

P3/

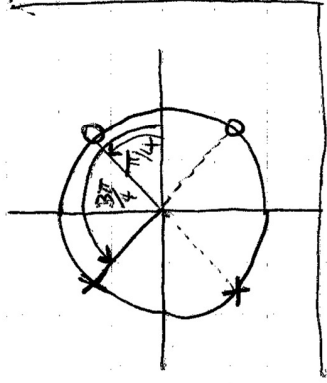
EL 3005

Pauta Control 1

$$\begin{aligned} a) Y(z) &= -\sqrt{2} z^{-1} V(z) - z^{-2} V(z) + X(z) - \sqrt{2} z^{-1} X(z) + z^{-2} X(z) \\ \Rightarrow H(z) &= \frac{1 - \sqrt{2} z^{-1} + z^{-2}}{1 + \sqrt{2} z^{-1} + z^{-2}} = \frac{z^2 - \sqrt{2} z + 1}{z^2 + \sqrt{2} z + 1} \end{aligned}$$

$$p_{1,2} = \frac{-\sqrt{2} \pm j\sqrt{2}}{2} \Rightarrow p_1 = e^{j\frac{3\pi}{4}} \wedge p_2 = e^{-j\frac{3\pi}{4}}$$

$$\text{Análogamente: } z_1 = e^{j\frac{\pi}{4}} \wedge z_2 = e^{-j\frac{\pi}{4}}$$



b) NO es estable. La ROC no contiene la circunferencia unitaria debido a p_1 y p_2 .

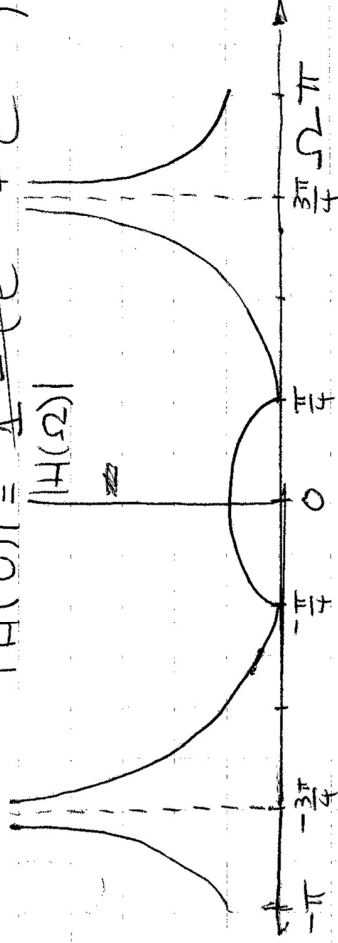
$$c) X[n] = e^{\pm j\frac{3\pi}{4}n} u[n]; \text{ acotada.}$$

$$H(z = e^{j\frac{3\pi}{4}}) = \frac{Y[n]}{(z - e^{j\frac{\pi}{4}})(z - e^{-j\frac{\pi}{4}})} \Big|_{z=e^{j\frac{3\pi}{4}}} = \frac{e^{j\frac{3\pi}{4}n}}{(z - e^{j\frac{3\pi}{4}})(z - e^{-j\frac{\pi}{4}})} \Big|_{z=e^{j\frac{3\pi}{4}}} = \infty$$

Salida no es acotada.

d) e) $|H(\omega)|$ es infinito en los polos y cero en los ceros.

$$|H(0)| = \frac{1}{1 - (e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}})} = \frac{1 - \sqrt{2} + 1}{4 + \sqrt{2} + 1} = \frac{2 - \sqrt{2}}{5 + \sqrt{2}}$$



f) La entrada corresponde a una suma de autofunciones, exponenciales complejas:

$$X[n] = e^{j0n} + 2e^{j\frac{3\pi}{4}n} - 3 \cdot 2^{\frac{n}{2}} \cdot 2^{\frac{n}{2}} e^{j\frac{\pi}{4}n} + 4 \cdot e^{-j\pi n}$$

$$Y[n] = H(0)e^{j0n} + 2H(\frac{3\pi}{4})e^{j\frac{3\pi}{4}n} - 3 \cdot 2^{\frac{n}{2}} H(\frac{\pi}{4})e^{j\frac{\pi}{4}n} + 4H(-\pi)e^{j\pi n}$$

$$H(0) = 3 - 2\sqrt{2}$$

$$H\left(\frac{3\pi}{2}\right) = \frac{-1 - \sqrt{2}j + 1}{-1 + \sqrt{2}j + 1} = -1$$

$$H\left(\frac{\pi}{4}\right) = 0$$

$$H(-\pi) = \frac{1 + \sqrt{2} + 1}{1 - \sqrt{2} + 1} = 3 + 2\sqrt{2}$$

$$\Rightarrow y[n] = (3 - 2\sqrt{2}) - 2e^{j\frac{3\pi}{2}n} + (3 + 2\sqrt{2})(-1)^n$$