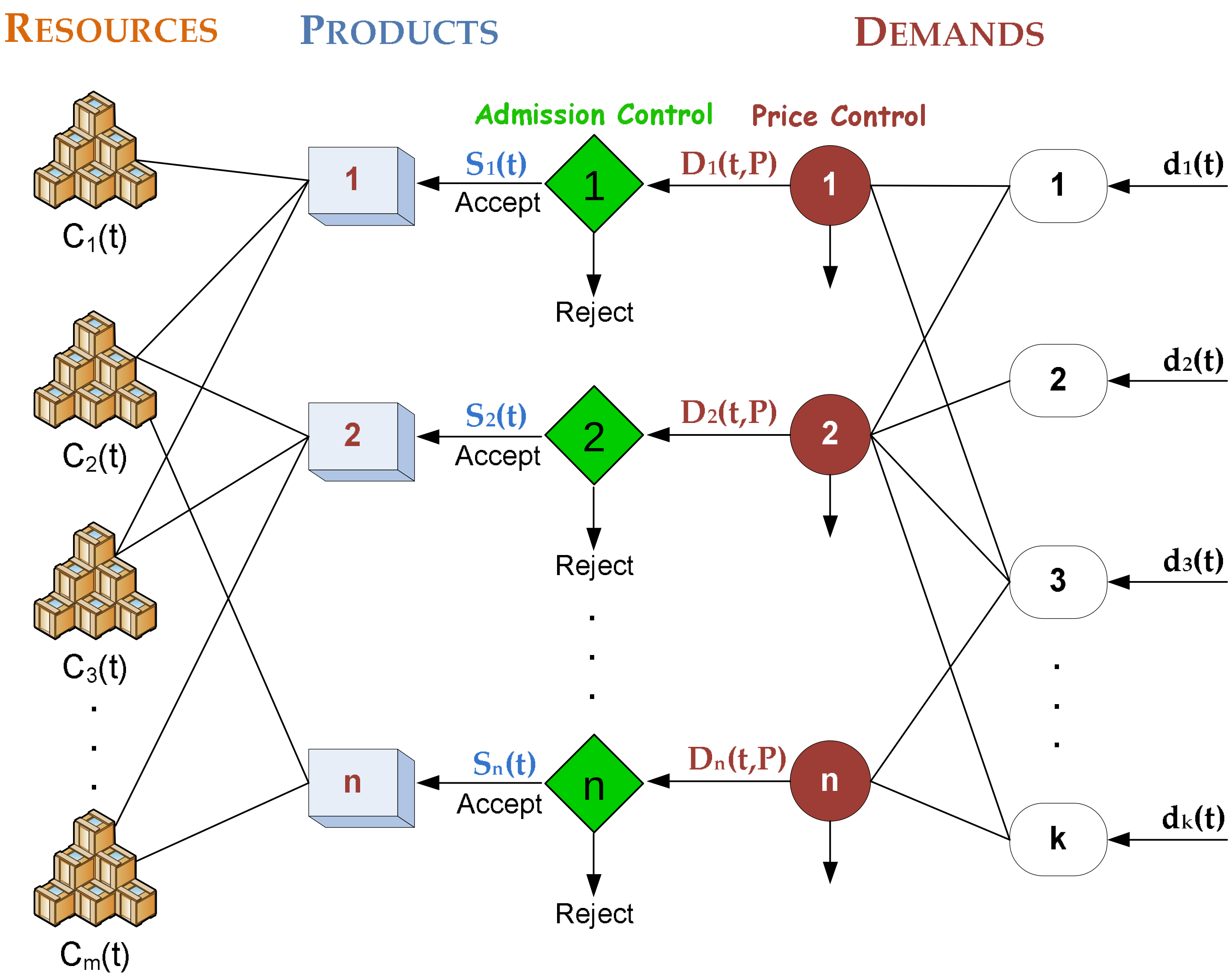


Classification of Models and Methods

- Quantity-based Revenue Management
  - Single-resource capacity control
  - Network capacity control
  - Overbooking

- Price-based Revenue Management
  - Dynamic pricing
    - ✓ Markdowns
    - ✓ Promotions
  - Auctions

The Revenue Management Problem



Revenue Management Taxonomy

	Elements	Descriptor
A	Resource	Discrete/Continuous
B	Capacity	Fixed/Nonfixed
C	Prices	Predetermined/Set Optimally/Set Jointly
D	Willingness to Pay	Buildup/Drawdown
E	Discount Price Classes	$1/2/3/\dots/I$
F	Reservation Demand	Deterministic/Mixed/Random-independent/ Random-correlated
G	Show-Up of Discount Reservation	Certain/Uncertain without Cancellation/ Uncertain with cancellation
H	Show-Up of Full-Price Reservation	Certain/Uncertain without Cancellation/ Uncertain with cancellation
I	Group Reservations	No/Yes
J	Diversion	No/Yes
K	Displacement	No/Yes
L	Bumping Procedure	None/Full-price/Discount/FCFS/Auction
M	Asset Control Mechanism	Distinct/Nested
N	Decision Rule	Simple Static /Advanced Static / Dynamic

Source: Weatherford & Bodily (1992), *Ops. Res.* **40**, 831-844.

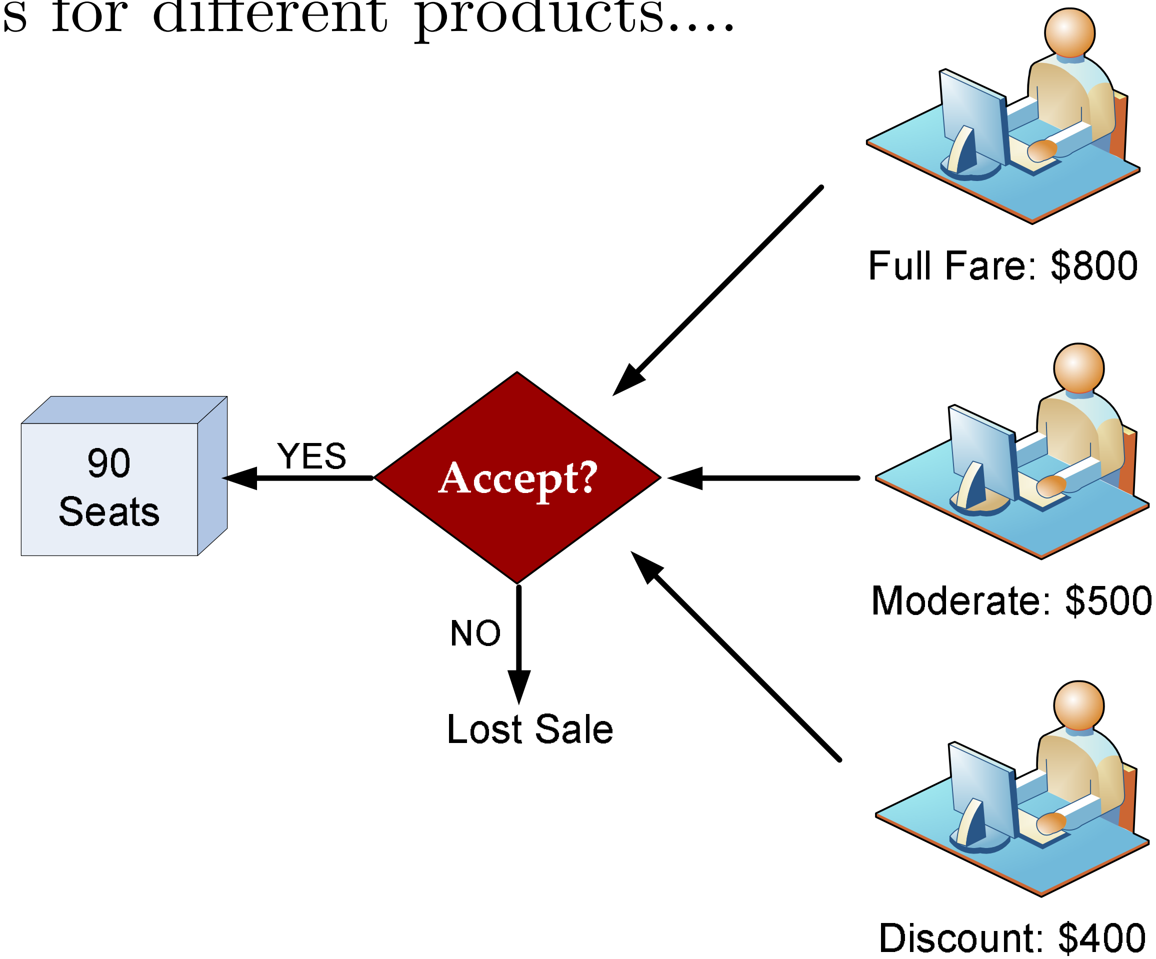
Example: A1-B1-C1-E3-N3

Quantity-Based Revenue Management

Single-Resource Capacity Control

Traditional airline/hotel quantity-based RM models:

Given requests for different products....

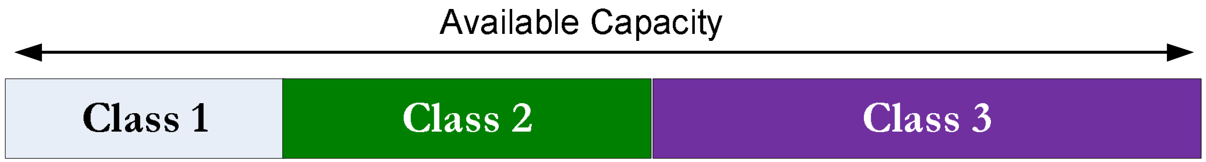


... decide which ones to accept or reject.

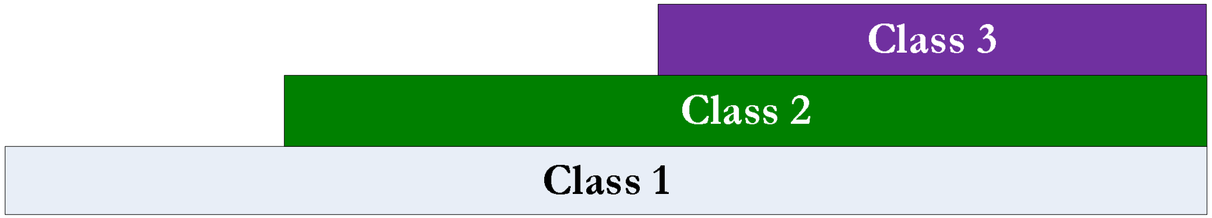
Types of Controls

BOOKING LIMITS: Maximum capacity assigned to a particular class at a given time.

- Partitioned: Partition available capacity into separate blocks (or buckets).
- Nested: Capacity is assigned in a hierarchical and overlapping manner.



Partitioned Booking Limits

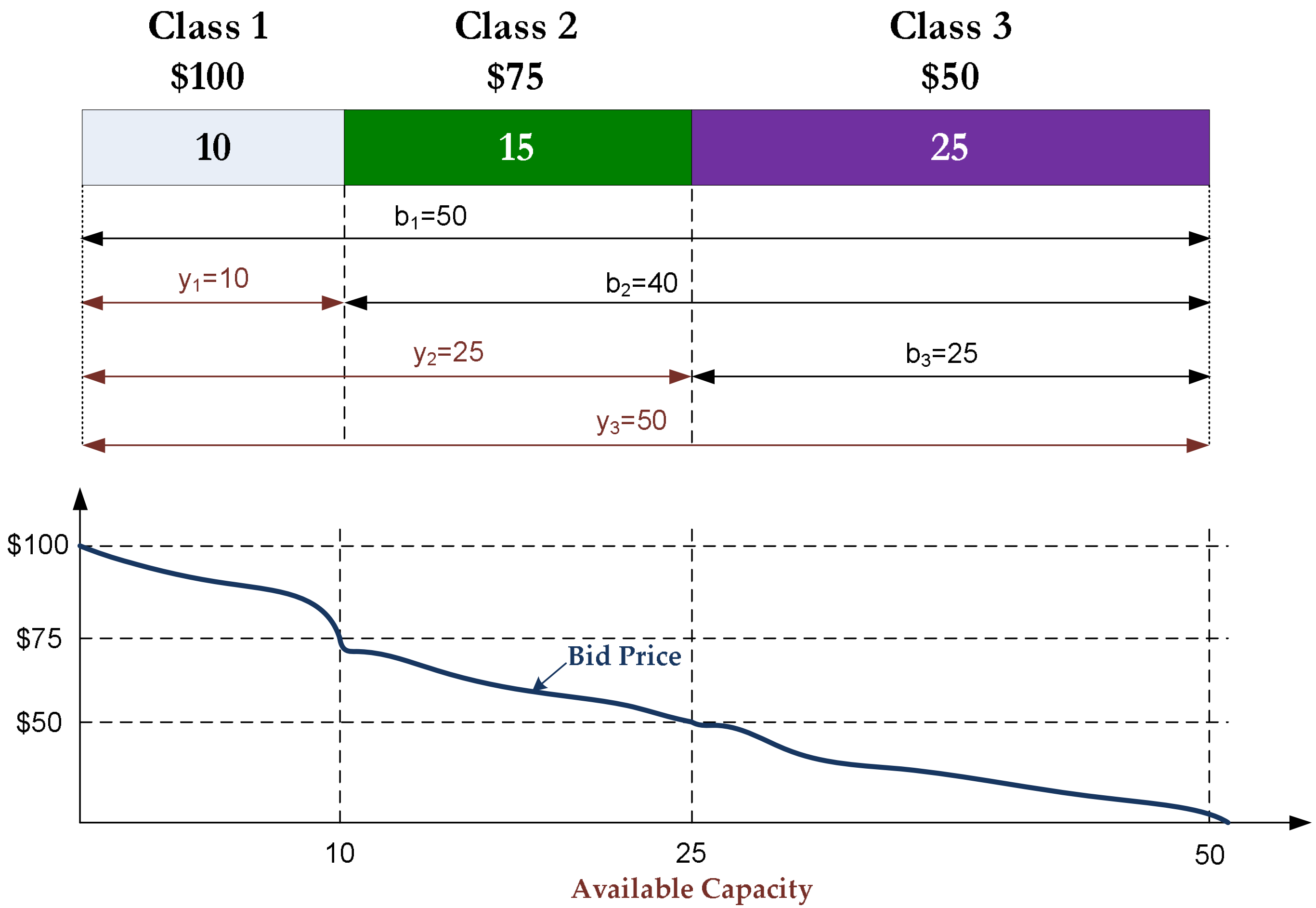


Nested Booking Limits

PROTECTION LEVELS: Total capacity minus booking limit

Types of Controls (contd')

Bid Prices: Threshold price for "Accept/Reject" requests.



Displacement Cost: $V(x) - V(x-1)$.

The "Static Model"

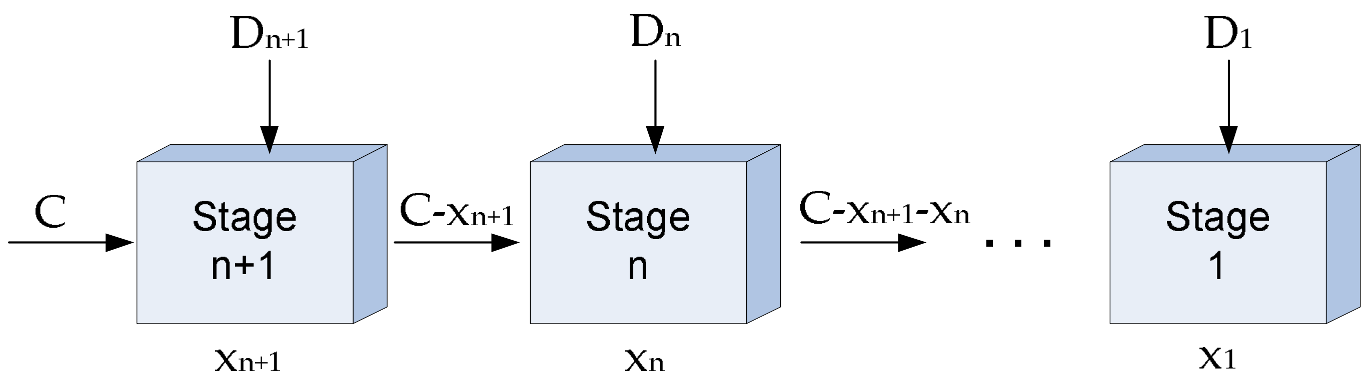
- D_i: Demand for class $i = 1, \ldots, n+1$ (continuous i.i.d r.v.)
- f_i: Fare (net contribution) of class i

$$f_1 > f_2 > \cdots > f_{n+1}$$

- x_i: Number of class i customers accepted (control)

$$0 \leq x_i \leq D_i$$

- Low-before-high order arrival...



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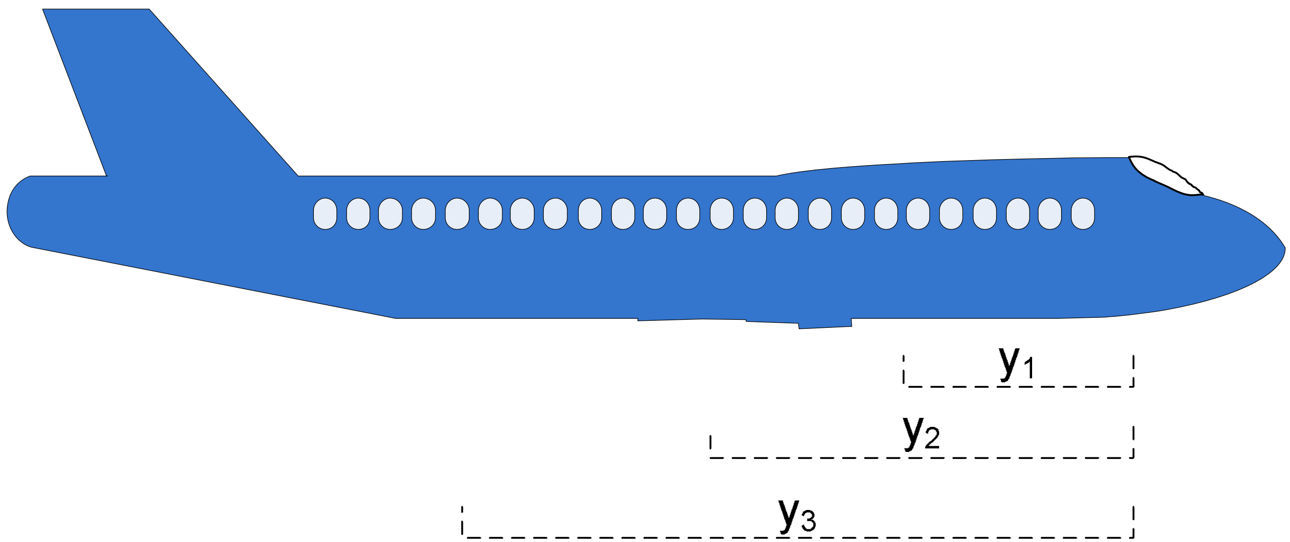
2) Nested allocation decision rule:

y_i: Nested protection level for class i and higher



Littlewood's Two-Class Model (AGIFORS'72)

– Fixed Capacity C.

– Two fares $f_1 > f_2$.

– Demand D_i is random with cdf F_i.

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Proposition. *The optimal protection level for class 1, y_1, satisfies*

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– BOOKING LIMIT: $b_2 = (C - y_1)^+$

– BID PRICE: $\pi(x) = f_1 \, \mathbb{P}(D_1 > x)$.

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Define $V_j(x) =$ optimal expected payoff-to-go if the available capacity is x and the classes $j, j-1, \ldots, 1$ are yet to arrive.

$$V_j(x) = \mathbb{E}\left[\max_{0 \le u \le \min\{x, D_j\}} \{f_j\, u + V_{j-1}(x-u)\}\right], \qquad V_0(x) = 0.$$

Define $\Delta V_j(x) := V_j(x) - V_j(x-1)$.

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PROPOSITION.

–) $\Delta V_j(x+1) \le \Delta V_j(x)$

–) $\Delta V_{j+1}(x) \ge \Delta V_j(x)$.

The n-Class Model (cont'd)

Computation of Optimal Protection Levels

(Brumelle & McGill, OR'93)

"Fill Events"

$$A_1(X,y) = \{D_1 > y_1\}$$

$$A_2(X,y) = \{D_1 > y_1 \cap D_1 + D_2 > y_2\}$$

$$\vdots$$

$$A_i(X,y) = \{D_1 > y_1 \cap D_1 + D_2 > y_2 \cap \ldots \cap D_1 + D_2 + \cdots + D_i > y_i\}$$

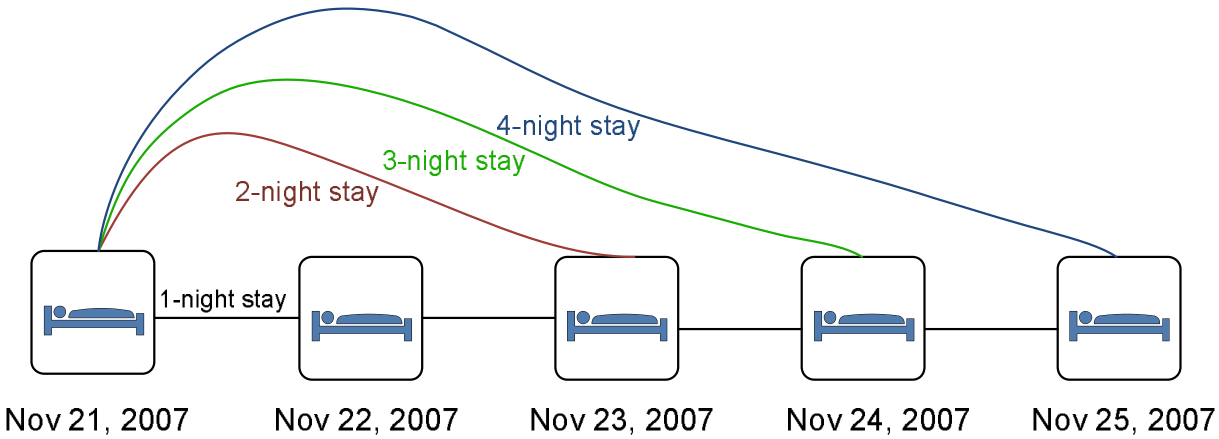
Optimality Conditions:

$$\mathbb{P}(A_i(X,y)) = \frac{f_{i+1}}{f_1}$$

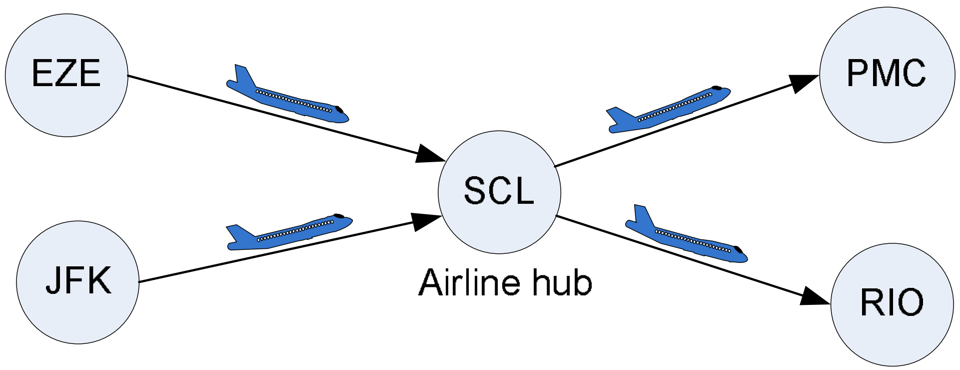
Solved via Monte-Carlo integration (Robinson OR'95)

Network Revenue Management

Network Capacity Control

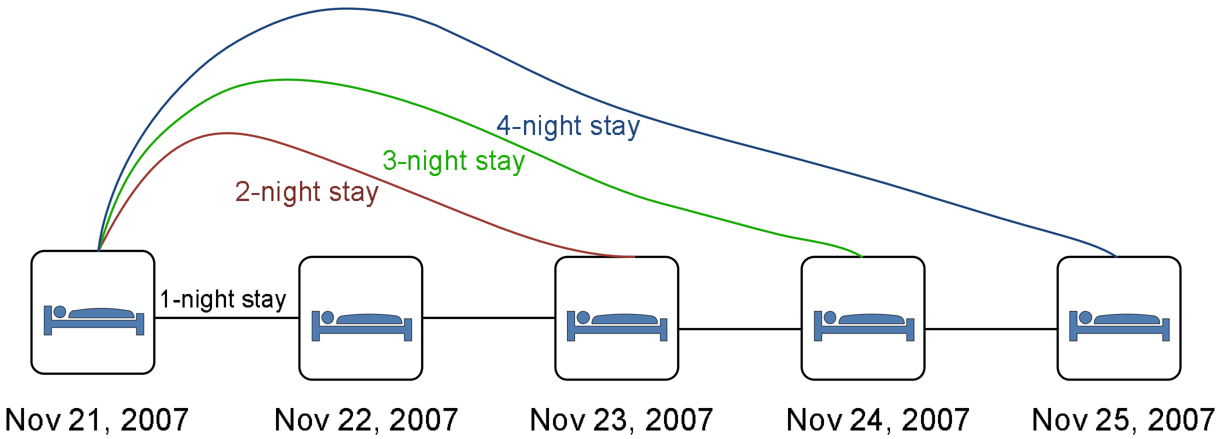


(i) Hotel length-of-stay network



(ii) Airline hub-and-spoke network

Network Capacity Control



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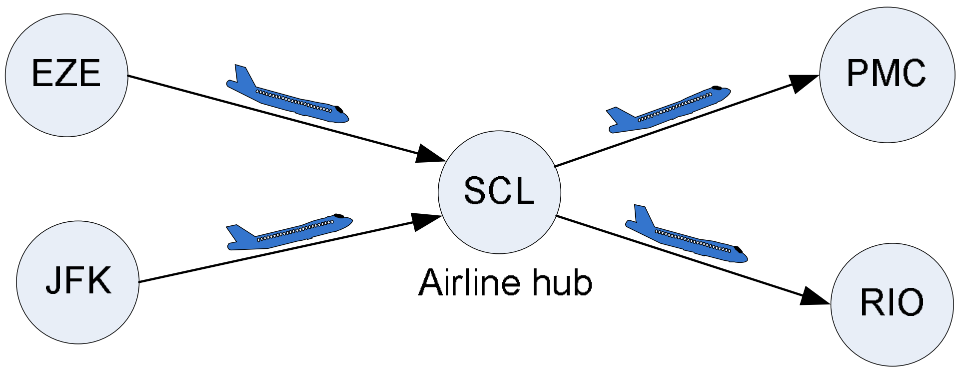


(ii) Airline hub-and-spoke network

Optimize accept/deny decisions for path requests

Network Capacity Control (cont'd)

- t: time period
- C_t: m-vector of leg capacities
- $A = [a_{ij}]$: $m \times n$-incidence matrix

$$a_{ij} = \begin{cases} 1 & \text{if itinerary } j \text{ uses leg } i \\ 0 & \text{otherwise} \end{cases}$$

- ξ_t: n-vector of randomly arriving revenues
- u_t: n-vector of 0-1 controls (accept/deny decisions)

Dynamic Program

$$V_t(C_t) = \max_u \mathbb{E}\left[\xi_t^T u_t(C_t, \xi) + V_{t-1}(C_t - A\, u_t(C_t, \xi))\right]$$

$$\text{subject to} \qquad C_t - A\, u_t(C_t, \xi) \geq 0$$

Network Capacity Control (cont'd)

STRUCTURE OF AN OPTIMAL CONTROL:

Accept revenue f_j for itinerary j if and only if

$$f_j \geq \underbrace{V_{t-1}(C) - V_{t-1}(C - A_j)}_{\text{displacement cost}}$$

ISSUES:

- Approximating control structure
- Approximating displacement cost

Network Capacity Control (cont'd)

Approximate control structures

I) Bid Prices:

Given values $\mu_i(C,t)$, $i = 1,\ldots,m$ for each leg, accept a request for itinerary $j = (i_1, i_2, \ldots, i_k)$ if

$$f_j \geq \mu_{i_1}(C,t) + \mu_{i_2}(C,t) + \cdots + \mu_{i_k}(C,t)$$

II) Displacement Adjusted Virtual Nesting (DAVN):

$$f_j - \mu_{i_1}(C,t) - \mu_{i_2}(C,t) - \cdots - \mu_{i_k}(C,t)$$

Compute displacement-adjusted revenue for each itinerary and apply the resulting revenues and demand in a single-leg model on each leg.

Network Capacity Control (cont'd)

Approximating the problem

Step 1: Use alternative model to approximate the value function

$$
\begin{aligned}
V_t(C) &\approx V_t^A(C) \\
V_t^A(C) &\approx \text{Optimal value of alternative model} \\
&\quad\ \text{given capacity} = C \text{ and time} = t
\end{aligned}
$$

Step 2: Use approximate value function to decide

Accept revenue f_j for itinerary j only if

$$f_j \geq V_t^A(C) - V_t^A(C - A_j) \approx \nabla^T V_t^A(C)\, A_j$$

Network Capacity Control (cont'd)

Example: Deterministic LP

$$V_t^{LP}(C) = \max_y \sum_j f_j \, y_j$$

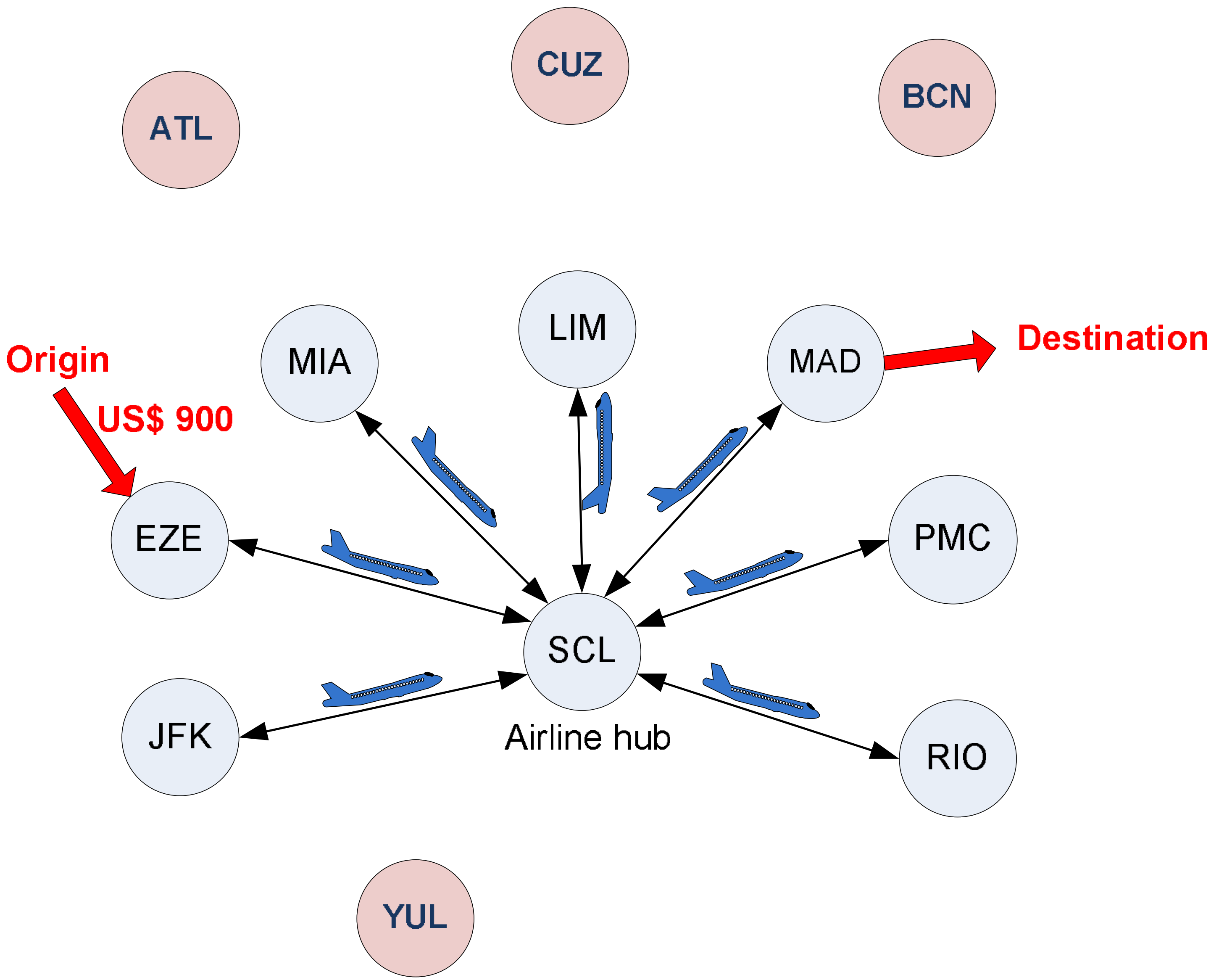
$$\text{subject to} \qquad A\,y \leq C$$

$$0 \leq y \leq \mathbb{E}[D]$$

Then, $\nabla V_t^{LP}(C) = \lambda$ (provided gradient exists) and we accept f_j if

$$\begin{aligned} f_j &\geq V_t^{LP}(C) - V_t^{LP}(C - A_j) \quad \text{(Bertsimas and Popescu, \textit{Trans. Sci.}, 93)} \\ &\approx \nabla^T V_t^{LP}(C)\, A_j \\ &= \sum_{i \in A_j} \lambda_i \quad \text{(Williamson'88)} \end{aligned}$$

Extension: Code-Share Revenue Management



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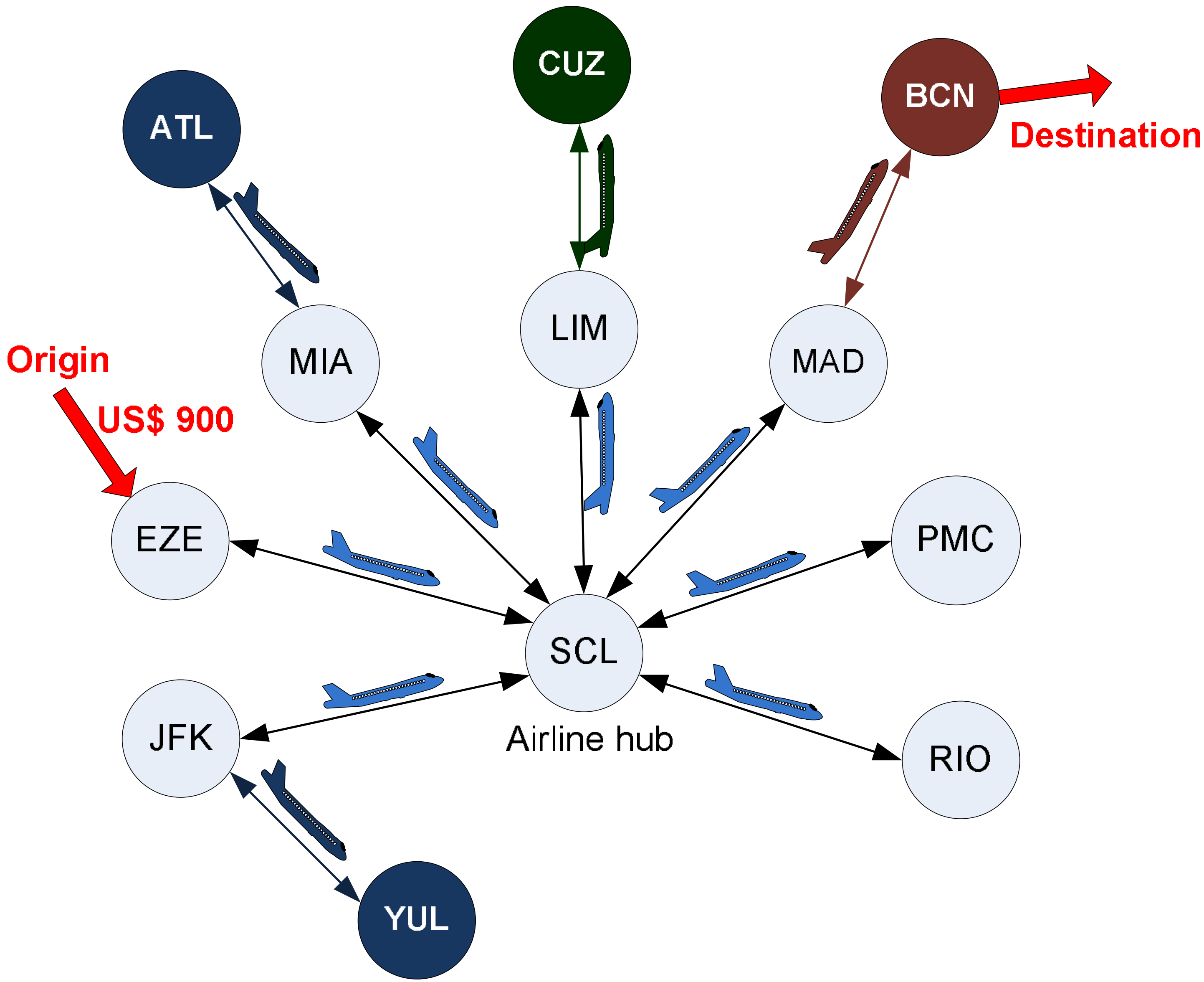

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