Investment Opportunities as Real Options: Getting Started on the Numbers

by Timothy A. Luehrman



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MANAGER'S TOOL KIT

Here's a way to apply option pricing to strategic decisions without hiring an army of Ph.D.'s.

Investment Opportunities as Real Options: Getting Started on the Numbers

by Timothy A. Luehrman

HE ANALOGY between financial options and corporate investments that create future opportunities is both intuitively appealing and increasingly well accepted. Executives readily see why investing today in R&D, or in a new marketing program, or even in certain capital expenditures (a phased plant expansion, say) can generate the possibility of new products or new markets tomorrow. But for many nonfinance managers, the journey from insight to action, from the puts and calls of financial options to actual investment decisions, is difficult and deeply frustrating.

Experts do a good job of explaining what option pricing captures that conventional discounted-cash-flow (DCF) and net-present-value (NPV) analyses do not. Moreover, simple option pricing for exchange-traded puts and calls is fairly straightforward, and many books present the basics lucidly. But at that point, most executives get stuck. Their interest piqued, they want to know How can I use option pricing on *my* project? and How can I use this with real numbers rather than with sterilized examples? Unfortunately, howto advice is scarce on this subject and mostly aimed at specialists, preferably with Ph.D.'s. As a result, corporate analyses that generate real numbers have been rare, expensive, and hard to understand.

The framework presented here bridges the gap between the practicalities of real-world capital projects

and the higher mathematics associated with formal option-pricing theory. It produces quantitative output, can be used repeatedly on many projects, and is compatible with the ubiquitous DCF spreadsheets that are at the heart of most corporate capital-budgeting systems. What this framework cannot supply is absolute precision: when a very precise number is required, managers will still have to call on technical experts with specialized financial tools. But for many projects in many companies, a "good enough" number is not only good enough but considerably better than the number a plain DCF analysis would generate. In such cases, forgoing some precision in exchange for simplicity, versatility, and explicability is a worthwhile trade.

We'll begin by examining a generic investment opportunity-a capital budgeting project - to see what makes it similar to a call option. Then we'll compare DCF with the option-pricing approach to evaluating the project. Instead of looking only at the differences between the two approaches, we will also look for points of commonality. Recognizing the differences adds extra insight to the analysis, but exploiting the commonalities is the key to making the framework understandable and compatible with familiar techniques. In fact, most of the data the framework uses come from the DCF spreadsheets that managers routinely prepare to evaluate investment proposals. And for option values, the framework uses the Black-Scholes option-pricing table instead of complex equations. Finally, once we've built the framework, we'll apply it to a typical capital-investment decision.

Mapping a Project Onto an Option

A corporate investment opportunity is like a call option because the corporation has the right, but not the obligation, to acquire something – let us say, the operating assets of a

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MANAGER'S TOOL KIT INVESTMENT OPPORTUNITIES AS REAL OPTIONS

new business. If we could find a call option sufficiently similar to the investment opportunity, the value of the option would tell us something about the value of the opportunity. Unfortunately, most business opportunities are unique, so the likelihood of finding a similar option is low. The only reliable way to find a similar option is to construct one.

To do so, we need to establish a correspondence between the project's characteristics and the five variables that determine the value of a simple call option on a share of stock. By mapping the characteristics of the business opportunity onto the template of a call option, we can obtain a model of the project that combines its characteristics with the structure of a call option. The option we will use is a *European* call, which is the simplest of all options because it can be exercised on

only one date, its expiration date. The option we synthesize in this way is not a perfect substitute for the real opportunity, but because we've designed it to be similar, it is indeed informative. The diagram "Mapping an Investment Opportunity onto a Call Option" shows the correspondences making up the fundamental mapping.

Many projects involve spending money to buy or build a productive asset. Spending money to exploit such a business opportunity is analogous to exercising an option on, for example, a share of stock. The amount of money expended corresponds to the option's *exercise price* (denoted for simplicity as *X*). The present value of the asset built or acquired corresponds to the *stock price* (*S*). The length of time the company can defer the investment decision without losing the opportunity cor-





responds to the option's *time to expiration* (*t*). The uncertainty about the future value of the project's cash flows (that is, the riskiness of the project) corresponds to the *standard deviation of returns* on the stock (σ). Finally, the time value of money is given in both cases by the *risk-free rate of return* (r_f). By pricing an option using values for these variables generated from our project, we learn more about the value of the project than a simple discounted-cash-flow analysis would tell us.

Linking NPV And Option Value

Traditional DCF methods would assess this opportunity by computing its net present value. NPV is the difference between how much the operating assets are worth (their present value) and how much they cost:

> NPV = present value of assets - required capital expenditure.

When NPV is positive, the corporation will increase its own value by making the investment. When NPV is negative, the corporation is better off not making the investment.

When are the project's option value and NPV the same? When a final decision on the project can no longer be deferred; that is, when the company's "option" has reached its expiration date. At that time, either

the option value = S - X

or

the option value = 0

whichever is greater. But note that

as well, because we know from our map that *S* corresponds to the present value of the project assets and *X* to the required capital expenditure. To reconcile the two completely, we need only observe that when NPV is negative, the corporation will not invest, so the project value is effectively zero (just like the option value) rather than negative. In short, both approaches boil down to the same number and the same decision. (See the diagram "When Are Conventional NPV and Option Value Identical?")

NPV = S - X

This common ground between NPV and option value has great practical significance. It means that corporate spreadsheets set up to compute conventional NPV are highly relevant for option pricing. Any spreadsheet that computes NPV already contains the information necessary to compute *S* and *X*, which are two of the five option-pricing variables. Accordingly, executives who want to begin using option pricing need not discard their current DCF-based systems.

When do NPV and option pricing diverge? When the investment decision may be deferred. The possibility of deferral gives rise to two additional sources of value. First, we would always rather pay later than sooner, all else being equal, because we can earn the time value of money on the deferred expenditure. Second, while we're waiting, the world can change. Specifically, the value of the operating assets we intend to acquire may change. If their value goes up, we haven't missed out; we still can acquire them simply by making the investment (exercising our option). If their value goes down, we might decide not to acquire them. That also is fine (very good, in fact) because, by waiting, we avoid making what would have turned out to be a poor investment. We have preserved the ability to participate in good outcomes and insulated ourselves from some had ones.

For both of these reasons, being able to defer the investment decision is valuable. Traditional NPV misses the extra value associated with deferral because it assumes the decision cannot be put off. In contrast, option pricing *presumes* the ability to defer and provides a way to quantify the value of deferring. So to value the investment, we need to develop two new metrics that capture these extra sources of value.

Quantifying Extra Value: NPVq. The first source of value is the interest you can earn on the required capital expenditure by investing later rather than sooner. A good way to capture that value is to suppose you put just enough money in the bank now so that when it's time to invest, that money plus the interest it has earned is sufficient to fund the required expenditure. How much money is that? It is the discounted present value of the capital expenditure. In option notation, it's the present value of the exercise price, or PV(X). To compute PV(X), we discount *X* for the requisite number of periods (*t*) at the risk-free rate of return (*r_f*):

 $\mathsf{PV}(X) = X \div (1 + r_f)^t.$

The extra value is the interest rate (r_f) times *X*, compounded over however many time periods (t) are involved. Alternatively, it is the difference between *X* and PV(*X*).

We know that conventional NPV is missing that extra value, so let's put it in. We have seen that NPV can be expressed in option notation as:

NPV = S - X.

Let's rewrite it using PV(X) instead of *X*. Thus:

"modified" NPV = S - PV(X).

Note that our modified NPV will be greater than or equal to regular NPV because it explicitly includes interest to be earned while we wait. It picks up one of the sources of value we are interested in.

Modified NPV, then, is the difference between S (value) and PV(X) (cost adjusted for the time value of money). Modified NPV can be positive, negative, or zero. However, it will make our calculations a lot easier if we express the relationship between cost and value in such a way that the number can never be negative or zero.

So instead of expressing modified NPV as the *difference* between *S* and PV(*X*), let's create a new metric: *S divided* by PV(*X*). By converting the difference to a ratio, all we are doing, essentially, is converting negative values to decimals between zero and one.¹ We'll call this new metric NPVq, where "q" reminds us that we are expressing the relationship between cost and value as a quotient:

$\mathsf{NPVq} = S \div \mathsf{PV}(X).$

Modified NPV and NPVq are not equivalent; that is, they don't yield the same numeric answer. For example, if S = 5 and PV(X) = 7, NPV = -2but NPVq = 0.714. But the difference in the figures is unimportant because we haven't lost any information about the project by substituting one metric for another. When modified NPV is positive, NPVq will be greater than one; when NPV is negative, NPVq will be less than one. Anytime modified NPV is zero, NPVq will be one. There is a perfect correspondence between them, as the diagram "Substituting NPVq for NPV" shows.

Quantifying Extra Value: Cumulative Volatility. Now let's move on to the second source of additional value, namely that while we're waiting, asset value may change and affect our investment decision for the better. That possibility is very important, but naturally it is more difficult to quantify because we are not actually sure that asset values will change or, if they do, what the future values will be. Fortunately, rather than measuring added value directly,

Substituting NPVg for NPV

We can rank projects on a continuum according to values for NPVq, just as we would for NPV. When a decision can no longer be deferred, NPV and NPVq give identical investment decisions, but NPVq has some mathematical advantages.



we can measure *uncertainty* instead and let an option-pricing model quantify the value associated with a given amount of uncertainty. Once again, we'll go through two steps. First, we'll identify a sensible way to measure uncertainty. Then we'll express the metric in a mathematical form that will be easier for us to use but will not cause us to lose any practical content.

The only way to measure uncertainty is by assessing probabilities. Imagine that the project's future value is to be drawn from an urn containing all possible future values, weighted according to their likelihood of occurring. That is, if a value of \$100 were twice as likely as \$75 or \$125, there would be twice as many \$100 balls in the urn as \$75 balls or \$125 balls.

How can we quantify this uncertainty? Perhaps the most obvious measure is simply the range of all possible values: the difference between the lowest and the highest possibilities. But we can do better than that by taking into account the relative likelihood of values between those extremes. If, for example, very high and very low values are less likely than "medium" or "average" values, our measure of uncertainty should reflect that. The most common probability-weighted measure of dispersion is *variance*, often denoted as sigma squared (σ^2). Variance is a summary measure of the likelihood of drawing a value far away from the average value in the urn. The higher the variance, the more likely it is that the values drawn will be either much higher or much lower than average. In other words, we might say that high-variance assets are riskier than low-variance assets.

Variance is an appealing measure of uncertainty, but it's incomplete. We have to worry about a time dimension as well: how much things can change while we wait depends on how long we can afford to wait. For business projects, things can change a lot more if we wait two years than if we wait only two months. So in option valuation, we speak in terms of variance *per period*. Then our measure of the total amount of uncertainty is variance per period times the number of periods, or $\sigma^2 t$.

This sometimes is called *cumula*tive variance. An option expiring in two years has twice the cumulative variance as an otherwise identical option expiring in one year, given the same variance per period. Alternatively, it may help to think of cumulative variance as the amount of variance in the urn times the number of draws you are allowed, which again is $\sigma^2 t$.

Cumulative variance is a good way to measure the uncertainty associated with business investments. Now we'll make two modifications. again for mathematical convenience, that won't affect the ability of the variable to tell us what we want to know about uncertainty. First, instead of using the variance of project *values*, we'll use the variance of project returns. In other words, rather than working with the actual dollar value of the project, we'll work with the percentage gained (or lost) per year. There is no loss of content because a project's return is completely determined by the project's value:

return =
(future value – present value)
present value.

The probability distribution of possible values is usually quite asymmetric; value can increase greatly but cannot drop below zero. Returns, in contrast, can be positive or negative, sometimes symmetrically positive or negative, which makes their probability distribution easier to work with.

Second, it helps to express uncertainty in terms of *standard deviation* rather than variance. Standard deviation is simply the square root of variance and is denoted by σ . It tells us just as much about uncertainty as variance does, but it has the advantage of being denominated in the same units as the thing being measured. In our business example, future asset values are denominated in units of currency – say, dollars – and returns are denominated in percentage points. Standard deviation, then, is likewise denominated in dollars or percentage points, whereas variance is denominated in *squared* dollars or *squared* percentage points, which are not intuitive. Since we are going to work with returns instead of values, our units will be percentage points instead of dollars.

To make these refinements to our measure of total uncertainty, we do the following:

First, stipulate that σ^2 denotes the variance of returns per unit of time on our project.

Second, multiply variance per period by the number of periods (*t*) to get cumulative variance ($\sigma^2 t$).

Third, take the square root of cumulative variance to change units, expressing the metric as standard deviation rather than variance. Let's call this last quantity *cumulative volatility* ($\sigma\sqrt{t}$) to distinguish it from cumulative variance.

Valuing the Option. Together, our two new call-option metrics, NPVq and $\sigma\sqrt{t}$, contain all the information needed to value our project as a European call option using the Black-Scholes model. They capture the extra sources of value associated with opportunities. And they are composed of the five fundamental option-pricing variables onto which we mapped our business opportunity. NPVq is actually a combination of four of the five variables: S, X, r_f, and t. Cumulative volatility combines the fifth, σ , with t. (See the diagram "Linking Our Metrics to the Black-Scholes Model.") By combining variables in this way, we get to work with two metrics instead of five. Not only is that easier for most of us to grasp, it also allows us to plot two-dimensional pictures, which can be helpful substitutes for equations in managers' discussions and presentations. Finally, each of the metrics has a natural business interpretation, which makes optionbased analysis less opaque to nonfinance executives.

The graph "Locating the Option Value in Two-Dimensional Space" shows how to use NPVq and $\sigma\sqrt{t}$ to obtain a value for the option. NPVq is on the horizontal axis, increasing from left to right. As NPVq rises, so does the value of the call option.

What causes higher values of NPVq? Higher project values (*S*) or lower capital expenditures (*X*). Note further that NPVq also is higher whenever the present value of *X* is lower. Higher interest rates (r_f) or longer time to expiration (t) both lead to lower present values of *X*. Any of these changes (lower *X* or higher *S*, r_f , or t) increases the value of a European call.

Cumulative volatility is on the vertical axis of the graph, increasing

from top to bottom. As $\sigma\sqrt{t}$ increases, so does call value. What causes higher values of $\sigma\sqrt{t}$? Greater uncertainty about a project's future value and the ability to defer a decision longer. Either of these changes (higher σ or t) likewise increases the value of a European call.

Plotting projects in this twodimensional space creates a visual representation of their relative option values. No matter where you start in the graph, call value increases



Locating the Option Value in Two-Dimensional Space

We can locate investment opportunities in this two-dimensional space.



when you move down, to the right, or in both directions at once. Projects in the lower-right corner of the graph are high on both NPVq and $\sigma\sqrt{t}$ metrics and their option value is high compared with projects in the upper-left corner.

Locating various projects in the space reveals their value relative to one another. How do we get absolute values? That is, how can we get a *number*? Having gotten this far, we find that getting a number is easy. Because NPVq and $\sigma\sqrt{t}$ contain all five Black-Scholes variables, we can fill in a table with Black-Scholes call values that correspond to *every pair* of NPVq and $\sigma\sqrt{t}$ coordinates. I call this "pricing the space," and the table does it for us.

The exhibit "Using the Black-Scholes Option-Pricing Model to 'Price the Space'" shows part of the filled-in Black-Scholes table. Each

number expresses the value of a specific call option as a percentage of the underlying project's (or asset's) value. For example, for a project whose NPVq equals 1.0 and $\sigma\sqrt{t}$ equals 0.5, the value given in the table is 19.7%. Any European call option for which NPVq is 1.0 and $\sigma\sqrt{t}$ is 0.5 will have a value equal to 0.197 times S. If the assets associated with a particular project have a value (S) of \$100, then the project viewed as a call option has a value of \$19.70. If *S* were \$10, the call option value would be \$1.97, and so forth. Option values in the table are expressed in relative terms, as percentages of S, rather than in absolute dollars, to enable us to use the same table for both big and small projects. It's also convenient not to have to manipulate the Black-Scholes equation every time we want to value a project. The Black-Scholes model is

used once, to generate the table itself.² After that, we need only locate our project in the table and multiply by a factor of S.

Why is the option value \$19.70 less than the asset value of \$100? We've been analyzing sources of extra value associated with being able to defer an investment. The key is to remember that extra refers to a comparison between option value and net present value (NPV), not to a comparison between option value and present value (S). In this example, we are not expecting the option value to be greater than S; we are expecting it to be greater than NPV, which is S minus capital expenditures (*X*). *S* equals \$100 here, but we didn't say what X was. Since NPVq equals 1.0 in this example, X must in fact be greater than \$100; otherwise NPVq would be greater than 1.0. For concreteness, suppose that this is a

U	sing	the	B	lac	k-S	ich	ol	es (Opt	tio	n-l	Pri	ciı	ng	M	od	el	to	"Pr	ice	the	S	pace	1
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Each number in the table gives the value of a European call for specified values of NPVq and $\sigma\sqrt{t}$, as a percentage of *S*, the value of project assets.

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Example:
Suppose $S = 100
<i>X</i> = \$105
t = 1 year
$r_{f} = 5\%$
σ = 50% per year
then NPVq $= 1.0$
and $\sigma \sqrt{t}$ = 0.50.
The table gives a value of 19.7%.
Interpretation:
Viewed as a call option, the project has a value of:
Call value = 0.197 × \$100 = \$19.70.
Compare this to its conventional

NPV: NPV = S - X= \$100 - \$105 = \$-5. Black-Scholes value of a European call option, expressed as a percentage of underlying asset value.

								NF	۷q							
		0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00	1.02	1.04	1.06	1.08
	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.6	1.2	2.0	3.1	4.5	6.0	7.5
	0.10	0.0	0.1	0.2	0.3	0.5	0.8	1.2	1.7	2.3	3.1	4.0	5.0	6.1	7.3	8.6
	0.15	0.5	0.7	1.0	1.3	1.7	2.2	2.8	3.5	4.2	5.1	6.0	7.0	8.0	9.1	10.2
	0.20	1.5	1.9	2.3	2.8	3.4	4.0	4.7	5.4	6.2	7.1	8.0	8.9	9.9	10.9	11.9
	0.25	2.8	3.3	3.9	4.5	5.2	5.9	6.6	7.4	8.2	9.1	9.9	10.9	11.8	12.8	13.7
	0.30	4.4	5.0	5.7	6.3	7.0	7.8	8.6	9.4	10.2	11.1	11.9	12.8	13.7	14.6	15.6
	0.35	6.2	6.8	7.5	8.2	9.0	9.8	10.6	11.4	12.2	13.0	13.9	14.8	15.6	16.5	17.4
	0.40	8.0	8.7	9.4	10.2	11.0	11.7	12.5	13.4	14.2	15.0	15.9	16.7	17.5	18.4	19.2
\overline{t}	0.45	9.9	10.6	11.4	12.2	12.9	13.7	14.5	15.3	16.2	17.0	17.8	18.6	19.4	20.3	21.1
2	0.50	11.8	12.6	13.4	14.2	14.9	15.7	16.5	17.3	18.1	18.9	19.7	20.5	21.3	22.1	22.9
	0.55	13.8	14.6	15.4	16.1	16.9	17.7	18.5	19.3	20.1	20.9	21.7	22.4	23.2	24.0	24.8
	0.60	15.8	16.6	17.4	18.1	18.9	19.7	20.5	21.3	22.0	22.8	23.6	24.3	25.1	25.8	26.6
	0.65	17.8	18.6	19.3	20.1	20.9	21.7	22.5	23.2	24.0	24.7	25.3	26.2	27.0	27.7	28.4
	0.70	19.8	20.6	21.3	22.1	22.9	23.6	24.4	25.2	25.9	26.6	27.4	28.1	28.8	29.5	30.2
	0.75	21.8	22.5	23.3	24.1	24.8	25.6	26.3	27.1	27.8	28.5	29.2	29.9	30.6	31.3	32.0
	0.80	23.7	24.5	25.3	26.0	26.8	27.5	28.3	29.0	29.7	30.4	31.1	31.8	32.4	33.1	33.8
	0.85	25.7	26.5	27.2	28.0	28.7	29.4	30.2	30.9	31.6	32.2	32.9	33.6	34.2	34.9	35.5
	0.90	27.7	28.4	29.2	29.9	30.6	31.3	32.0	32.7	33.4	34.1	34.7	35.4	36.0	36.6	37.3

one-year option – that is, suppose we can defer the decision for one year – and that the risk-free rate of return (r_f) is 5%. Then for NPVq to equal 1.0, X must be \$105 Recall that:

NPVq =
$$S \div PV(X)$$

= \$100 ÷ (\$105 ÷ 1.05).

Thus the conventional NPV is actually negative:

NPV =
$$S - X$$

= \$100 - \$105
= -\$5.

And an option value of \$19.70 really is substantially greater than conventional NPV.

Using the Framework: An Example in Seven Steps

To illustrate how to apply the framework, consider this example of a hypothetical, but representative, capital investment. Division managers at a company we'll call Franklin Chemical are proposing a phased expansion of their manufacturing facilities. They plan to build a new, commer-

cial-scale plant immediately to exploit innovations in process technology. And then they anticipate further investments, three years out, to expand the plant's capacity and to enter two new markets. The initial investment is obviously strategic because it creates the opportunity for subsequent growth. Yet executives responsible for the company's capital budget are unimpressed by the project because its NPV is essentially zero. (Cash flow projections and NPV calculations for the projected investment are shown in the table "Franklin Chemical's Initial Calculations for a Proposed Expansion.") In fact, in the annual jockeying for funds, this program may not beat out competing alternatives. Its champions are frustrated and feel sure that the company's conventional NPV approach is missing something.

They are right.

This project has considerable option value because the initial expenditure of \$125 million buys the right to expand (or not) three years later. This is important because the expen-

Figures are in \$millions and have been rounded.

Year	0	1	2	3	4	5	6		
Operating projections									
revenues		455.0	551.0	800.0	1080.0	1195.0	1255.0		
-cost of good sold		341.3	414.9	596.0	811.1	893.9	941.3		
= gross profit		113.8	136.1	204.0	268.9	301.1	313.8		
-SG&A expense		110.4	130.0	219.2	251.6	280.3	287.4		
= operating profit		3.3	6.1	-15.2	17.3	20.8	26.3		
Cash flow calculation									
EBIT (1-tax rate)		2.2	4.0	-10.0	11.5	13.7	17.4		
+ depreciation		19.0	21.0	21.0	46.3	48.1	50.0		
-capital expenditures	100.0	8.1	9.5	307.0	16.0	16.3	17.0		
-increase NWC	25.0	4.1	5.5	75.0	7.1	8.0	9.7		
= free cash flow, assets	-125.0	9.0	10.0	-371.0	34.7	37.5	40.7		
+ terminal value, assets	(perpetu	ity value	with 5%	per year	growth)		610.3		
Discount to present value									
× discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507		
= PV (by year)	-125.0	8.0	8.0	-264.1	22.0	21.3	329.8		

Franklin Chemical's Initial Calculations for a Proposed Expansion

ditures in the third year are large – three times the initial investment.

Step 1 is to recognize the option and describe it. It takes practice to recognize the options that may be buried in conventional projects. However, there are at least two easy ways to see the option in our example. The first is simply to look beyond the numbers and examine the project's description. It surely says something about the two-phased

Recognizing the Option

Patterns in the total cash flows and/or in expenditure items such as capital expenditures and investments in net working capital provide clues about the structure of the option.





nature of the program by way of justifying the large outlays in year 3. The other is to examine the pattern of the project cash flows over time. The cash flows in the chart are very uneven: two figures are an order of magnitude larger than the other five, and both of these are negative. A graph of the capital-expenditures line would clearly show the spike in spending in year 3. Such a large sum is almost surely *discretionary*. That is, the company can choose not to make the investment, based on how things look when the time comes. This is a classic expansion option, sometimes called a growth option. (See the graphs "Recognizing the Option.")

Franklin's project has two major parts. The first part is to spend \$125 million now to acquire some operating assets. The second part is an option to spend an additional sum, more than \$300 million, three years from now to acquire the additional capacity and enter the new markets. The option here is a call option, owned by the company, with three years to expiration, that can be exercised by investing certain amounts in net working capital (NWC) and fixed assets. Viewing the project in this way, we want to evaluate the following: NPV (entire proposal) = NPV (phase I assets) + call value (phase 2 assets).

Phase 1 refers to the initial investment and the associated cash flows. It can be valued using NPV as usual. *Phase 2* refers to the opportunity to expand, which may or may not be exploited in year 3. To value phase 2, we will use the framework outlined above to synthesize a comparable call option and then value it.

Step 2 is to map the project's characteristics onto call option variables. This mapping will create the synthetic option we need and indicate where in the DCF spreadsheet we need to go to obtain values for the variables. The value of the underlying assets (S) will be the present value of the assets acquired when and if the company exercises the option. The exercise price (X) will be the expenditures required to acquire the phase 2 assets. The time to expiration (t) is three years, according to

How to Estimate Cumulative Volatility

The variable in our option-pricing model that managers are least accustomed to estimating is variance (σ^2), or standard deviation (σ), which we used to get our metric for cumulative volatility ($\sigma\sqrt{t}$). For a real option, σ cannot be found in a newspaper or in a financial statement, and most people do not have highly developed intuition about, say, the annualized standard deviation of returns on assets associated with entering a new market. In the example given in the text, we assume σ is 40% per year. Is that reasonable? Here are several sound approaches to creating or judging estimates of σ :

Take a(n educated) guess.

Assets to which you would assign higher hurdle rates because of their higher-than-average systematic risk are also likely to have higher values of σ . How high is "high" for standard deviation? Returns on broad-based U.S. stock indexes had a standard deviation of approximately 20% per year for most of the past 15 years; exceptions (upward spikes) were associated with events like the 1987 stock-market crash and the 1990-1991 Persian Gulf crisis. Individual stocks generally have a higher standard deviation than the market as a whole; returns on General Motors' stock, for example, have a σ of about 25% per year. Individual projects within companies can be expected to have a still higher σ . When I work with manufacturing assets and have no specific information at all about σ , I begin by examining a range: from 30% to 60% per year.

Gather some data.

For some businesses, we can estimate volatility using historical data on investment returns in the same or related industries. Alternatively, we might compute what is called *implied volatility* using current prices of options traded on organized ex-

changes. The idea is to observe a market price for an option whose parameters are all known except for σ . We then use a model like Black-Scholes to figure out what σ must be given all the other variables. Today we can get implied volatility for shares of a very large number of companies in many industries. Often it is possible to use implied volatilities from options on stocks to infer σ for assets in corresponding industries. The quality and availability of such data have improved enormously in the past ten years.

Simulate σ .

Spreadsheet-based projections of a project's future cash flows, together with Monte Carlo simulation techniques, can be used to synthesize a probability distribution for project returns. Once you have the synthesized distribution, the computer can quickly calculate the corresponding standard deviation. Simulation software packages for desktop computers are commercially available and work with the same popular spreadsheet applications that generate your company's DCF models. These tools also have become far more widely available and much easier to use in recent years.

the projections given in the DCF analysis, although we might want to quiz the managers involved to determine whether the decision actually could be made sooner or later. The three-year risk-free rate of interest (r_f) is 5.5% (which is the market rate of interest on a three-year U.S. government bond). Note for comparison that the risk-adjusted discount rate being applied in the spreadsheet is 12%. Finally, the standard deviation of returns on these operating assets (σ) is not given anywhere in the spreadsheet. For now, we'll assume that figure is 40% per year, a value that is neither particularly high nor low. The insert "How to Estimate Cumulative Volatility" explains ways to obtain values for σ in cases like these.

Step 3 is to rearrange the DCF projections for two purposes: to separate phase 1 from phase 2 and to isolate values for S and X. I generally find it easier to work on S and X first. That requires making a judgment about what spending is discretionary versus nondiscretionary or what spending is routine versus extraordinary. It also requires making a similar judgment about which cash inflows are associated with phase 1 as opposed to phase 2.

In this project, expenditures on net working capital and fixed assets obviously are lumpy. The very large sums in year 3 clearly are discretionary and form part of the exercise price (X). The smaller sums in other years are plausibly routine and may be netted against phase 2 cash inflows, ultimately to be discounted and form part of S, the value of the phase 2 assets.

Sometimes it is easy to separate phase I cash flows from phase 2 cash flows because whoever prepared the DCF analysis built it up from detailed, phase-specific operating projections. When that's the case, as it is in our example, all we have to do is use the disaggregated detail underlying the summary DCF analysis, and the table "Franklin's Projections Rearranged" presents this information for our example. At other times, we have to allocate cash flows to each phase. A common expedient is simply to break out the phase I cash inflows and terminal value. Then, phase 2 cash inflows and terminal value are whatever is left over. Note that when we discount cash flows for the two phases separately, we obtain the same NPV as before.

Step 4 is to establish a benchmark for phase 2's option value based on the rearranged DCF analysis. Having separated phases 1 and 2, we can get a conventional discounted-cashflow NPV for each, which can be seen in the table showing Franklin Chemical's rearranged calculations. This table shows that phase I alone has a positive NPV of \$16.3 million while phase 2's NPV is -\$16.2 million. The sum of the two is the same NPV, \$0.1 million, that we obtained originally. Already, we have a quantitative option-related insight. The value of the whole proposal must be *at least* \$16.3 million because the option value of phase 2, whatever it turns out to be, cannot be less than zero. In fact, if the option value of phase 2 is significant, the project as a whole will have a much higher value than \$16.3 million, to say nothing of the \$0.1 million we started with. This insight is available only when we separate the project's two phases and realize that we will have a choice about whether or not to undertake phase 2.

Our first DCF benchmark for phase 2 is -\$16.2 million. Actually, though, phase 2's conventional NPV is worse than that, and it's worth a digression to see why. The DCF valuation contains a common mistake. It discounted the discretionary spending in year 3 at the same 12% risk-adjusted rate that had already

Franklin's Projections Rearranged

The DCF projections have been rearranged to separate phases 1 and 2 and isolate S and X.

Year Phase 1	0	1	2	3	4	5	6
cash flow	0.0	9.0	10.0	11.0	11.6	12.1	12.7
+ terminal value							191.0
-investment	-125						
× discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507
= PV (each year)	-125.0	8.0	8.0	7.8	7.3	6.9	103.2

NPV (sum of years) **16.3** This is the conventional NPV of phase 1.

Phase 2

cash flow	0.0	23.1	25.4	28.0
+ terminal value				419.3
-investment	-382			
× discount factor (12%)	0.712	0.636	0.567	0.507
= PV (each year)	-271.9	14.7	14.4	226.6

NPV (sum of years) -16.2 This is the conventional NPV of phase 2.

Phases 1 and 2								
cash flow	0.0	9.0	10.0	11.0	34.7	37.5	40.7	
+ terminal value							610.3	
-investment	-125			-382				
× discount factor (12%)	1.000	0.893	0.797	0.712	0.636	0.567	0.507	
= PV (each year)	-125.0	8.0	8.0	-264.1	22.0	21.3	329.8	
NPV (sum of years) 0.1 This is the same conventional NPV we got previously.								
			Figur	res are in \$n	nillions an	d have bee	n rounded	

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been applied to the project's cash flows. That rate is almost certainly too high because such expenditures are rarely subject to the same operating and product-market forces that make the project's cash flows risky. Construction costs, for example, may be uncertain, but they are usually much more dependent on engineering factors, weather conditions, and contractors' performance than on customers' tastes, competitive conditions, industry capacity utilization, and such. Overdiscounting future discretionary spending leads to an optimistically biased estimate of NPV. To see the magnitude of this effect, discount the year 3 expenditures of \$382 million at 5.5% instead of 12% (again, it's as if we were putting investment funds into treasury bonds between now

and year 3). Then, phase 2 has a conventional DCF value of -\$69.6 million, not -\$16.2 million, and the NPV for the whole project goes from \$0.1 million to -\$53.4 million, a very substantial difference (See the table "Getting the Right Benchmark.")

Step 5 is to attach values to the option-pricing variables. Having reformulated the DCF spreadsheet, we can now pull values for *S* and *X* from it. *X* is the amount the company will have to invest in net working capital and fixed assets (capital expenditures) in year 3 if it wants to proceed with the expansion: \$382 million. *S* is the present value of the new phase 2 operating assets. In other words, it's the DCF value now (at time zero) of the cash flows those assets are expected to

produce in the fourth year and beyond. The same table shows that to be \$255.7 million. The other optionpricing variables have already been mentioned: *t* is 3 years, r_f is 5.5%, and σ is 40% per year.

Step 6 is to combine the five option-pricing variables into our two option-value metrics: NPVq and $\sigma\sqrt{t}$. In this case:

$$\mathsf{NPVq} = S \div \mathsf{PV}(X) = \frac{\$255.7}{382 \div (1.055)^3} = 0.786$$

And

 $\sigma\sqrt{t} = 0.4 \times \sqrt{3} = 0.693.$

The exhibit "Deriving the Option-Value Metrics for Franklin's Project" shows how all five variables were derived and how they combine to form our two metrics.



Step 7 is to look up call value as a percentage of asset value in our Black-Scholes option-pricing table. The table does not show values that correspond exactly to our computed value for NPVq and $\sigma\sqrt{t}$, but by interpolating we can see that the value of our synthesized call option is about 19% of the value of the underlying assets (S). Accordingly, the dollar value of the option is 0.19 times \$255.7 million, which equals \$48.6 million. Recall the value of the entire proposal is given by: NPV (entire proposal) = NPV (phase I assets) + call value (phase 2 assets). Filling in the figures gives: NPV (entire proposal) = \$16.3 million + \$48.6 million = \$64.9 million.

Our final estimate of \$64.9 million is a long way from the original figure for NPV of \$0.1 million and even further from -\$53.4 million. Yet the option-pricing analysis uses the same inputs from the same spreadsheet as the conventional NPV. What looks like a marginal-toterrible project through a DCF lens is in fact a very attractive one. This seems especially so when we compare \$64.9 million with the required initial investment of \$125 million; the value associated with the option to expand the plant in year 3 is fully half again as much as the initial investment. Few projects look so good.

What should you do next? All the things you would usually do when evaluating a capital project. Perform sensitivity analyses. Check and update assumptions. Examine particularly interesting or threatening scenarios. Compare and interpret the analysis in light of other historical or contemporary investments and transactions. In addition, now that you've begun pricing synthesized real options, you may want to consider adding a few other items to your list. Some are discussed further in the insert "How Far Can You Extend the Framework?" They include checking for project features that would make it more suitable to use an American rather than a European option. They also include checking for clear disadvantages associated with deferring investment, such as competitive preemption, which would offset some or all of the sources of value associated with waiting. Some of these concerns can be handled by straightforward modifications to the framework. Others require more sophisticated modeling than either this framework alone or conventional NPV generally can provide. Even in those cases, though, a naïvely formulated option value will augment whatever insight may be drawn from a DCF treatment alone. Remember that simply by recognizing the structure of the problem we gleaned an important insight about the value of the project in our example (that it had to be at least \$16.3 million) before we actually priced the option.

Does the framework really work? Yes. Even though we have taken some liberties, we know more about our project after using it than we did before. And if it seemed worthwhile, we could further refine our initial estimate of option value. But the key to getting useful insight from option

/ariable	Project Characteristic	Source in DCF Spreadsheet	Value	Metric
S	The present value of the project assets asso- ciated with phase 2.	Separate phase 1 from phase 2 cash flows; compute the DCF value of phase 2 alone.	\$255.7	
x	Spending required in year 3 to obtain the phase 2 assets.	Identify large discretionary portions of capital expenditures and net-working-capital spending associated with phase 2 only, in year 3.	\$382	$NPVq = \frac{\$255.7}{\$382 \div (1.055)^3} = 0.786$
t	Length of time phase 2 spending may be deferred.	Appears in spreadsheet to be three years; check with managers.	3 years —	
r _f	Time value of money at same horizon (t) as option.	Obtain market rate of interest on three-year U.S. government bond, assumed here to be 5.5%.	<u>5.5%</u>	$\sigma \sqrt{t} = 0.4 \times \sqrt{3} = 0.693$
σ	Standard deviation per year on phase 2 assets.	Not in DCF spreadsheet; obtain from similar traded assets, from implied volatilities on traded options, or try a range. Assume 40% per year to start.	40% per year	

How Far Can You Extend the Framework?

Real corporate projects will present immediate challenges to some of the simplifications underlying this framework. Can the framework be souped up to handle more complex problems? Or does its very simplicity present insurmountable limitations? When might it generate seriously misleading information?

Bells and whistles can be added to the framework fairly easily, although they require extra data, and the details are beyond the scope of this article. For example, the framework assumes that the amount and timing of a project's capital expenditures (X) are certain. But what if they are not? That's usually the case with business opportunities. The framework can be adapted to handle those circumstances, but the adaptation helps only if we can describe the uncertainty. Specifically, we need to know the probability distribution of *X* and the joint probability distribution of S and X. That is, it matters whether X tends to be high when S tends to be high; whether the opposite is true (that is, X tends to be high when S is low and vice versa); or whether they are both not only uncertain but also unrelated.

As another example, suppose the uncertainty (or variance) associated with a project changes over time. That's fairly common, and it makes sense that it should affect our estimate of cumulative volatility. Once again, if we know *how* the variance changes over time, or if we can make plausible guesses, the framework can be adapted without much trouble. The insert "Where to Find Additional Help" cites some readings that address how such real-world problems affect option values.

Both of these examples describe situations in which the real limitation is not the framework but rather the data or our knowledge of the project's parameters. Even when we know that we lack necessary data, the framework can help by showing us what the effect on value would be if the data were one thing or another. We might conclude that it's worthwhile to gather or create better data.

Some other real-world complications are thornier. The framework does a good job of capturing the extra value associated with deferring an investment decision. But what if there are particular costs associated with deferral? For example, companies trying to be first to market with the next generation of a hot product will incur large costs if deferral allows a competitor to preempt them. Anytime there are predictable costs to deferring, the option to defer an investment is less valuable, and we would be foolish to ignore those costs. If such additional costs were the only issue, we could easily handle them in our framework as long as we knew when the decision to invest or not to invest finally would be made. But in the real world, we often don't know. Companies may not be compelled to invest at a certain moment, but rather may have discretion to time their investments. So the problem is to decide not only whether to invest but also when.

In effect, many real options are American rather than European. American options can be exercised at any time prior to expiration; European options may be exercised only at expiration. The optionpricing table embedded in this framework prices European options. American options are more valuable than European options whenever the costs associated with deferral are predictable. When that's the case, I recommend using the framework to set the lower boundary of the option value and then supplementing that figure with another table or a spreadsheet-based algorithm to convert European option values to the corresponding American option values. That is not quite as easy as merely rewriting the pricing table because it would take a three-dimensional or possibly four-dimensional table to accommodate the extra variables needed. But it can be set up in a spreadsheet. In short, the combination of costs associated with deferral and a company's ability to time investments calls for some added analysis. But the framework needs to be augmented, not scrapped.

Finally, the Black-Scholes option-pricing model that generated the numbers in the table makes some simplifying assumptions of its own. They include assumptions about the form of the probability distribution that characterizes project returns. They also include assumptions about the tradability of the underlying project assets; that is, about whether those assets are regularly bought and sold. And they include assumptions about the ability of investors to continually adjust their investment portfolios. When the Black-Scholes assumptions fail to hold, this framework still yields qualitative insights but the numbers become less reliable. Consequently, it may be worthwhile to consult an expert about alternative models to improve the quantitative estimates of option value.

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Where to Find Additional Help

Graduate-level corporate-finance textbooks cover the basics of option pricing, beginning from first principles:

Zvi Bodie and Robert C. Merton, *Finance* (Upper Saddle River, N.J.: Prentice-Hall, 1998).

Richard A. Brealey and Stewart Myers, *Principles of Corporate Finance*, fifth edition (New York: McGraw-Hill, 1996).

Stephen A. Ross, Randolph W. Westerfield, and Jeffrey Jaffe, *Corporate Finance*, fourth edition (Chicago: Richard D. Irwin, 1996).

Other books go beyond the basics to treat specialized problems and present more advanced models for option pricing. Well-known titles include:

John C. Hull, *Options, Futures, and Other Derivatives,* third edition (Upper Saddle River, N.J.: Prentice-Hall, 1997).

David G. Luenberger, *Investment Science* (New York: Oxford University Press, 1998).

Paul Wilmott, Jeff Dewynne, and Sam Howison, *Option Pricing* (Oxford, England: Oxford Financial Press, 1993).

A few books focus on real options in particular. Each takes a somewhat different approach to modeling corporate opportunities:

Martha Amram and Nalin Kulatilaka, *Real Options: Managing Strategic Investment in an Uncertain World* (Boston, Mass.: Harvard Business School Press, forthcoming 1998).

Avinash K. Dixit and Robert S. Pindyck, *Investment Under Uncertainty* (Princeton, N.J.: Princeton University Press, 1994).

Lenos Trigeorgis, *Real Options: Managerial Flexibility* and Strategy in Resource Allocation (Cambridge, Mass.: MIT Press, 1996).

Shorter readings on selected related topics include:

Avinash K. Dixit and Robert S. Pindyck, "The Options Approach to Capital Investment," HBR May–June 1995.

W. Carl Kester, "Today's Options for Tomorrow's Growth," HBR March–April 1984.

Timothy A. Luehrman, "What's It Worth? A General Manager's Guide to Valuation," HBR May–June 1997.

Lenos Trigeorgis, ed., *Real Options in Capital Investment: Models, Strategies, and Applications* (Westport, Conn.: Praeger, 1995).

Finally, expanded versions of the option-pricing table published in this article are available in the Brealey and Myers text cited above and in Luehrman, "Capital Projects as Real Options: An Introduction," HBS case no. 295-074. pricing sooner rather than later is to build on, rather than abandon, the DCF-based NPV analysis your company already uses. Had we set out to value the option from scratch, it would have been more difficult and taken longer. It also would have been hard to tell how well we had done and when to stop working on it. Option pricing should be a complement to existing capital-budgeting systems, not a substitute for them. The framework presented here is a way to start where you are and get somewhere better.

1. There are other mathematical advantages, beyond the scope of this article, associated with using the quotient instead of the difference. Also, students of economics will recognize the similarity between NPVq and the famous "Tobin's q," which measures the ratio of the value of an asset to its replacement cost.

2. This form of option-pricing table is nearly as old as the Black-Scholes model itself. I first ran into it as an M.B.A. student using Richard Brealey and Stewart Myers's text, *Principles of Corporate Finance* (New York: McGraw-Hill, 1981).

In the September–October 1998 issue of HBR, Timothy Luehrman extends the framework presented here. In "Strategy as a Portfolio of Real Options," he explores how managers can use option pricing to improve the way they make decisions about the sequence and timing of strategic investments.

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