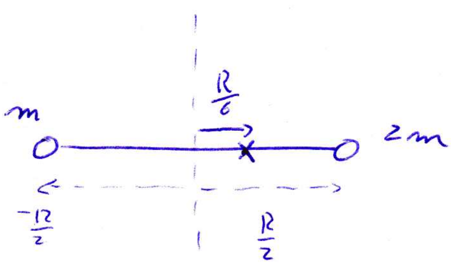
CLASE AUXILIAR

UNIDAD 4A : ESTATICA

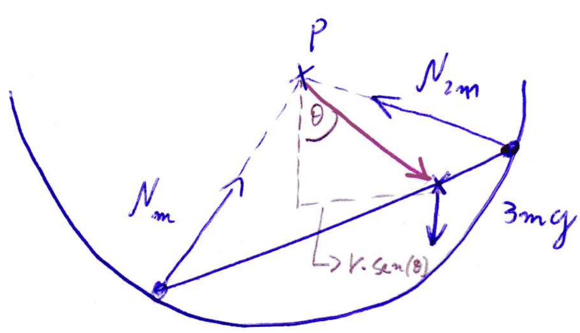
Problema 1

Se calcula el C.M.



$$X_{cm} = \frac{\sum x_i m_i}{\sum m_i} = \frac{-\frac{R}{2}m + \frac{R}{2}2m}{3m} = \frac{R}{6}$$

Con esto se hace el DCL:

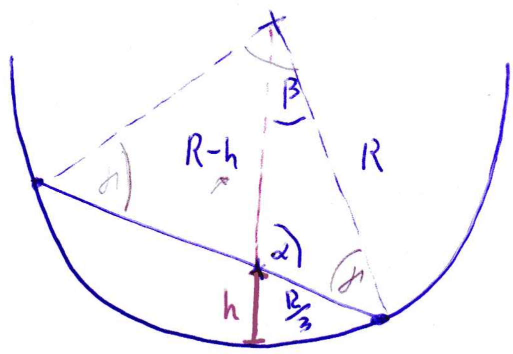


Si se pone el sistema de referencia en P la unica fuerza que hace torque es el peso pues las normales son paralelas al brazo ($\measuredangle \vec{R}, \vec{N} = 0$)

$$\Rightarrow \tau = R \cdot sen(\theta)\, 3mg = 0 \Rightarrow sen(\theta) = 0 \Rightarrow \theta = 0$$

Como el angulo θ es 0 la posicion es la siguiente

$180 : 3 = 60$



Ahora buscamos h, lo que queda es solo geometria

$$\frac{\operatorname{sen}(\alpha)}{R} = \frac{\operatorname{sen}(\beta)}{\frac{R}{3}} \quad \text{(Teo. Seno)}$$

$180 - \alpha = \gamma + \beta$ $\quad$ $\pi - \alpha = \beta + \frac{\pi}{3}$ (geometria triangulos)

$\gamma = 60°$ $\quad$ $\alpha + \beta + \gamma = \pi$

$$\frac{1}{3}\operatorname{sen}(\alpha) = \operatorname{sen}\left(\frac{2}{3}\pi - \alpha\right) = \operatorname{sen}\left(\frac{2}{3}\pi\right)\cos(\alpha) - \cos\left(\frac{2}{3}\pi\right)\operatorname{sen}(\alpha)$$

$$\operatorname{sen}\left(\frac{2}{3}\pi\right) = \operatorname{sen}\left(\frac{\pi}{3} + \frac{\pi}{3}\right) = 2\cos\left(\frac{\pi}{3}\right)\operatorname{sen}\left(\frac{\pi}{3}\right) = 2\,\frac{1}{2}\,\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\operatorname{sen}\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{2}{3}\pi\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{3}\right) = \cos^2\left(\frac{\pi}{3}\right) - \operatorname{sen}^2\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

y queda:

$$\cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$$

$$\frac{1}{3}\operatorname{sen}(\alpha) = \frac{\sqrt{3}}{2}\cos(\alpha) + \frac{1}{2}\operatorname{sen}(\alpha)$$

$$\frac{1}{3} - \frac{1}{2} = \frac{2-3}{6}$$

$$-\frac{1}{6}\operatorname{sen}(\alpha) = \frac{\sqrt{3}}{2}\cos(\alpha) \longrightarrow \frac{\operatorname{sen}(\alpha)}{\cos(\alpha)} = -3\sqrt{3}$$

$$\alpha = \arctan(-3\sqrt{3}) \quad //$$

y para h: $\dfrac{\operatorname{sen}(\alpha)}{R} = \dfrac{\operatorname{sen}\left(\frac{\pi}{3}\right)}{R-h}$ $\ast \rightarrow$ Ver en la figura

$$\longrightarrow \quad h = R\left(1 - \frac{\sqrt{3}}{2\operatorname{sen}\alpha}\right) \quad //$$

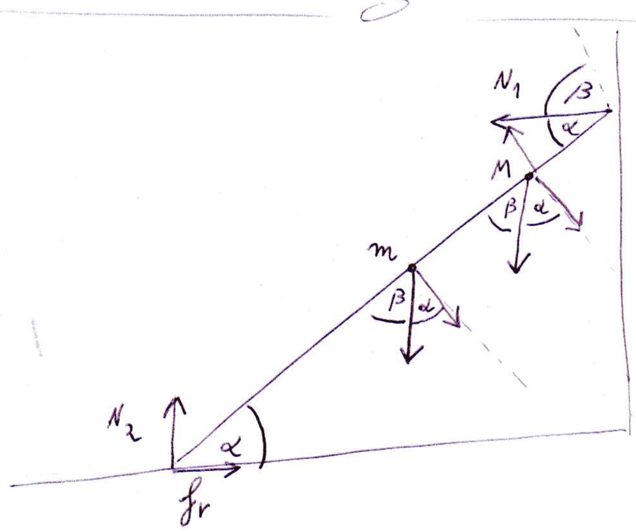
P1

$m_0 = -\rho \pi r^2 \cdot D$
$m_1 = \rho \pi R^2 \cdot D$

$$C.M = \frac{\cancel{(\rho \pi R^2 D)\cdot 0}^{\,0} + (-\rho \pi r^2 \cdot D)\cdot (R-r)}{\rho \pi R^2 \cdot D - \rho \pi r^2 \cdot D} = \frac{-\cancel{\rho \pi D}\cdot r^2 (R-r)}{\cancel{\rho \pi D}\cdot (R^2 - r^2)}$$

$$= \frac{r^2}{(R^2 - r^2)}$$

P3



F_x: $f_r - N_1 = 0 \Rightarrow f_r = N_1$; $f_r = \mu \cdot N_2 = \mu (m + M) g$

F_y: $N_2 - mg - Mg = 0 \Rightarrow N_2 = (m + M) g$

τ: $\frac{L}{2} mg \cos(\alpha) - r \cdot Mg \cos(\alpha) - r N_1 \operatorname{sen}(\alpha) = 0$

$\Rightarrow$ $\frac{L}{2} m\cancel{g} \cos(\alpha) - r \cdot M\cancel{g} \cos(\alpha) - r \mu (m + M)\cancel{g} \operatorname{sen}(\alpha) = 0$