P1

$$e_k^n(x) = \begin{cases} 2^{n/2} & \frac{k-1}{2^n} \le x < \frac{k-1/2}{2^n} \\ -2^{n/2} & \frac{k-1/2}{2^n} \le x < \frac{k}{2^n} \\ 0 & \text{otro caso} \end{cases}$$

a) Graficar $\sum_{k=1}^{2^n} e_k^n(x)$ para $n=1,2$.

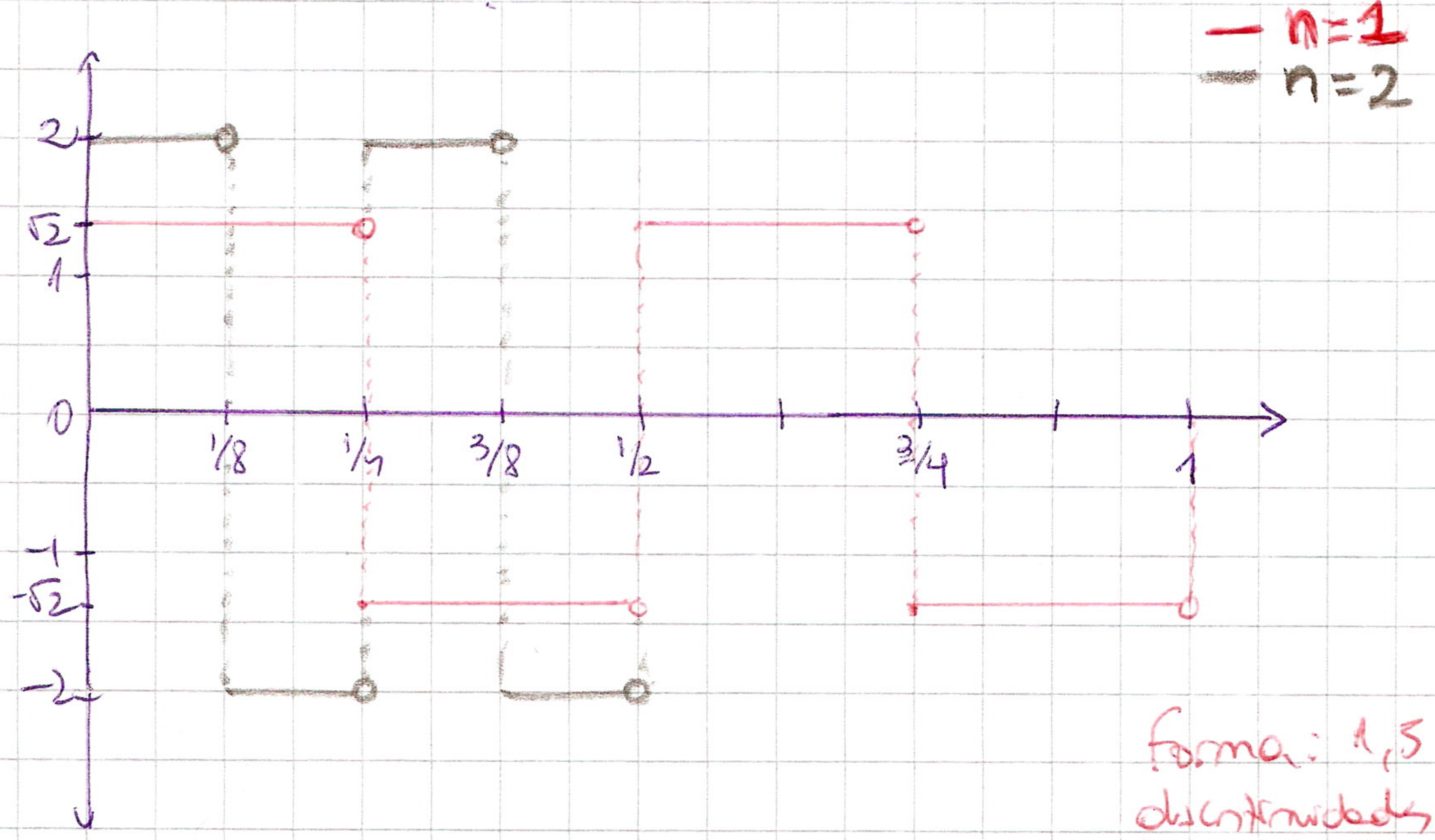
$n=1$:

$$\sum_{k=1}^{2} e_k^1(x) = \begin{cases} \sqrt{2} & 0 \le x < \frac{1}{4} \\ -\sqrt{2} & \frac{1}{4} \le x < \frac{1}{2} \\ \sqrt{2} & \frac{1}{2} \le x < \frac{3}{4} \\ -\sqrt{2} & \frac{3}{4} \le x < 1 \end{cases}$$

$n=2$:

$$\sum_{k=1}^{4} e_k^2(x) = \begin{cases} 2 & 0 \le x \le \frac{1}{8} \\ -2 & \frac{1}{8} \le x \le \frac{1}{4} \\ 2 & \frac{1}{4} \le x \le \frac{3}{8} \\ -2 & \frac{3}{8} \le x \le \frac{2}{4} \end{cases}$$

Entonces se tiene:



forma: 1,5 ptos
discontinuidades: 1,5 ptos

b) Pd$_t$ $\int_{-\infty}^{\infty} e_k^n e_{k'}^{n'} dx = \delta_{kk'}\delta_{nn'}$ $\qquad \delta_{ij} = \begin{cases} 1 & i=j \\ 0 & \sim \end{cases}$

Primero $k \neq l$:

$$\int_{-\infty} e_k^n e_l^n dx = \int_{\frac{k-1}{2^n}}^{\frac{k}{2^n}} e_k e_l^n dx = 0$$

$$= \int_{\frac{k-1}{2^n}}^{\frac{k}{2^n}} e_k^n \cdot 0 \, dx + \int_{\frac{l-1}{2^n}}^{\frac{l}{2^n}} 0 \cdot e_l^n dx = 0$$ ← pues sólo tienen valores dentro de sus intervalos y $k \neq l$.

(1,5 ptos)

Segundo $k = l$

$$\int_{-\infty}^{\infty} e_k^n e_k^n dx = \int_{\frac{k-1}{2^n}}^{\frac{k}{2^n}} (e_k^n)^2 dx = \int_{\frac{k-1}{2^n}}^{\frac{k-\frac{1}{2}}{2^n}} 2^n dx + \int_{\frac{k-\frac{1}{2}}{2^n}}^{\frac{k}{2^n}} 2^n dx$$

$$= 2^n \left(\frac{k}{2^n} - \frac{(k-1)}{2^n} \right) = 2^n \cdot \frac{1}{2^n} = 1$$

(1,5 ptos)

$\Rightarrow e_k^n$ ortonormal.