PAUTA CONTROL 3 EL3005/Primavera/2010

P1

a) $$\hat{X}_L(\Omega) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL]\,e^{-j\Omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,e^{-j\Omega kL} = X(\Omega L)$$

$$\Rightarrow \boxed{\hat{X}_L(\Omega) = X(\Omega L)}$$

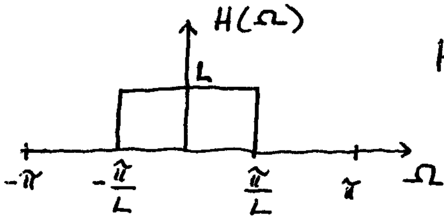
$\rightarrow$ $\hat{X}_L(\Omega)$ no se puede escribir como $H(\Omega)X(\Omega) \Rightarrow$ NO es LTI

b) $$y[n] = x_a\left(\frac{nT_s}{L}\right) \underset{\substack{\uparrow \\ \text{Teo. del Muestreo}}}{=} \sum_{k=-\infty}^{\infty} x[k]\,h_{PB}\Big(\underbrace{\frac{nT_s}{L} - kT_s}_{(n-kL)\frac{T_s}{L}}\Big)$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-kL]$$

donde $h[n] = h_{PB}\left(\frac{nT_s}{L}\right)$. En forma explícita:

$$h[n] = \frac{\sin\left(\frac{\pi}{L}n\right)}{\pi n} L \xrightarrow{\text{DTFT}}$$

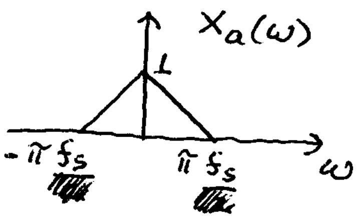


$$H(\Omega) = \frac{L}{T_s} H_{PB}\left(\frac{L}{T_s}\Omega\right)$$

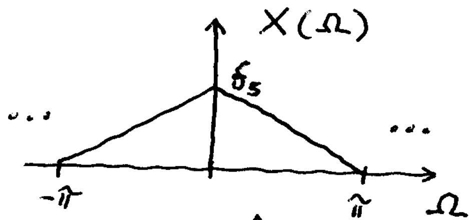
Y se tiene que $h[n-kL] = h[n] \circledast \delta[n-kL]$

$$\Rightarrow y[n] = \Big\{\underbrace{\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL]}_{\hat{x}_L[n]}\Big\} \circledast h[n] = \hat{x}_L[n] \circledast h[n]$$

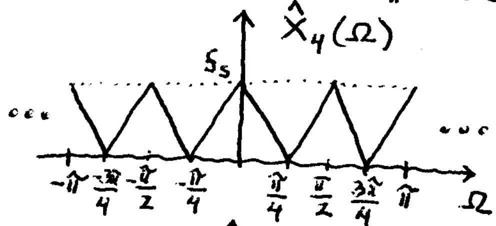
c)



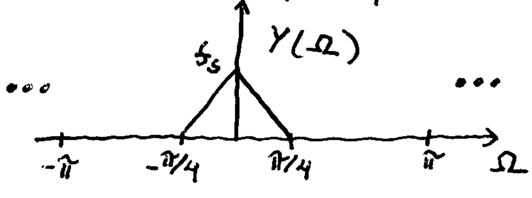
$\Rightarrow$



$$X(\Omega) = f_s X_a(f_s \Omega)$$
$$-\pi \leq \Omega \leq \pi$$



$$\hat{X}_4(\Omega) = X(4\Omega)$$



$$Y(\Omega) = \hat{X}_4(\Omega) H(\Omega)$$

P2 |

a) $$H(\Omega)=\sum_{n=0}^{M-1} h[n]e^{-j\Omega n} = e^{-j\Omega\frac{M-1}{2}}\sum_{n=0}^{M-1} h[n]e^{-j\Omega(n-\frac{M-1}{2})}$$

$$= e^{-j\Omega\frac{M-1}{2}}\left\{\sum_{n=0}^{\frac{M-1}{2}-1} h[n]e^{-j\Omega(n-\frac{M-1}{2})} + \underbrace{\sum_{n=\frac{M-1}{2}}^{M-1} h[n]e^{-j\Omega(n-\frac{M-1}{2})}}_{\sum_{n'=0}^{\frac{M-1}{2}} \underbrace{h[M-1-n']}_{h[n']} e^{+j\Omega(n'-\frac{M-1}{2})}}\right\} \quad n'=M-1-n$$

$$= e^{-j\Omega\frac{M-1}{2}}\left\{h[\tfrac{M-1}{2}] + \sum_{n=0}^{\frac{M-1}{2}-1} h[n]\left(e^{-j\Omega(n-\frac{M-1}{2})}+e^{j\Omega(n-\frac{M-1}{2})}\right)\right\}$$

$$= \underbrace{e^{-j\Omega\frac{M-1}{2}}}_{\text{respuesta lineal en la fase}}\left\{\underbrace{h[\tfrac{M-1}{2}] + \sum_{n=0}^{\frac{M-1}{2}-1} h[n]\,2\cos\left(\Omega(n-\tfrac{M-1}{2})\right)}_{\text{real y positivo (ignorando posibles valores negativos en la banda de detención)}}\right\}$$

→ Magnitud del desfase: $\frac{M-1}{2}$

b) $h[n]\in\mathbb{R} \Rightarrow H(\Omega)=H^*(-\Omega) \Rightarrow |H(2\pi-\Omega)|=|H(\Omega)|$

$= H^*(2\pi-\Omega) \Rightarrow \left|H\left(2\pi-\frac{2\pi k}{M}\right)\right| = \left|H\left(\frac{2\pi k}{M}\right)\right| \quad k=1,\dots,\frac{M-1}{2}$

$\Rightarrow \underline{A[M-k]=A[k]} \quad k=1,\dots,\frac{M-1}{2}$

c) $$H\left(\frac{2\pi k}{M}\right)=\sum_{n=0}^{M-1} h[n]e^{-j\frac{2\pi k}{M}n} = \mathrm{DFT}_M\{h[n]\} \qquad k=0,\dots,M-1$$

$$\Rightarrow h[n] = \mathrm{IDFT}_M\left\{H\left(\frac{2\pi k}{M}\right)\right\} = \frac{1}{M}\sum_{k=0}^{M-1} H\left(\frac{2\pi k}{M}\right)e^{j\frac{2\pi k}{M}n} \qquad n=0,\dots,M-1$$

$$= \frac{1}{M}\sum_{k=0}^{M-1} A[k]e^{-j\left(\frac{2\pi k}{M}\right)\frac{M-1}{2}} e^{j\frac{2\pi k}{M}n}$$

$$= \frac{1}{M}\left[\sum_{k=0}^{\frac{M-1}{2}} A[k]e^{j\frac{2\pi k}{M}(n-\frac{M-1}{2})} + \underbrace{\sum_{k=\frac{M-1}{2}+1}^{M-1} A[k]e^{j\frac{2\pi k}{M}(n-\frac{M-1}{2})}}_{\sum_{k'=1}^{\frac{M-1}{2}} \underbrace{A[M-k']}_{A[k']} e^{-j\frac{2\pi k'}{M}(n-\frac{M-1}{2})}}\right] \quad k'=M-k$$

$$\underline{= \frac{1}{M}\left[A[0] + 2\sum_{k=1}^{\frac{M-1}{2}} A[k]\cos\left(\frac{2\pi k}{M}\left(n-\frac{M-1}{2}\right)\right)\right]} \qquad n=0,\dots,M-1$$