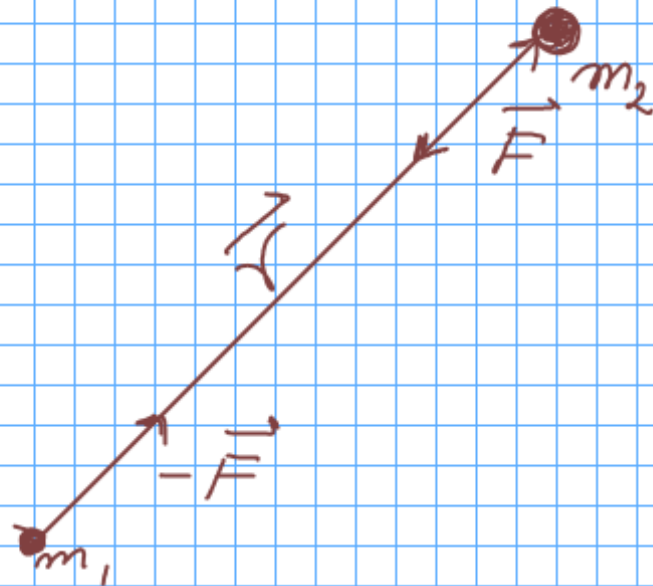


GRAVEDAD

Ley de Gravitación Universal



$$\vec{F} = -G \frac{m_1 m_2}{r^2} \hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$G = 6.6732 \times 10^{-8} \frac{\text{dyne cm}^2}{\text{g}^2} \quad (\text{C. g. s.})$$

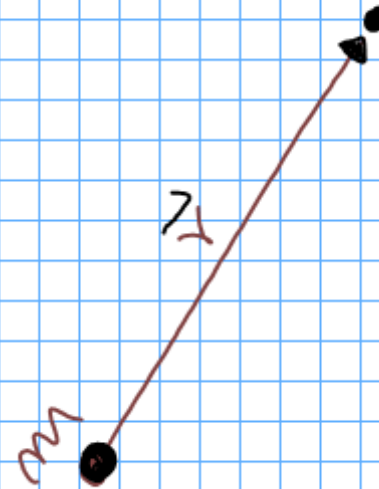
$$= 6.6732 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2} \quad (\text{M. K. S.})$$

Gravedad, Campo gravitatorio

$$\frac{\vec{F}}{m_2} = -\frac{G m_1}{r^2} \hat{r} \quad \text{campo de } m_1$$

Para una masa m en general, su campo gravitatorio es:

$$\vec{g} = -\frac{G m}{r^2} \hat{r}$$



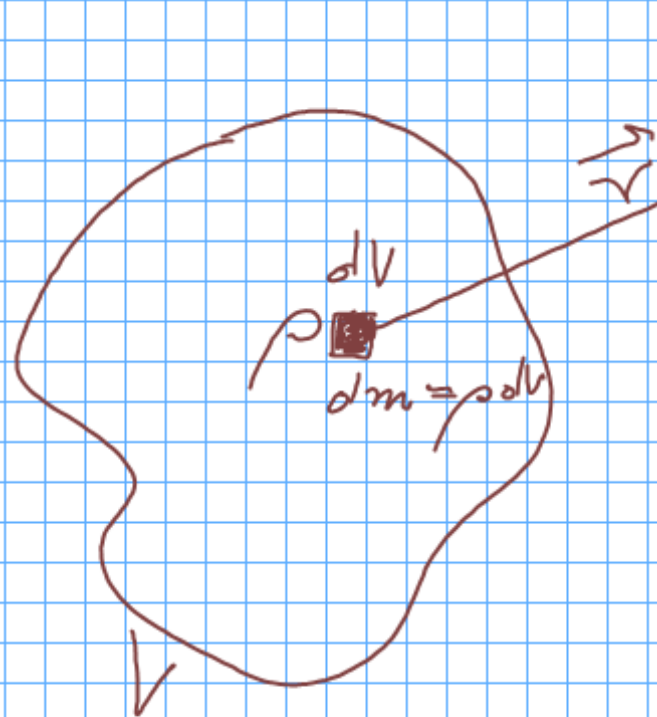
Unidades

$$\text{MKS, } g \left[\frac{\text{m}}{\text{s}^2} \right]; \text{ cgs, } g \left[\frac{\text{cm}}{\text{s}^2} \right]$$

$$1 \text{ gal} = 1 \frac{\text{cm}}{\text{s}^2}, \quad 1 \text{ mgal} = 10^{-3} \text{ gal}$$

$$g(\text{sup. Tierra}) \sim 980000 \text{ mgal}$$

Distribución de masas

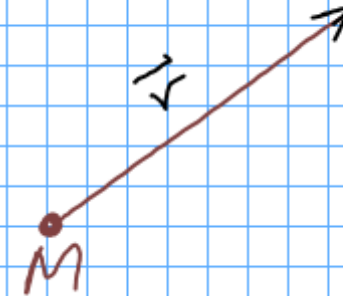
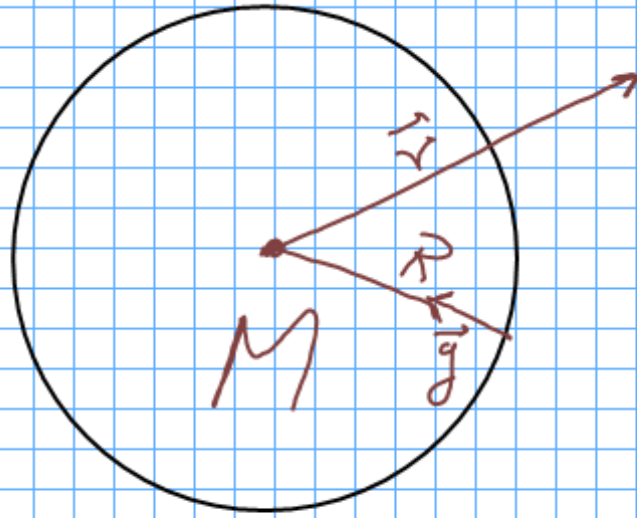


$$d\vec{g} = -G \frac{dm}{r^2} \hat{r}$$

$$\vec{g} = \int_V -G \frac{dm}{r^2} \hat{r}$$

$$\vec{g} = -G \int_V \frac{\rho dV}{r^2} \hat{r}$$

$$= -G \int_V \frac{\rho dV}{r^3} \vec{r}$$

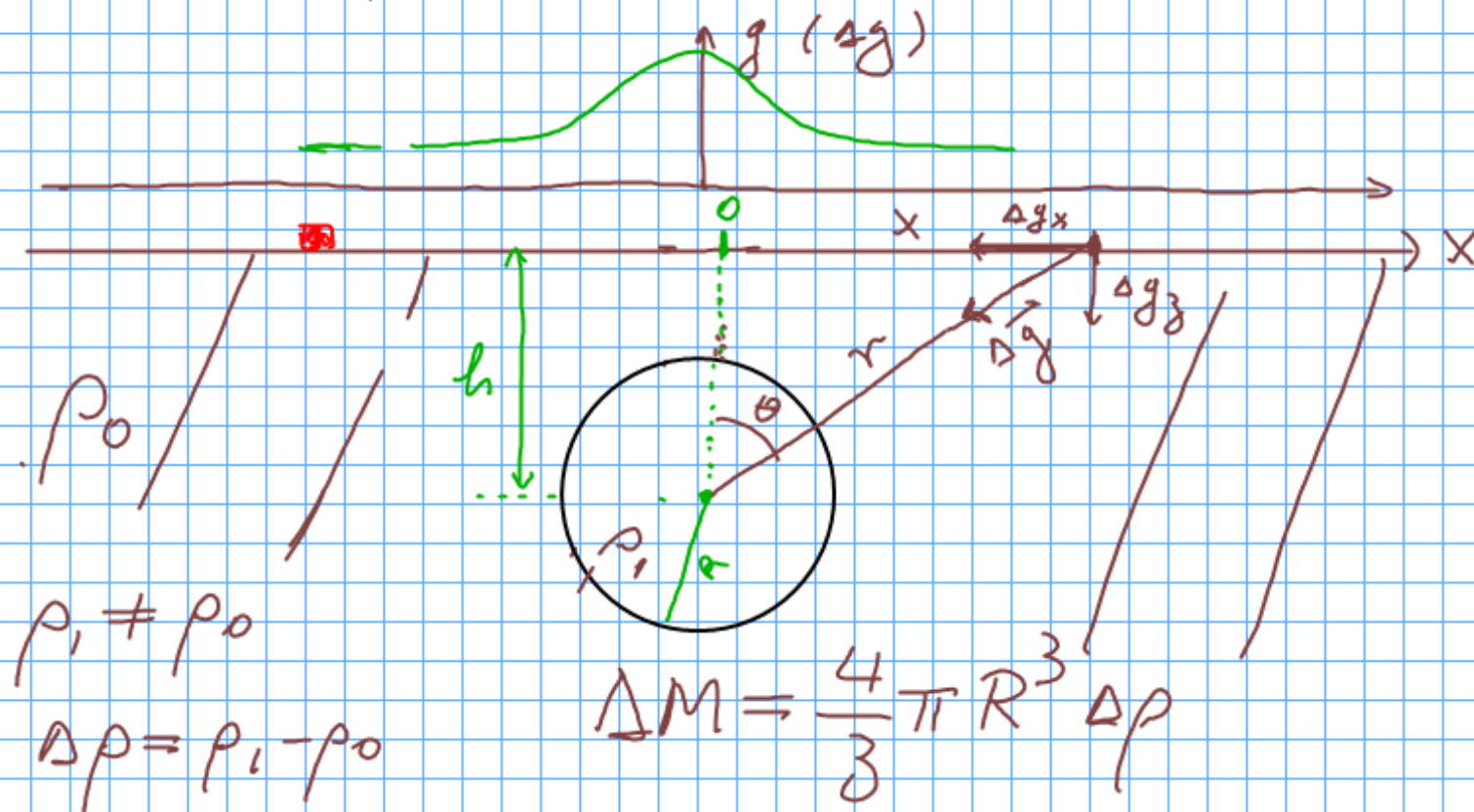


Distribución de masa radialmente
simétrica

$$g = \frac{GM}{R^2}$$

$$\rho = \rho(r)$$

Efecto gravitatorio (anomalía) de esfera a profundidad h y contraste de densidad $\Delta\rho$

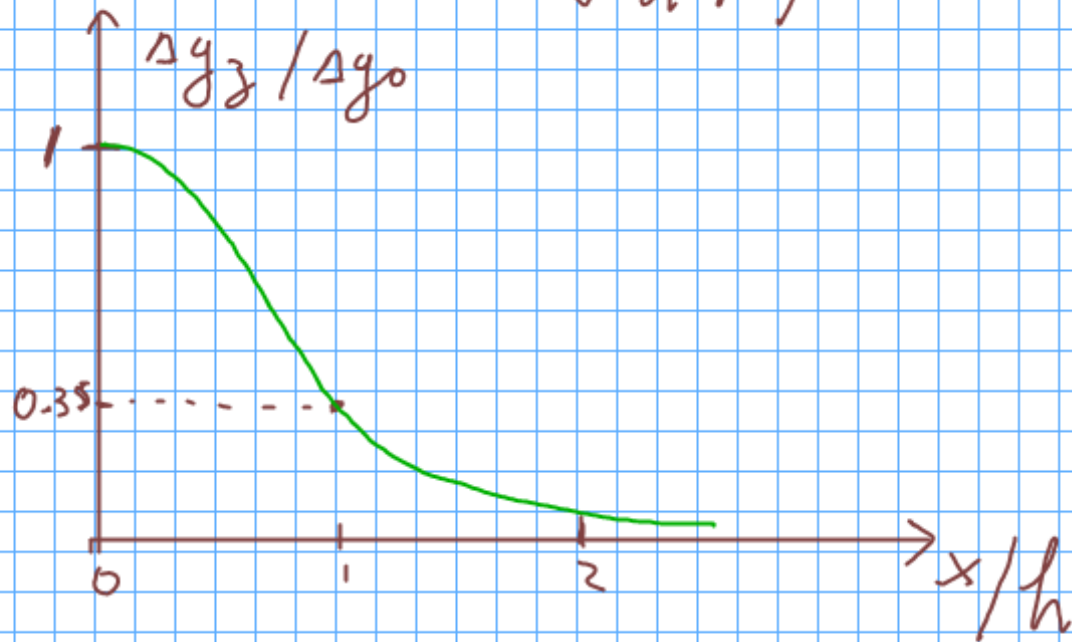


$$\Delta g = \frac{G \Delta M}{r^2} = G \frac{4}{3} \pi R^3 \Delta\rho \frac{1}{r^2}$$

$$\Delta g_z = \Delta g \cos\theta = \Delta g \frac{h}{r} \quad , \quad r = \sqrt{h^2 + x^2}$$

$$\Delta g_z = \frac{4}{3} \frac{\pi G R^3 \Delta \rho}{h^2} \frac{1}{\sqrt{1 + \left(\frac{x}{h}\right)^2}^3} = \Delta g_z(x)$$

$$\Delta g_z = \frac{\Delta g_0}{\left(\sqrt{1 + \left(\frac{x}{h}\right)^2}\right)^3}, \quad \Delta g_0 = \Delta g_z(x=0)$$



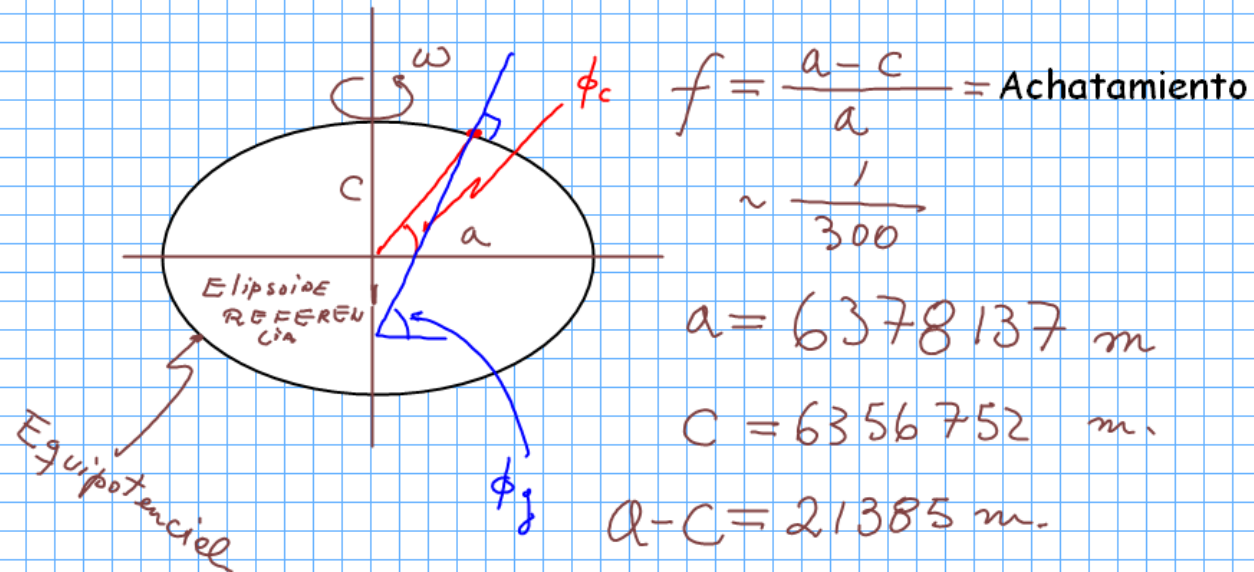
Ejemplo más o menos realista

$$h = 1 \text{ km} = 10^5 \text{ cm}, \quad R = 500 \text{ m.}$$

$$\Delta \rho = 1 \frac{\text{g}}{\text{cc}}$$

$$\Delta g_0 = \frac{4\pi \times 6.67 \times 10^{-8} \times (0.5 \times 10^5)^3 \times 1}{3 \times (10^5)^2} = 3.5 \text{ mgal.}$$

Forma normal de la Tierra, gravedad normal



$\phi_c =$ Latitud Geocentrica

$\phi_g =$ Latitud Geográfica (ϕ)

$$\tan \phi_c = \frac{c^2}{a^2} \tan \phi_g$$

Gravedad normal, teórica o de referencia

Para el elipsoide equipotencial de referencia se tiene (sin demostración. Ver Physical Geodesy, Heiskanen & Moritz):

$$g = g_e \left[\frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \right]$$

$g_e =$ Gravedad ecuatorial

$\phi =$ Latitud Geográfica

$k, e \ll 1$, dependen de M, a, f, ω

$$k = \frac{C_{gp} - a g_e}{a g_e}, \quad e = \sqrt{\frac{a^2 - c^2}{a^2}} = \text{Excentricidad}$$

$$g_e = 9.780327 \text{ m/s}^2$$

$$g_p = 9.832186 \text{ m/s}^2$$

$$f' = \frac{g_p - g_e}{g_e} \sim 0.0053 = 1/188.68$$

$$g_p - g_e = 5186 \text{ mgal}$$

Gravedad normal, otras fórmulas de referencia

Fórmula internacional de gravedad (1930):

$$g = 9.78049 (1 + 0.0052884 \sin^2 \phi - 0.0000059 \sin^2 2\phi)$$

Fórmula internacional de gravedad (1967):

$$g = 9.78031846 (1 + 0.0053024 \sin^2 \phi - 0.0000058 \sin^2 2\phi)$$

GRS80 ----> WGS84

$$g = 9.7803267714 \frac{1 + 0.00193185138639 \sin^2 \phi}{\sqrt{1 - 0.00669437799013 \sin^2 \phi}}$$

Variación de gravedad con latitud

$$g \sim g_e (1 + A \sin^2 \phi)$$

$$\frac{dg}{d\phi} = g_e A 2 \sin \phi \cos \phi = g_e A \sin 2\phi$$

En términos de distancia en la dirección N-S
(dS):

$$dS = R_T d\phi, \quad d\phi = \frac{dS}{R_T}$$

$$\frac{dg}{dS} = \frac{g_e A}{R_T} \sin 2\phi$$

$$\text{Usando: } R_T = 6371 \text{ km}, g_e = 9.78032534$$

$$A = 0.0053$$

$$\frac{g_e A}{R_T} = 0.814 \frac{\text{mgal}}{\text{km}}$$

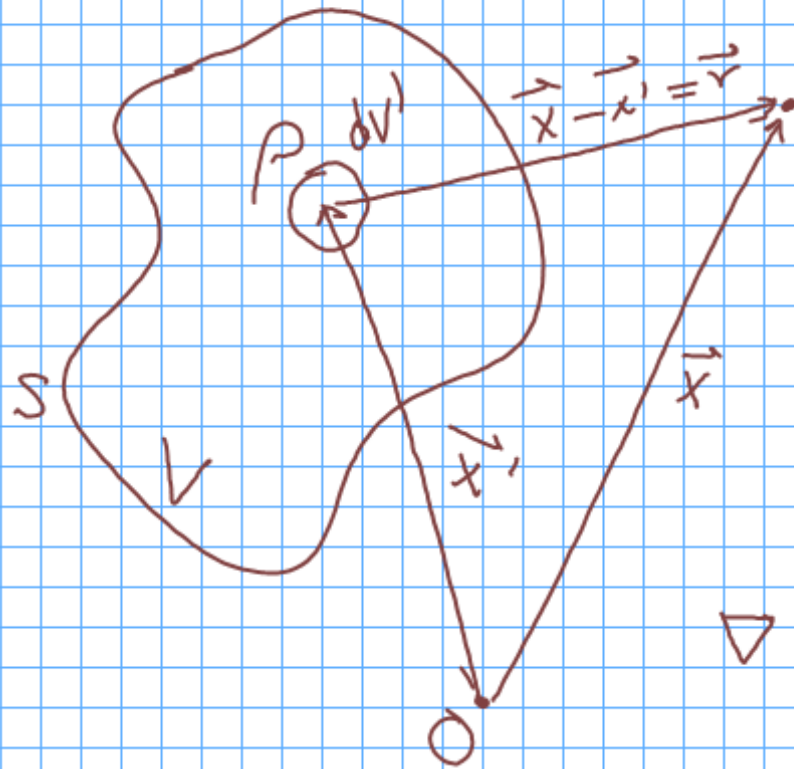
$$\frac{dg}{ds} = 0.814 \sin 2\phi \frac{\text{mgal}}{\text{km (N-S)}}$$

Elipsoide, Geoide, Superficie terrestre



$$h = H + N, \quad N \lesssim 100 \text{ m}$$

ASPECTOS TEÓRICOS ADICIONALES,



$$\vec{g}(\vec{x}) = -G \int_V \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} \rho(\vec{x}') dv'$$

$$= G \int_V \left[\nabla \left[\frac{1}{|\vec{x} - \vec{x}'|} \right] \right] \rho dv'$$

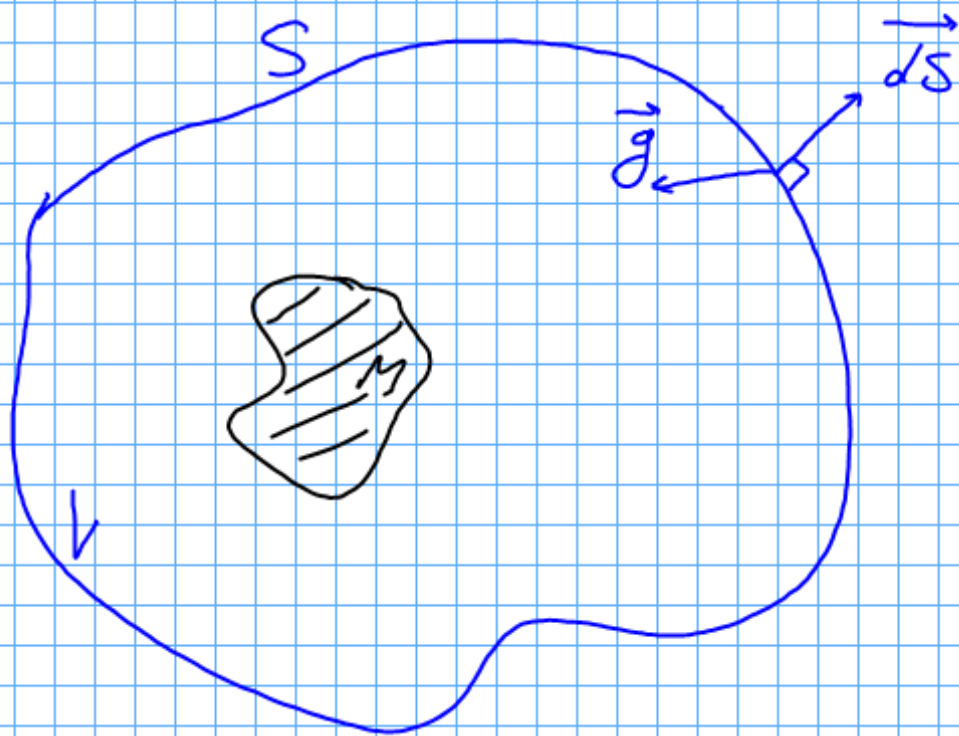
$\nabla =$ operador del gradiente sobre \vec{x}
(x, y, z)

$$\vec{g}(\vec{x}) = -\nabla \left[-G \int_V \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} dv' \right] \quad U$$

$$\vec{g}(\vec{x}) = -\nabla U, \quad U = -G \int_V \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} dV'$$

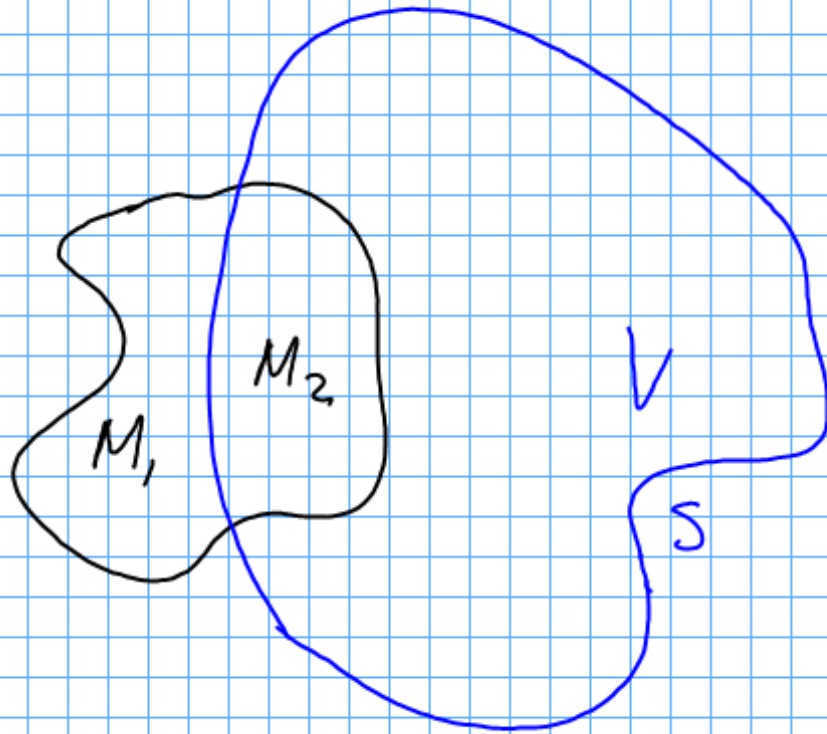
$U =$ Potencial gravitatorio

LEY DE GAUSS (GRAVEDAD):

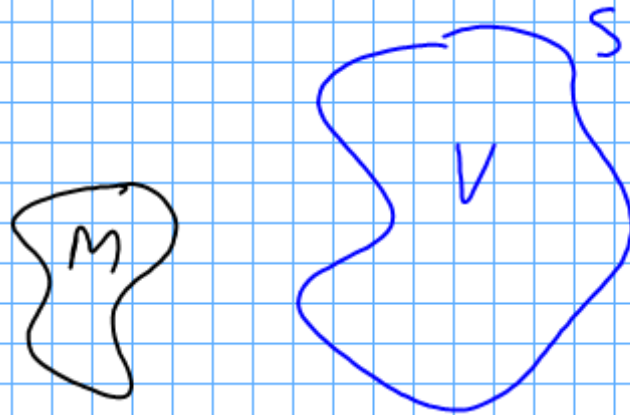


$$\int_S \vec{g} \cdot \vec{ds} = -4\pi G M$$

$M =$ Masa total encerrada por S , o contenida en V .

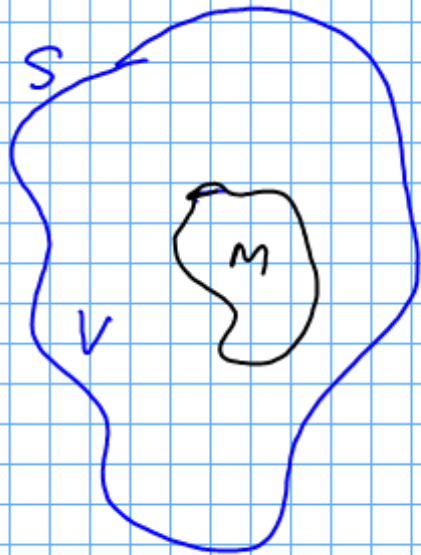


$$\int_S \vec{g} \cdot d\vec{S} = -4\pi G M_2$$



$$\int_S \vec{g} \cdot d\vec{S} = 0$$

LEY DE GAUSS, FORMA DIFERENCIAL.



$$\int_S \vec{g} \cdot \vec{ds} = -4\pi GM = -4\pi G \int_V \rho dV$$

Teo. DIVERGENCIA:

$$\int_S \vec{g} \cdot \vec{ds} = \int_V \nabla \cdot \vec{g} dV = -4\pi G \int_V \rho dV (*)$$

$$\left[\text{si } \vec{g} = (g_x, g_y, g_z), \quad \nabla \cdot \vec{g} = \frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \frac{\partial g_z}{\partial z} \right]$$

Como (*) es válida para cualquier V, entonces los integrandos Tienen que ser iguales:

$$\nabla \cdot \vec{g} = -4\pi G \rho$$

$$\nabla \cdot \vec{g} = -4\pi G\rho$$

$$\vec{g} = -\nabla U \quad \therefore \nabla \cdot (-\nabla U) = -4\pi G\rho$$

$$\therefore \nabla^2 U = 4\pi G\rho, \text{ Ec. Poisson}$$

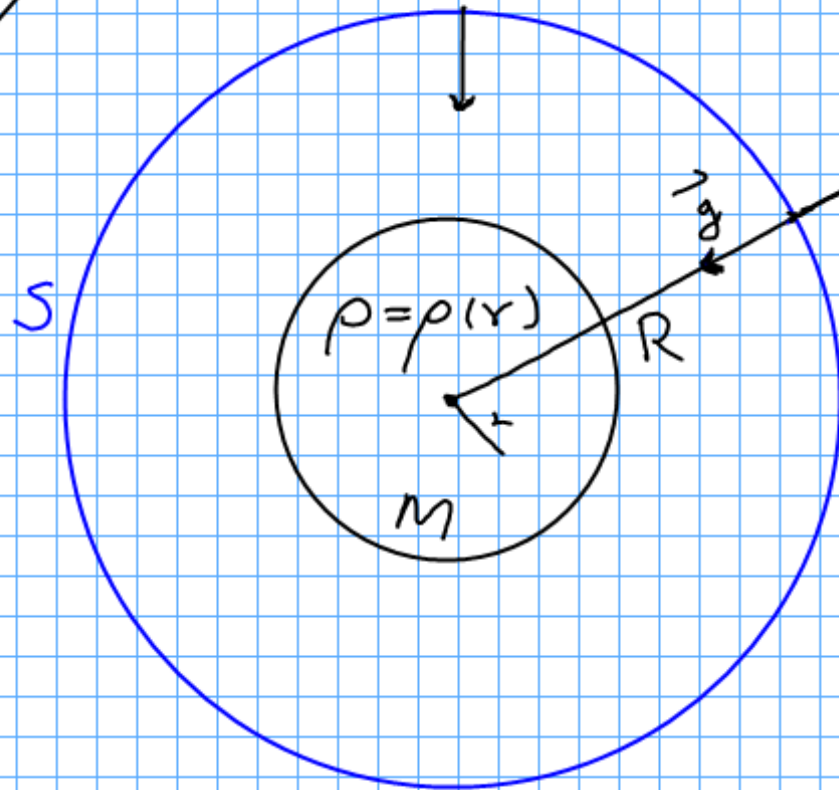
$$\left[U = U(x, y, z), \nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right]$$

En una zona donde $\rho = 0$.

$$\nabla^2 U = 0, \text{ Ec. de LAPLACE.}$$

APLICACIONES ELEMENTALES DE LEY DE GAUSS.

a) DISTRIBUCIÓN esférica de masa.

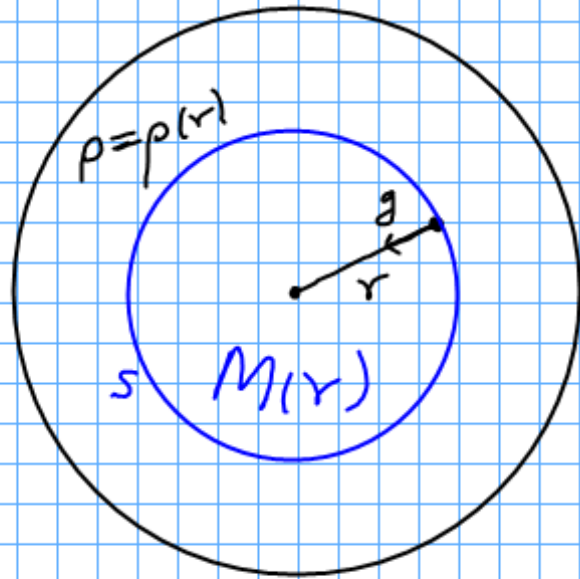


$$\int_S \vec{g} \cdot \vec{ds} = - \int_S g ds$$

$$= -g \int_S ds$$

$$\rightarrow -4\pi GM = -g 4\pi R^2$$

$$g = \frac{GM}{R^2}$$



$$g(r) = \frac{GM(r)}{r^2}$$

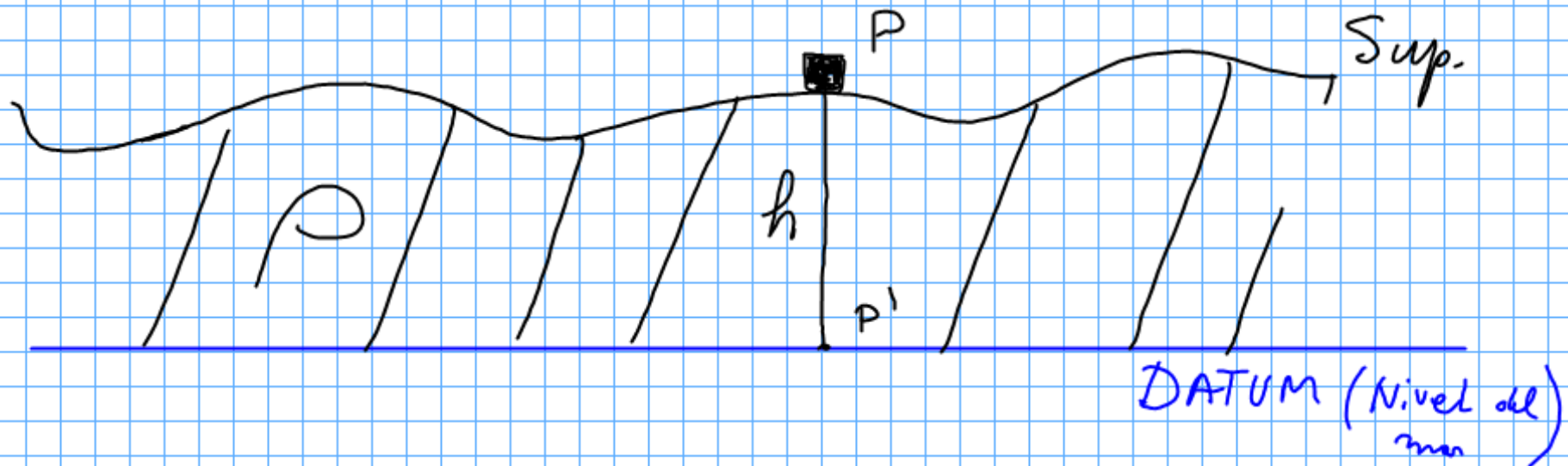
b) Placa de espesor h , infinitamente extendida lateralmente



$$g = 2\pi G\rho h.$$

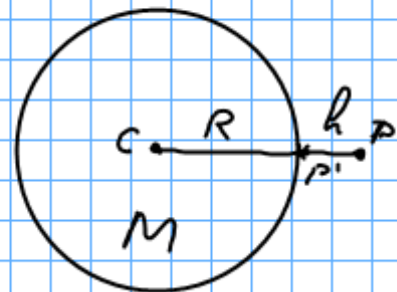
Independiente de la altura sobre la placa.

MEDICIONES DE GRAVEDAD, CORRECCIONES Y ANOMALIAS.



CORRECCIONES

1) AIRE LIBRE (POR ALTURA) (dg_A)



$$\text{En } P', \quad g = g_0 = \frac{GM}{R^2}$$

En P por otro lado

$$g = \frac{GM}{(R+h)^2} = \frac{GM}{R^2} \left(1 + \frac{h}{R}\right)^{-2} = g_0 \left(1 + \frac{h}{R}\right)^{-2}$$

$\frac{h}{R} \ll 1$. Usando la expansión binomial en 1er orden se tiene:

$$g \approx g_0 \left(1 - 2\frac{h}{R}\right)$$

$$g - g_0 = dg = -\frac{2g_0}{R} h.$$

$$dg_A = \frac{2g_0}{R} h$$

Usando: $g_0 = 9.8063 \text{ m/s}^2$, $R = 6371 \text{ km}$.

Se tiene

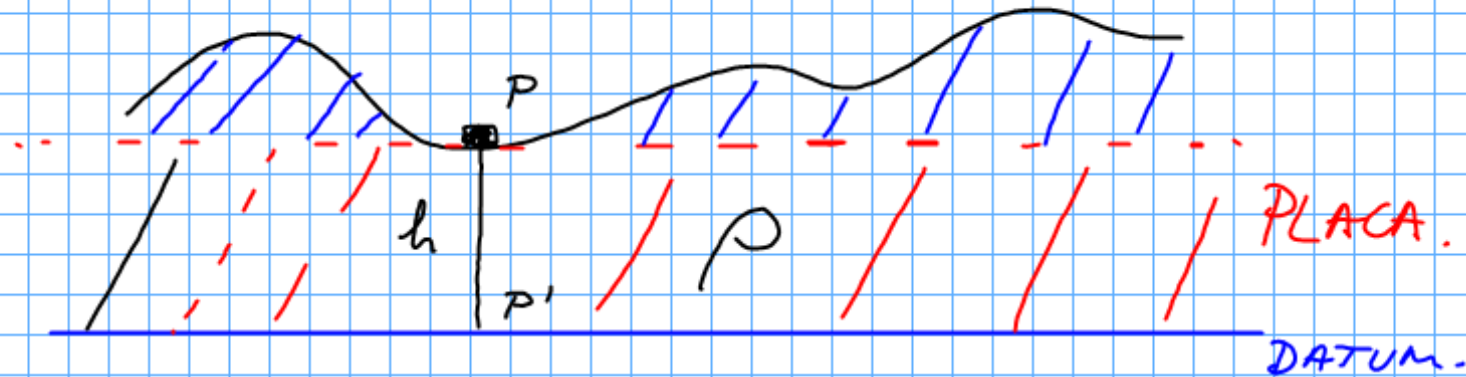
$$dg_A = 0.3078 \frac{\text{mgel}}{\text{m}}$$

Un cálculo exacto utilizando WGS84,

	dg_A (mgel/m)	
	$h=0$ (Sobre elipsoide)	$h=4 \text{ km}$.
Ecuador	0.3088	0.3082
Polo.	0.3083	0.3078

El valor intermedio que normalmente se toma (constante)

$$dg_A = 0.3086 \text{ mgel/m}$$

2) BOUGUER ($d_g A$) (PLACA)

$$d g_B = 2 \pi G \rho h.$$

$$d g_B [\text{mgal}] = 0.0419 \rho \left[\frac{\text{g}}{\text{cc}} \right] h (\text{m})$$

HABITUALMENTE se usa $\rho = 2.67 \frac{\text{g}}{\text{cc}}$

$$d g_B [\text{mgal}] = 0.1119 h (\text{m})$$

3) CORRECCIÓN POR LATITUD.

- Si TRABAJAMOS ABSOLUTAMENTE. (g total)

$$g_0 = g_e \left[\frac{1 + k \operatorname{sen}^2 \phi}{\sqrt{1 - e^2 \operatorname{sen}^2 \phi}} \right]$$

- SI TRABAJAMOS RELATIVAMENTE.

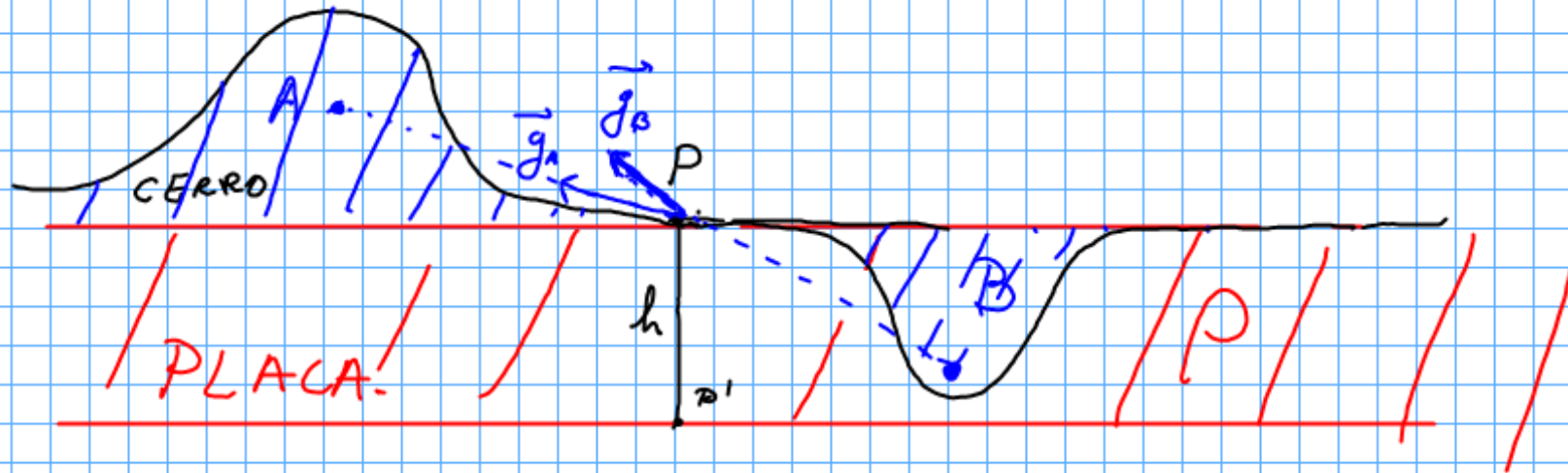
$$\frac{dg}{dS_{N-S}} \sim \frac{g_e A \operatorname{sen} 2\phi}{R_T}, \quad A \sim 0.0053$$

$$g_e = 9.78032534$$

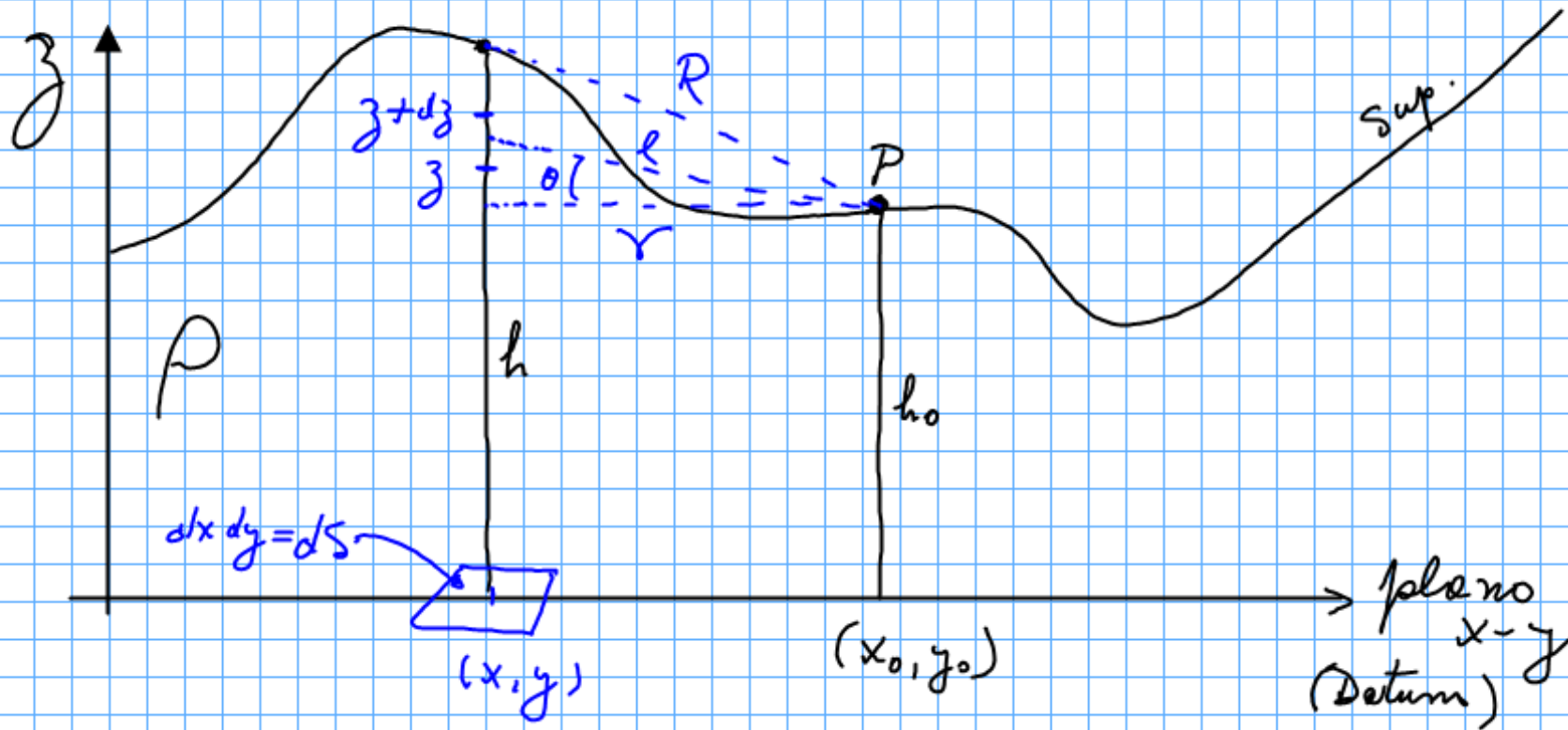
$$\frac{dg}{dS_{N-S}} \sim 0.814 \operatorname{sen} 2\phi \text{ (mgal)} \quad R_T = 6371 \text{ km}$$

S_{N-S} en km.

4) CORRECCIÓN TOPO GRÁFICA (d_{GT})



Definiendo $d_{GT} \geq 0$, d_{GT} se debe siempre
sumar al valor observado.



$h(x, y) = \text{Topografía}$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}, \quad R = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

La contribución de el elemento entre z y $z+dz$:

$$dg = |d\vec{g}| = \frac{G dm}{l^2} = \frac{G \rho dS dz}{l^2}$$

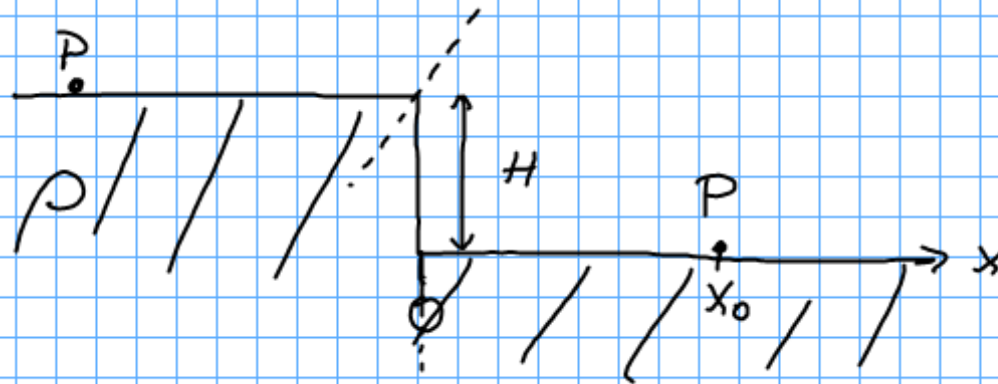
$$dg_z = dg \sin \theta = G \rho dS \frac{(z - h_0)}{[r^2 + (z - h_0)^2]^{\frac{3}{2}}} dz$$

$$dg_T = \rho G \int_{(x-y)} dS \int_{h_0}^h \frac{(z - h_0) dz}{[r^2 + (z - h_0)^2]^{\frac{3}{2}}}$$

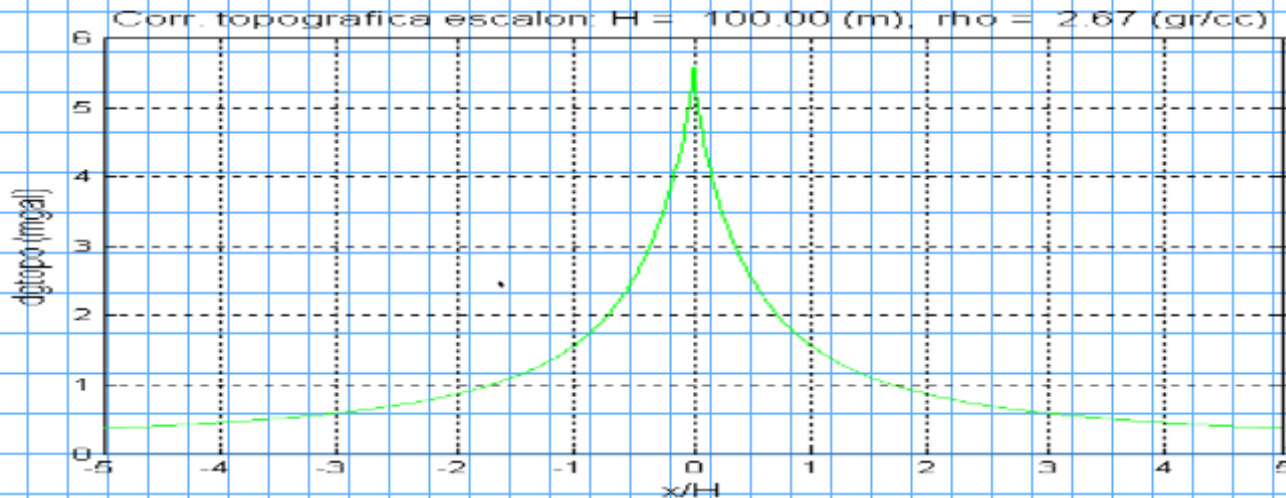
Integrando en z se tiene:

$$dg_T(x_0, y_0) = \rho G \int_{(x-y)} dS \left[\frac{1}{r} - \frac{1}{R} \right]$$

Ej: CORR. TOPOGRÁFICA DE ESCALÓN

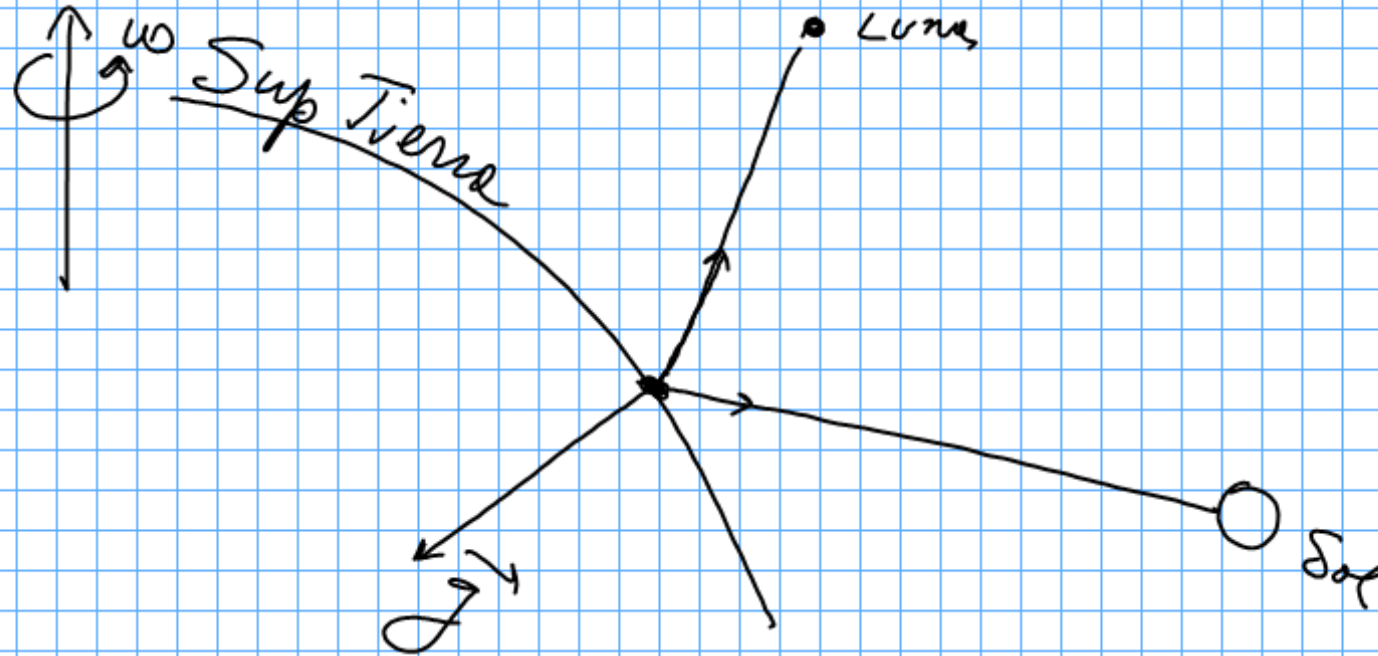


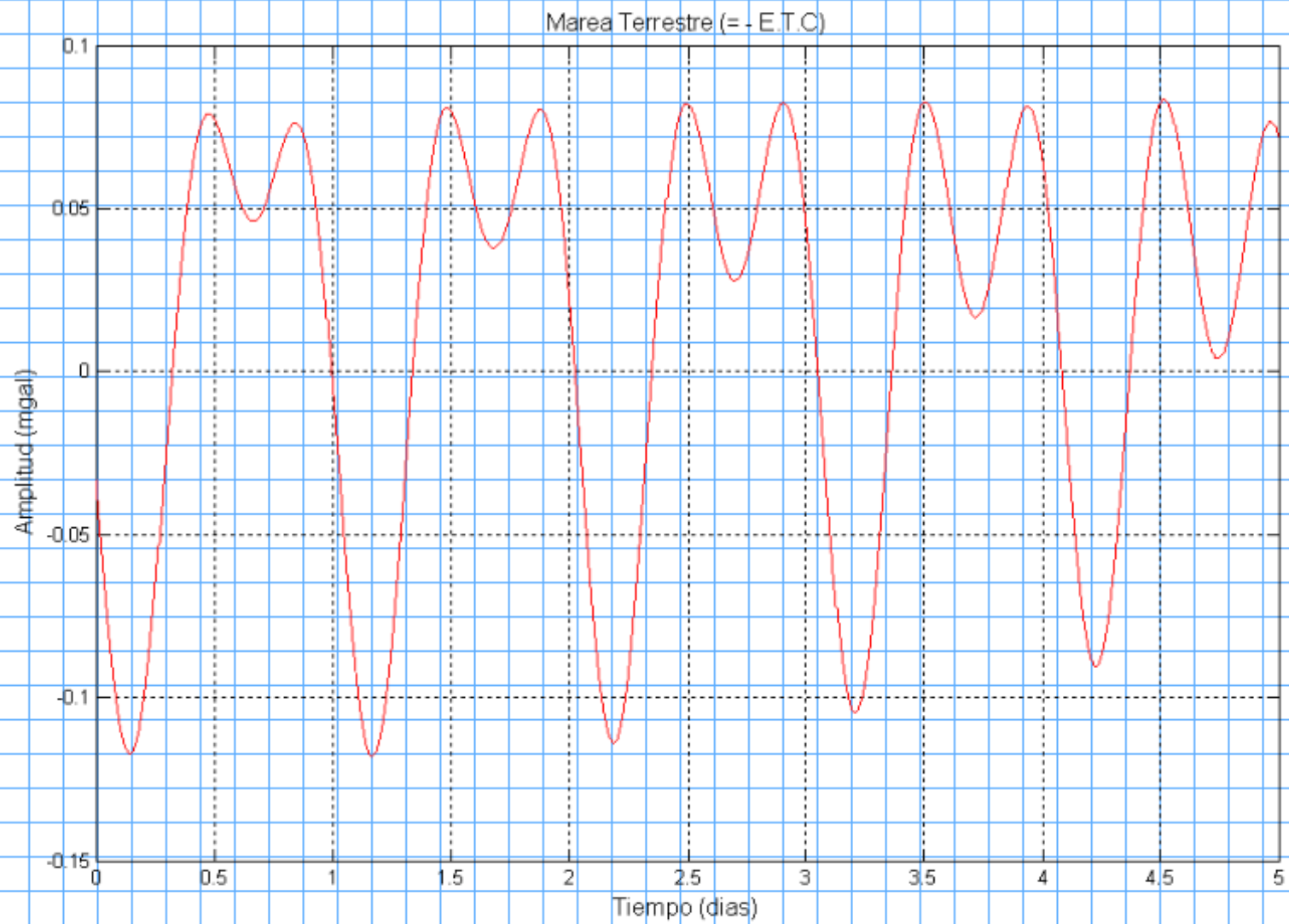
$$dg_T(x') = 2\rho G H \left[\arctan\left(\frac{1}{x'}\right) + x' \ln \frac{x'}{\sqrt{1+x'^2}} \right]$$



$$x' = \frac{x_0}{H}$$

5) Corrección por mareas





ANOMALÍAS GRAVIMÉTRICAS.

$$dg_A = \frac{2g}{R_T} h \longrightarrow 0.3086 \text{ mgal/m}$$

$$dg_B = 2\pi G\rho h \longrightarrow 0.1119 \text{ mgal/m} \left(\rho = 2.67 \frac{\text{g}}{\text{cc}}\right)$$

1) AIRE LIBRE.

g_{obs} contiene corrección por mareas. y por deriva instrumental.

g_0 Gravedad teórica o normal (Formulas en 3)

$$\Delta g_{AL} = g_{\text{obs}} + dg_A - g_0$$

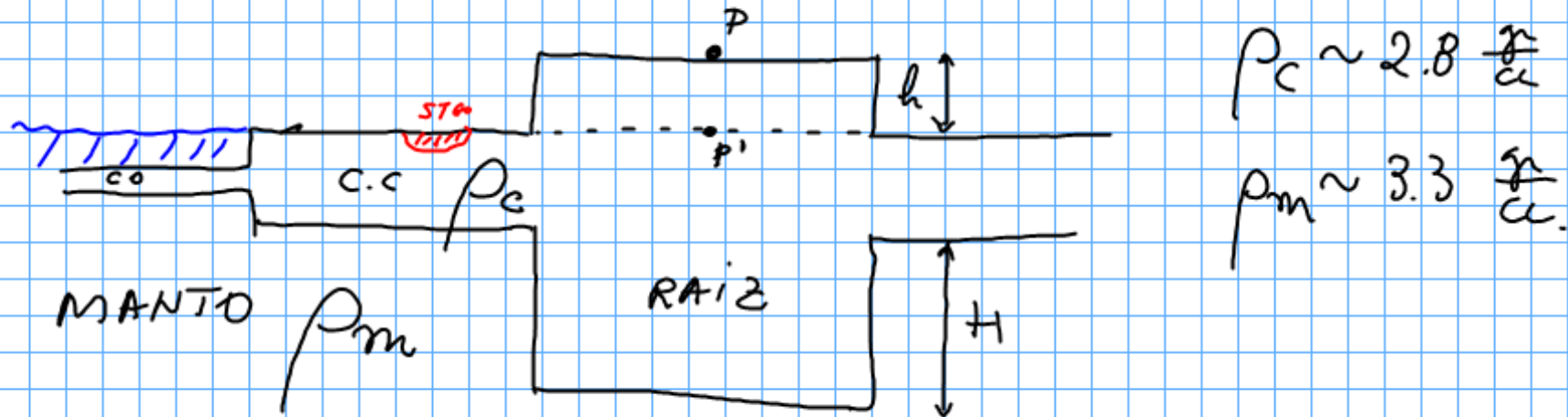
2) ANOMALIA DE BOUGUER (SIMPLE)

$$\Delta g_B = g_{obs} + dg_A - dg_B - g_0$$

3) ANOMALIA DE BOUGUER (COMPLETA)

$$\Delta g_B = g_{obs} + dg_A - dg_B + dg_T - g_0$$

TENDENCIA REGIONAL DE GRAN ESCALA.



ISOSTACIA: $H = \frac{\rho_c}{\rho_m - \rho_c} h \sim 5.6 h$

EFFECTO GRAVIMÉTRICO DE LA RAIZ.

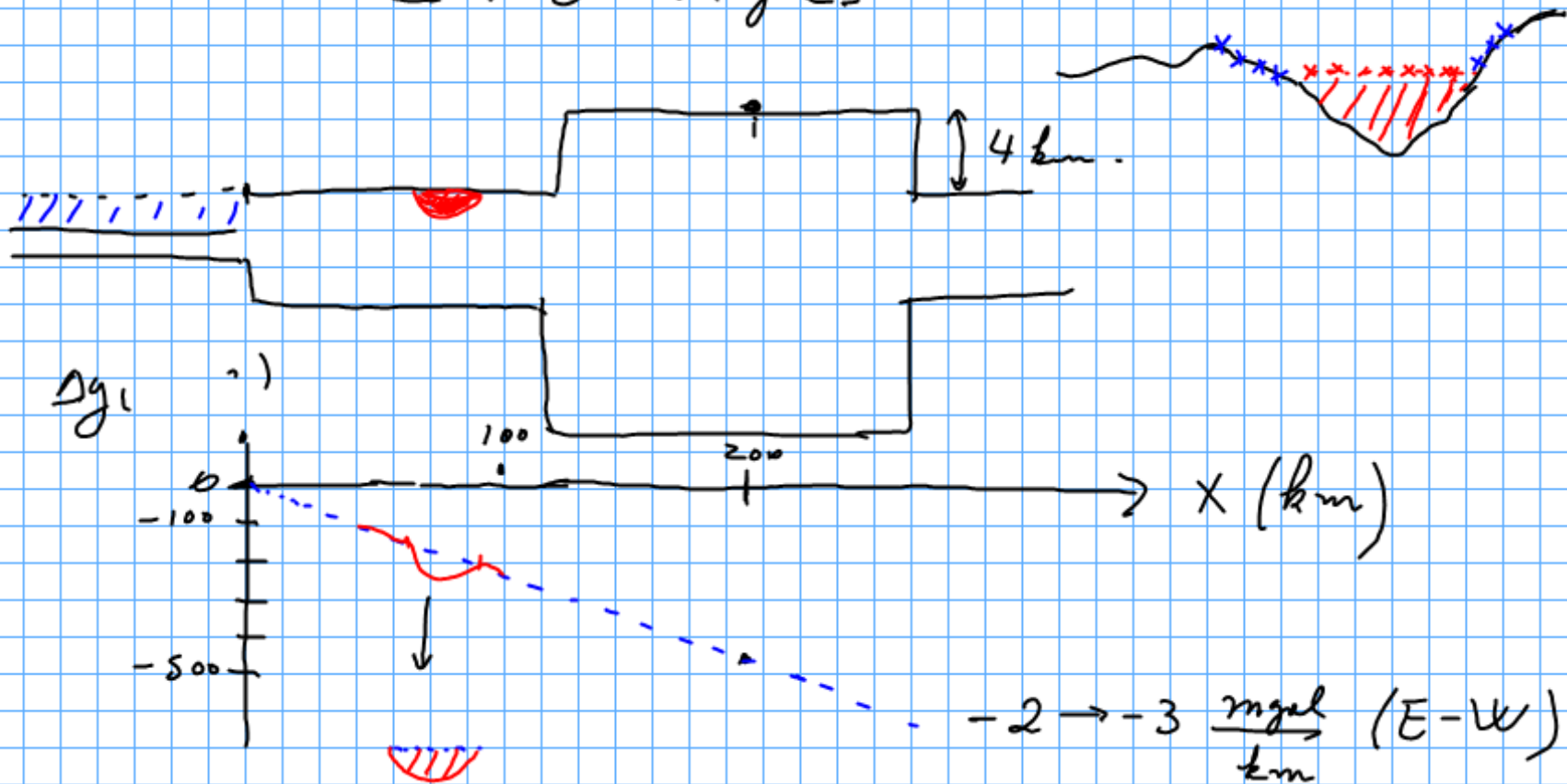
$$\Delta g_H = 2\pi G \Delta \rho H = 2\pi G \frac{\rho_c}{\rho_m - \rho_c} h$$

$$= 2\pi G \rho_c h$$

Para $h = 1 \text{ km}$, $\rho_c = 2.8$

$$\Delta g_H = 0.042 \rho \times 1000 = 0.042 \times 2.8 \times 1000$$

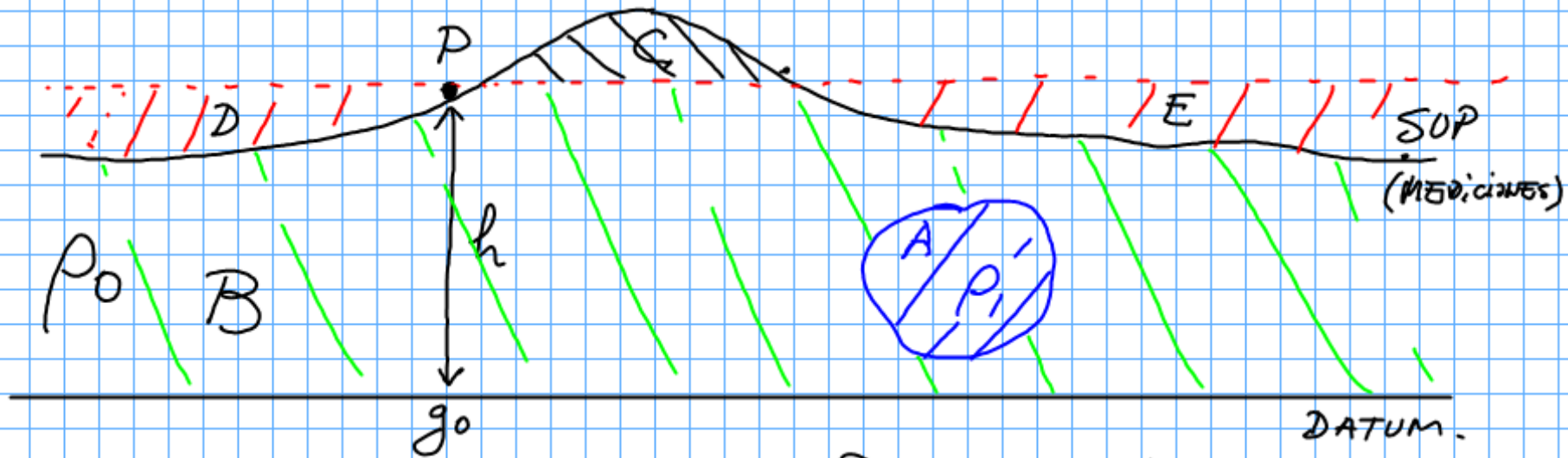
$$= 118 \text{ mgel.}$$



DENSIDAD DE ROCAS,

ROCAS SEDIMENTARIAS	$\rho \left[\frac{\text{gr}}{\text{cc}} \right]$
BARROS.	1.2 - 1.5
ARENAS	1.7 - 2.0
CALISAS, DOLOMITAS, MARMOL	2.5 - 2.8
ARENISCA	2.2 - 2.5
PIZARRAS	2.5 - 2.7
ROCAS IGNEAS.	
GRANITO	2.67
GRANODIORITA	2.72
DIORITA	2.84
BASALTO	2.75 - 2.9
GABRO	2.98
PERIDOTITA, DUNITA, PIROXENITA	3.2 - 3.3
CUARSITA, MICA, GRAISS, ESQUINITAS ROCAS METAMORFICAS, PORNEDRO	2.75

INTERPRETACIÓN ANOMALIAS GRAVIMÉTRICAS,



A_1 = gravedad de A en P, cuando éste se considere de densidad $\rho = 1$, $B_1 = \dots$, $C_1 = \dots$

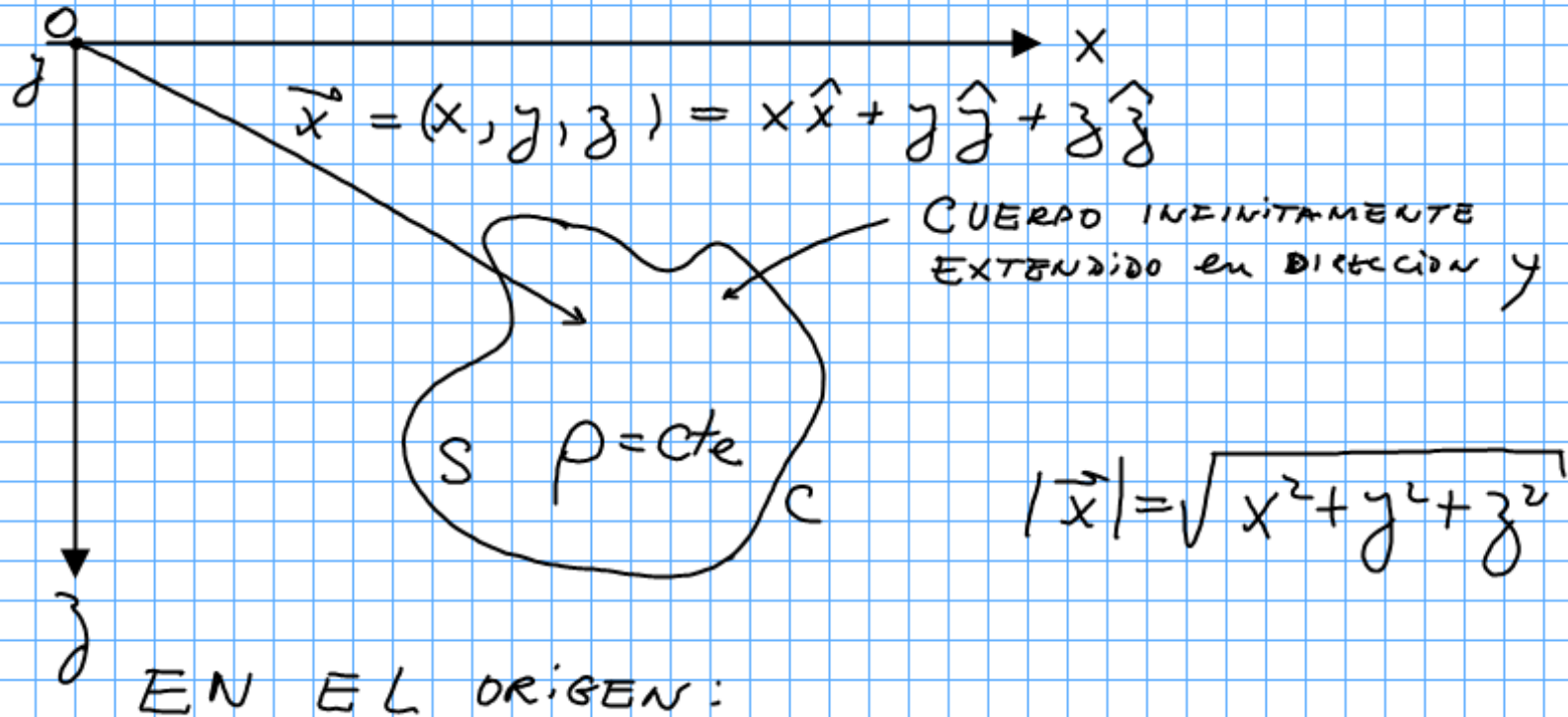
$$\begin{aligned}
 g_p &= g_0 - \gamma h + A_1 \rho_1 + B_1 \rho_0 + C_1 \rho_0 + \\
 &\quad A_1 \rho_0 - A_1 \rho_0 + D_1 \rho_0 - D_1 \rho_0 + E_1 \rho_0 - E_1 \rho_0 \\
 &= g_0 - \gamma h + \underbrace{(A_1 + B_1 + D_1 + E_1)}_{\text{PLACA (CB)}} \rho_0 + A_1 (\rho_1 - \rho_0) \\
 &\quad + \underbrace{(C_1 - D_1 - E_1)}_{\text{CT}} \rho_0
 \end{aligned}$$

$$g_p + \gamma_3 - C_B + C_T - g_0 = A_1 (\rho_1 - \rho_0)$$

ANOMALIA BOUGUER (COMPLETA)

La anomalía de Bouguer (completa) aísla la atracción diferencial ($\rho_1 - \rho_0$) del cuerpo anómalo en la superficie de medición.

Modelamiento gravimétrico (anomalías) 2-D



$$\vec{g} = \rho G \int_V \frac{\vec{x}}{|\vec{x}|^3} dV, \quad dV = dx dy dz$$

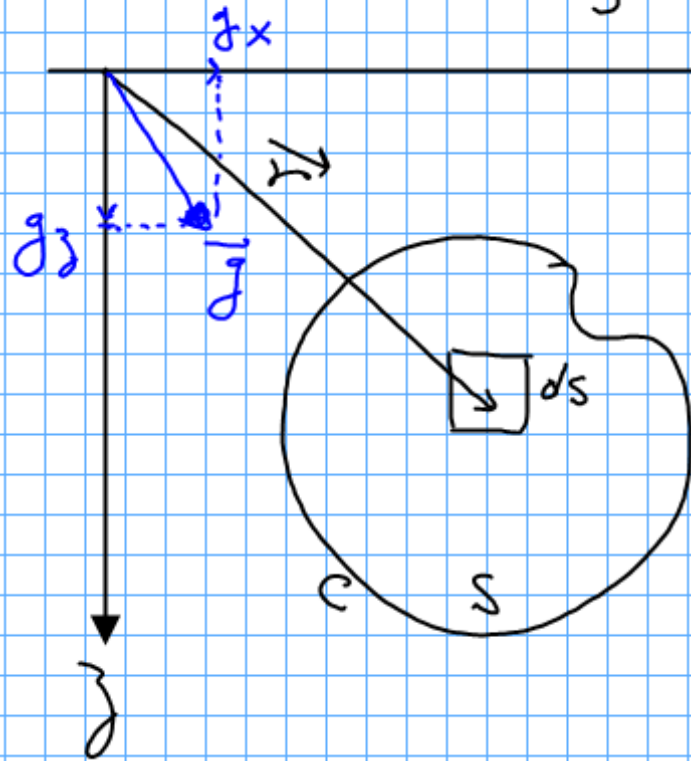
$$\vec{g} = \rho G \int_S dx dz \int_{-\infty}^{+\infty} \frac{\vec{X}}{|\vec{X}|^3} dy \quad (\text{impar}) \neq 0$$

$$= \rho G \int_S dx dz \left[(x\hat{x} + z\hat{z}) \int_{-\infty}^{+\infty} \frac{dy}{|\vec{X}|^3} + \hat{y} \int_{-\infty}^{+\infty} \frac{y dy}{|\vec{X}|^3} \right]$$

$$= \rho G \int_S dx dz \hat{y} \int_{-\infty}^{+\infty} \frac{y dy}{|\vec{X}|^3}$$

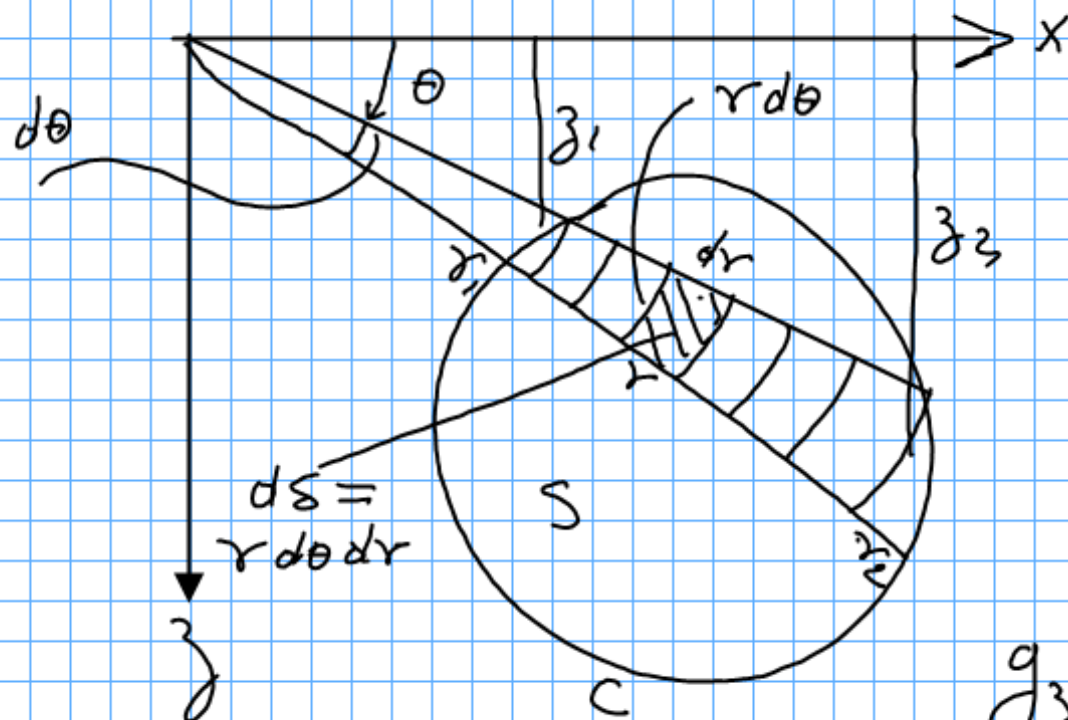
$$\int_{-\infty}^{+\infty} \frac{y dy}{|\vec{X}|^3} = \int_{-\infty}^{+\infty} \frac{y dy}{(r^2 + y^2)^{3/2}} = \frac{2}{r^2}$$

$$\vec{g} = 2G\rho \int_S dx dz \frac{\vec{r}}{r^2} = 2G\rho \int_S \frac{\vec{r}}{r^2} dS$$



$$g_z = 2G\rho \int_S \frac{z dS}{r^2}$$

$$g_x = 2G\rho \int_S \frac{x dS}{r^2}$$



$$g_z = 2G\rho \int_S \frac{z dS}{r^2}$$

$$= 2G\rho \int d\theta \int_{r_1}^{r_2} \frac{z}{r^2} r dr$$

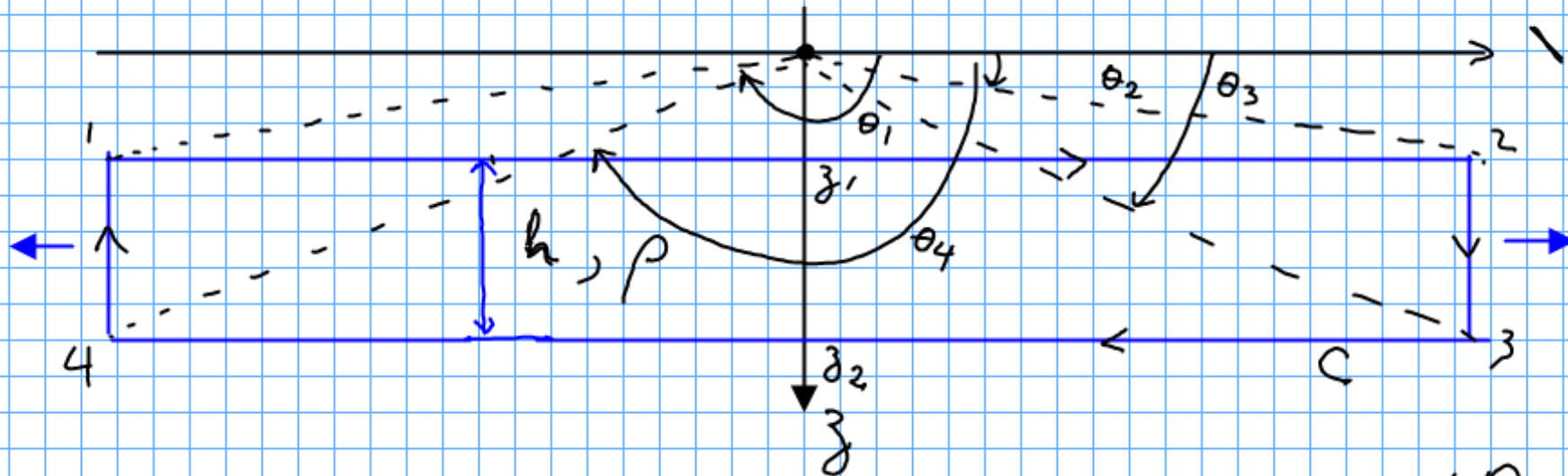
$$g_z = 2G\rho \int d\theta \int_{r_1}^{r_2} \sin\theta dr$$

$$= 2G\rho \int d\theta \sin\theta (r_2 - r_1)$$

$$\therefore g_z = 2G\rho \int d\theta (z_2 - z_1) \rightarrow$$

$$g_z = 2G\rho \oint_C z d\theta, \quad g_x = 2G\rho \oint_C x d\theta$$

Ejemplo, placa infinita. (Para el caso de un contorno poligonal arbitrario, ver Talwani et al., JGR V64, pp 49-??, 1959)



$$g_z = 2G\rho \oint_C z d\theta = 2G\rho \left[z_1 \int_{\theta_1}^{\theta_2} d\theta + \int_{\theta_2}^{\theta_3} z d\theta + \right.$$

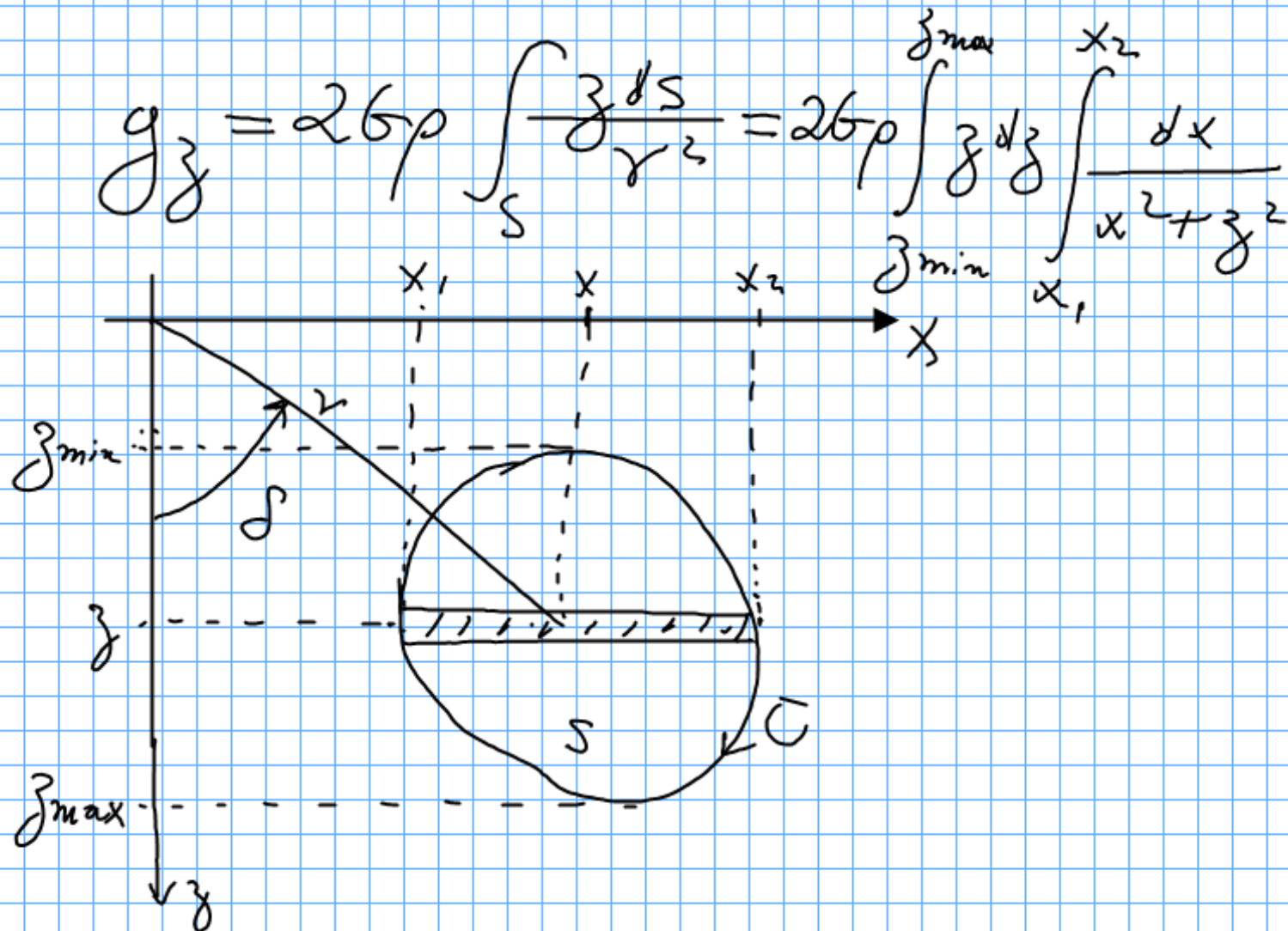
$$\left. z_2 \int_{\theta_3}^{\theta_4} d\theta + \int_{\theta_4}^{\theta_1} z d\theta \right]$$

Para una placa infinita

$$\theta_2 = \theta_3 = 0, \theta_1 = \theta_4 = \pi$$

$$g_z = 2G\rho [z_1(\theta - \pi) + z_2(\pi - 0)] = 2\pi G\rho h.$$

Punto de vista alternativo (Blakely)

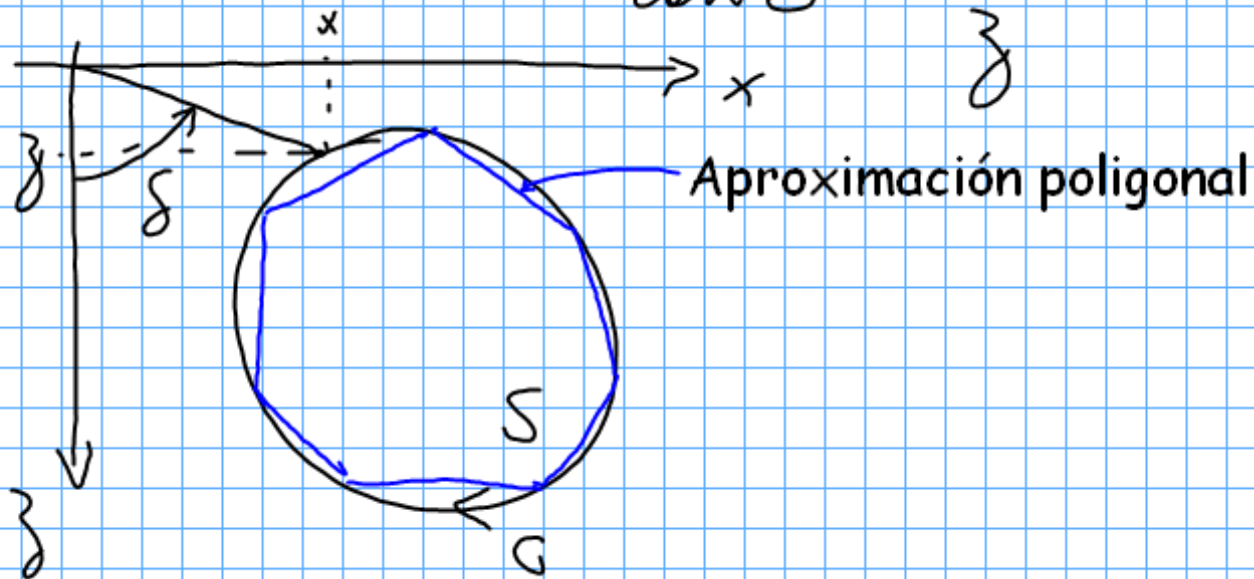


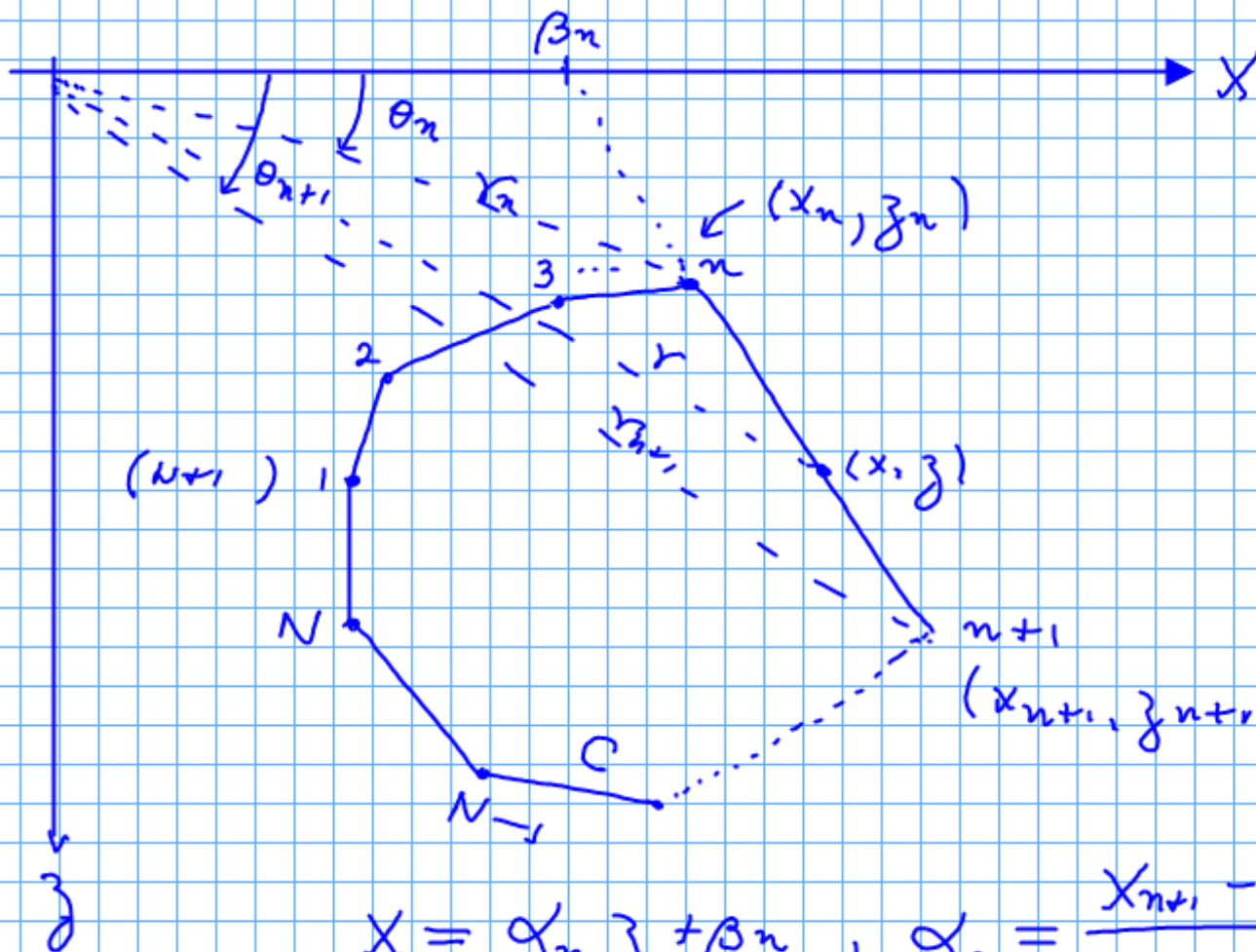
La integral en x es elemental:

$$g_z = 2G\rho \int_{z_{\min}}^{z_{\max}} \left[\arctan\left(\frac{x_2}{z}\right) - \arctan\left(\frac{x_1}{z}\right) \right] dz$$

$$\longrightarrow g_z = 2G\rho \int_C \arctan\left(\frac{x}{z}\right) dz = 2G\rho \int_C \delta dz$$

$$\tan \delta = \frac{x}{z}$$





$$x = \alpha_n z + \beta_n, \quad \alpha_n = \frac{x_{n+1} - x_n}{z_{n+1} - z_n}$$

$$\beta_n = x_n - \alpha_n z_n = x_{n+1} - \alpha_n z_{n+1}$$

$$g_z = 26\rho \sum_{n=1}^N \int_{z_n}^{z_{n+1}} \operatorname{atan} \left[\frac{\alpha_n z + \beta_n}{z} \right] dz$$

Después de algebrae

$$g_z = 2G\rho \sum_{n=1}^N \frac{\beta_n}{1+\alpha_n^2} \left[\ln \frac{r_{n+1}}{r_n} - \alpha_n (\theta_{n+1} - \theta_n) \right]$$

(Blakely, ec. 9.11)

