

a) Veamos que:

$$\begin{aligned}
 \sum_{k \in \mathbb{Z}} k u(n-k) &= \sum_{k \in \mathbb{Z}} (x(k) - x(k-1)) u(n-k) \\
 &= \sum_{k=-\infty}^n (x(k) - x(k-1)) = \sum_{k=-\infty}^n x(k) - \sum_{k=-\infty}^n x(k-1) \\
 &= \sum_{k=-\infty}^n x(k) - \sum_{k=-\infty}^{n-1} x(k) = x(n) \\
 \Rightarrow \sum_{k \in \mathbb{Z}} k u(n-k) &= x(n) \quad \forall n
 \end{aligned}$$

b) p.d.g. $(x(n)) \odot (y(n)) = (y(n)) \odot (x(n))$

$$\begin{aligned}
 (x(n)) \odot (y(n)) &= \sum_{k \in \mathbb{Z}} [x(k) - x(k-1)] y(n-k) \\
 &= \sum_{k \in \mathbb{Z}} x(k) y(n-k) - \sum_{k \in \mathbb{Z}} x(k-1) y(n-k) \\
 &= \sum_{k \in \mathbb{Z}} x(k) y(n-k) - \sum_{k \in \mathbb{Z}} x(k) y(n-k-1) \\
 &= \sum_{k \in \mathbb{Z}} x(k) [y(n-k) - y(n-k-1)] = \sum_{u \in \mathbb{Z}} x(n-u) [y(u) - y(u-1)] \\
 &= (y(n)) \odot (x(n))
 \end{aligned}$$

d.g.

$$\begin{aligned}
 & (\alpha x_1(n) + \beta x_2(n)) \odot y(n) = \alpha x_1(n) \odot y(n) + \beta x_2(n) \odot y(n) \quad (2) \\
 & = \sum_k (\alpha x_1(k) + \beta x_2(k) - \alpha x_1(k-1) - \beta x_2(k-1)) y(n-k) \\
 & = \alpha \sum_k (x_1(k) - x_1(k-1)) y(n-k) + \beta \sum_k (x_2(k) - x_2(k-1)) y(n-k) \\
 & = \alpha x_1 \odot y(n) + \beta x_2 \odot y(n)
 \end{aligned}$$

c) p.d.g: $y(n) = s(n) \odot x(n)$ donde $s(n) = h(n) * u(n)$
 $y(n) = h(n) * x(n)$

veamos que $y(n) = h(n) * x(n) = h(n) * (x(n) \odot u(n))$ por parte 1)

$\Rightarrow y(n) = h(n) * (u(n) \odot x(n)) \leftarrow \odot \text{ conmuta}$

$\Rightarrow y(n) = (h(n) * u(n)) \odot x(n)$ por *

$$y(n) = s(n) \odot x(n)$$

Veamos *:

$(h(n) * u(n)) \odot x(n) = x(n) \odot (h(n) * u(n))$ del de *

$$= \sum_{k \in \mathbb{Z}} x(n) \odot (h(k) u(n-k)) = \sum_{k \in \mathbb{Z}} x(n) \odot (h(k) u(n-k))$$

linealidad de $\odot$

$$= \sum_{k \in \mathbb{Z}} x(n) \odot (h(k) u(n-k)) = \sum_{k \in \mathbb{Z}} \sum_{p \in \mathbb{Z}} [x(p) - x(p-1)] h(k) u(n-k-p)$$

$$= \sum_{k \in \mathbb{Z}} \sum_{p \in \mathbb{Z}} \{ [x(p) - x(p-1)] u(n-k-p) \} h(k)$$

del $\odot$ = $\sum_{k \in \mathbb{Z}} \{ x(n-k) \odot u(n-k) \} h(k)$

del * = $(x(n) \odot u(n)) * h(n)$

con * y $\odot$ = $(u(n) \odot x(n)) * h(n)$