# Señales y Sistemas I

## Pauta P3 Control 1. Otoño 2010

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i) sistema LTI

$$x(n) \longrightarrow \boxed{LTI} \longrightarrow y(n) = h(n) * x(n)$$

sea $x(n) = \phi_{K,N}(n) = e^{j\frac{2\pi K}{N}n}$

$$\Rightarrow y(n) = \sum_{p=-\infty}^{\infty} h(p)\,\phi_{K,N}(n-p)$$ (0,5) / def de conv.

$$= \sum_{p=-\infty}^{\infty} h(p)\, e^{j\frac{2\pi K}{N}(n-p)}$$

$$= \left(\sum_{p=-\infty}^{\infty} h(p)\, e^{-j\frac{2\pi K}{N}p}\right) \underbrace{e^{j\frac{2\pi K}{N}n}}_{\phi_{K,N}(n)}$$ (0,5) despeje adecuado

pero sabemos que

$$X(\omega) = DTFT(x(n)) = \sum_{K=-\infty}^{\infty} x(K)\, e^{-j\omega K}$$ (0,5) reconocer DTFT

$$\Rightarrow \sum_{p=-\infty}^{\infty} h(p)\, e^{-j\frac{2\pi K}{N}p} = H(2\pi K/N) = DTFT(h(n))\Big|_{\omega = 2\pi K/N}$$

$$\Rightarrow \boxed{y(n) = H(2\pi K/N)\,\phi_{K,N}(n)}$$ (0,5) concluir

ii) $x(n)$ es $N$-periodica $\Rightarrow x(n+N) = x(n)$

ahora: $y(n) = \sum_{p=-\infty}^{\infty} h(p)\, x(n-p)$ (*) (0,5) def conv.

$$\Rightarrow y(n+N) = \sum_{p=-\infty}^{\infty} h(p)\, x((n+N)-p) \quad (0{,}5)$$

$$= \sum_{p=-\infty}^{\infty} h(p)\, x((n-p)+N) \quad \text{/usamos } x \text{ es } N\text{-periódica} \quad (0{,}5)$$

$$= \sum_{p=-\infty}^{\infty} h(p)\, x(n-p) \quad \text{/esto es } y(n) \ (*)$$

$$= y(n)$$

$\Rightarrow y(n+N) = y(n)$ $\therefore$ $y$ es $N$-periódica (0,5)

iii)

$$x(n) = \sum_{k=0}^{N-1} c_k^x e^{j\frac{2\pi k}{N} n} = \sum_{k=0}^{N-1} c_k^x \phi_{k,N}(n)$$

$$\Rightarrow y(n) = T[x(n)]$$

$$= T\left[\sum_{k=0}^{N-1} c_k^x \phi_{k,N}(n)\right] \quad \text{/pero el sistema es L.T.I.} \quad (0{,}5)$$

$$= \sum_{k=0}^{N-1} c_k^x\, T[\phi_{k,N}(n)] \quad \text{/usamos resultado de i)} \quad (0{,}5)$$

$$= \sum_{k=0}^{N-1} c_k^x\, H(2\pi k/N)\, \phi_{k,N}(n) \quad \text{/} y(n) = \sum c_k^y e^{j\frac{2\pi k}{N} n}$$

$$\Rightarrow \boxed{c_k^y = H(2\pi k/N)\, c_k^x}$$

relación entre

Bonus:

notemos que $x^N(n) = x(n) \quad \forall n \in \{0, \dots, N-1\}$

Ahora $C_k = \frac{1}{N} \sum_{n=0}^{N-1} x^N(n) e^{-j\frac{2\pi k}{N}n}$

$\Rightarrow C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k}{N}n} = \frac{1}{N} X\left(w = \frac{2\pi k}{N}\right)$

$x^N(n) = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi k}{N}n} \quad \forall n \in \mathbb{Z}$

$\Rightarrow$

$x(n) = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi k}{N}n} \quad \forall n = 0, \dots, N-1$

ahora. la T.F. de $x(n)$ queda como

$$X(w) = \sum_{n \in \mathbb{Z}} x(n) e^{-jwn} = \sum_{n=0}^{N-1} x(n) e^{-jwn}$$

pero $x(n) = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi k}{N}n} \Rightarrow X(w) = \sum_{n=0}^{N-1} \left( \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi k}{N}n} \right) e^{-jwn}$

$$\Rightarrow X(w) = \sum_{k=0}^{N-1} c_k \underbrace{\left[ \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi k}{N} - w\right)n} \right]}_{\theta_k(w)}$$

donde $\theta_k(w) = \begin{cases} \dfrac{e^{j\left(\frac{2\pi k}{N} - w\right)N} - 1}{e^{j\left(\frac{2\pi k}{N} - w\right)} - 1} = \dfrac{e^{-jwN} - 1}{e^{j\left(\frac{2\pi k}{N} - w\right)} - 1} & \text{si } \frac{2\pi k}{N} - w \neq 2\pi k' \\ N & \sim \end{cases}$

$\Rightarrow \boxed{X(w) = \sum_{k=0}^{N-1} C_k \, \theta_k(w)} \longrightarrow$ T.F. de $x(n)$ queda caract. por. $C_N = \{c_0, \dots, c_{N-1}\}$