HUNTER ROUSE AND SHIELDS DIAGRAM

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Abstract— Hunter Rouse (1906-1996) was one of the great hydraulics educators and researchers in the 20th century. He was well-known to sediment researchers by his sediment concentration distribution equation. However, his contributions to the Shields diagram fall into oblivion in recent textbooks. The purpose of this article is to permanently record Rouse's contributions to the Shields diagram by providing a Shields-Rouse equation.

I. INTRODUCTION

Hunter Rouse (Fig. 1) was born in Toledo, Ohio in 1906, which means that he was only two years younger than modern fluid mechanics - the birth of the boundary layer theory that was fathered by Prandtl in Germany in 1904. Rouse grew up in the golden era of the fast development of modern fluid mechanics. When he studied in MIT in the early 1930s, modern fluid mechanics had been widely applied in mechanical engineering. However, hydraulics was still in its primitive stage - empirical subject. Rouse believed that further advances in hydraulics required a better theoretical understanding of fluid flow. He then pinpointed his life's target: injecting the fluid-mechanical concepts that had been proved so fruitful in aeronautics into the study of hydraulics (Mutel 1998). Rouse was very successful for this target by his research, teaching, education innovation, and writing. He received many awards, medals and recognitions from National Academy of Engineering, ASCE and other organizations. When he was 85 in 1991, ASCE presented him the John Frits Medal "for pioneering the application of fluid mechanics to hydraulics, fusing theory and experimental techniques to firm the basis for modern engineering hydraulics." He died at the age of 90 in Sun City, Arizona in October 1996.

Rouse also pioneered the study of the history of hydraulics by his two classic books *History of Hydraulics* (Rouse and Ince 1957) and *Hydraulics in the United States 1776-1976* (Rouse 1976) in which he recorded many hydraulicians who made important contributions. In particular, some of them would fall oblivion if without his books. However, his contributions to the Shields diagram, one of the important laws in sediment transport, fall oblivion in recent textbooks and among young researchers. The purpose of this article is to permanently record Rouse's contributions to the Shields diagram by providing a Shields-Rouse equation.

Fig. 1. Hunter Rouse in 1986 (Albertson and Papadakis 1986)

II. ROUSE AND SHIELDS DIAGRAM

Both Rouse and Shields were pioneers of modern sediment transport and river mechanics. Unlike Rouse, Shields could not find a position in fluids after he returned to his motherland U.S. with a Ph.D. from Germany in 1936. He finally abandoned hydraulics and engaged in paper machinery industry. For details about Shields, readers are referred to Kennedy (1995) and Buffington (1999). It was Rouse who promoted Shields' (1936) work in hydraulics field. Rouse contributed to the Shields diagram in the following aspects. (1) He saved Shields' work from World War II (Kennedy 1995). (2) He introduced it to sediment researchers. (3) His physical interpretation of the Shields diagram and active uses made it become one of the fundamental laws in sediment transport. (4) He introduced an auxiliary parameter in the Shields diagram and made its calculation simple.

III. SHIELDS-ROUSE EQUATION

A. Shields equation

Shields' work has been extensively extended for six decades (Chien and Wan 1999, p.311-352). The latest development can be found in Papanicolaou et al. (2002), Dancey et al. (2002) and Dey (2003). This section only emphasizes the mathematical expression of the original Shields diagram.

Shields was one of the first who applied similarity approach to sediment studies. Since there are many uncertainties in turbulent flows and loose boundary materials, it is very difficult to formulate critical shear stress analytically. Shields then turned to a similarity approach or dimensional analysis to arrive at two important dimensionless variables (Shields 1936): dimensionless shear stress,

$$\frac{\tau}{(s-1)\,\rho g d} \tag{1}$$

and particle Reynolds number,

$$\frac{u_* d}{\nu} \tag{2}$$

In (1) and (2), τ = bed shear stress, s = specific gravity of sediment, ρ = density of water, g = gravitational acceleration, d = sediment diameter, $u_* = \sqrt{\tau/\rho}$ = shear velocity, and ν = kinematic viscosity of water.

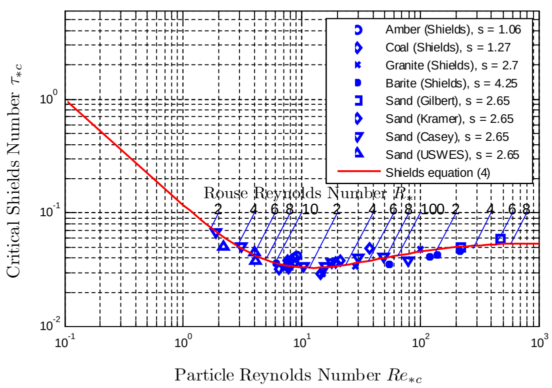

Fig. 2. Shields diagram for sediment incipient motion [Data source: Table 3 in Buffington (1999)]

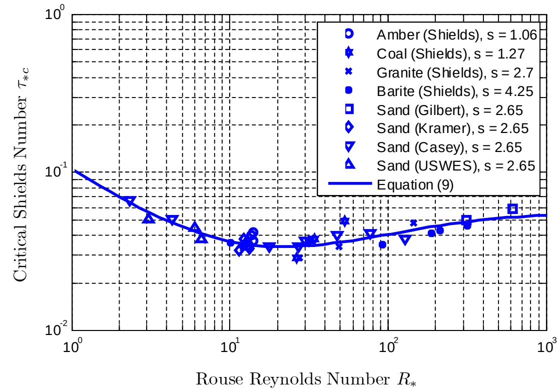

Fig. 3. Shields-Rouse diagram for sediment incipient motion

Dimensionless parameter (1) is now often termed as Shields number.

Introducing the critical shear stress τ_c, Shields claimed that when a sediment particle is about to move, the following dynamic similarity law must be true,

$$\frac{\tau_c}{(s-1)\,\rho g d} = f\left(\frac{u_{*c} d}{\nu}\right) \tag{3}$$

in which $u_{*c} = \sqrt{\tau_c/\rho}$. Shields determined equation (3) experimentally, shown in Fig. 2 where the data points were recalculated by Buffington (1999). With the wide applications of numerical modeling, a mathematical expression for (3) is necessary. To this end, Guo (1990) proposed the following empirical equation:

$$\tau_{*c} = \frac{0.11}{\mathrm{Re}_{*c}} + 0.054\left[1 - \exp\left(-\frac{4\,\mathrm{Re}_{*c}^{0.52}}{25}\right)\right] \tag{4}$$

in which

$$\tau_{*c} = \frac{\tau_c}{(s-1)\,\rho g d}$$

and

$$\mathrm{Re}_{*c} = \frac{u_{*c} d}{\nu}$$

Equation (4), which was also reported in Guo (1997), is compared with experimental data in Fig. 2. To permanently record Shields contribution, this article recommends (4) to be called the Shields equation. However, like the diagram, equation (4) is implicit in terms of τ_{*c} and then difficult to use in practice.

B. Shields-Rouse equation

When Rouse introduced the Shields diagram to sediment researchers, he introduced the following auxiliary parameter

$$\mathrm{R}_* = \frac{d\sqrt{0.1\,(s-1)\,gd}}{\nu} \tag{5}$$

to the Shields diagram (Rouse 1939, Vanoni 1975, Simons and Senturk 1977), shown in Fig. 2, and made its calculation explicitly. That is, first calculate the value of R_* from (5); then find the cross point of the auxiliary line and the Shields diagram from Fig. 2; finally the value of τ_{*c} corresponding to the cross point is the required critical Shields number. In other words, an explicit relationship between τ_{*c} and R_* exists. Equation (5) is called the Rouse Reynolds number in this article.

Guo (1990) has shown that by considering $\tau_{*c} = 0.1\,\mathrm{Re}_{*c}^2/\mathrm{R}_*^2$, (4) can be rewritten as

$$\frac{0.1\,\mathrm{Re}_{*c}^2}{\mathrm{R}_*^2} = \frac{0.11}{\mathrm{Re}_{*c}} + 0.054\left[1 - \exp\left(-\frac{4\,\mathrm{Re}_{*c}^{0.52}}{25}\right)\right] \tag{6}$$

which reduces to

$$\mathrm{Re}_{*c} = \left(\frac{0.11}{0.1}\right)^{2/3} \mathrm{R}_*^{2/3} \tag{7}$$

for small Reynolds number $\mathrm{Re}_{*c} \to 0$, and

$$\mathrm{Re}_{*c} = \sqrt{\frac{0.054}{0.1}}\,\mathrm{R}_* \tag{8}$$

for large Reynolds number $\mathrm{Re}_{*c} \to \infty$.

The first term of (4) is important only for small Reynolds number and the exponential function is important only for relative large Reynolds number. For a first approximation, Guo (1990) substituted (7) into the first term of (4) and (8) into the exponential function, respectively, to result in

$$\tau_{*c} = \frac{0.1}{\mathrm{R}_*^{2/3}} + 0.054\left[1 - \exp\left(-\frac{\mathrm{R}_*^{0.52}}{10}\right)\right] \tag{9}$$

The numerical coefficient 1/10 in the exponential function has been adjusted slightly to confirm with measured values. Equation (9), together with experimental data, is shown in Fig. 3. Noteworthy is that (9) is nothing but an expression of Rouse's idea. To permanently record Rouse's contributions to the Shields diagram, this article recommends that (9) is called the Shields-Rouse equation. The Rouse Reynolds number R_* and Fig. 3 are widely used in coastal engineering (Nielsen 1992, You 2000) although it was credited to Madsen and Grant (1975) who just changed the factor $\sqrt{0.1}$ to 1/4 in (5).

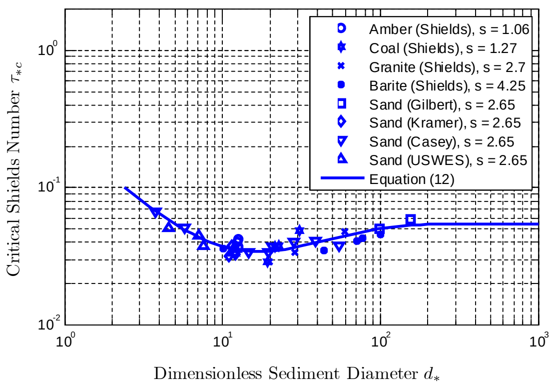

Fig. 4. Shields-Rouse equation in terms of d_*

C. Shields-Rouse equation in dimensionless diameter d_*

River hydraulicians prefer a dimensionless diameter d_* in the study of sediment initiation. For example, Gessler (1971) proposed the following relationship

$$\tau_{*c} = f(d_*) \tag{10}$$

in which d_* is defined as

$$d_* = \left[\frac{(s-1)g}{\nu^2}\right]^{1/3} d \tag{11}$$

The introduction of d_* is exactly analogous to Rouse's auxiliary parameter or Rouse Reynolds number, i.e., eliminating the critical shear stress from the abscissa.

Similar to the development of (9), Guo (1990) showed that relation (10) can be approximated by

$$\tau_{*c} = \frac{0.23}{d_*} + 0.054\left[1 - \exp\left(-\frac{d_*^{0.85}}{23}\right)\right] \tag{12}$$

in which the numerical exponent 0.85 and coefficient 1/23 have been adjusted slightly to confirm with measured data. Equation (12) is compared with experimental data in Fig. 4.

IV. Concluding Remarks

At the end of this article, it must be pointed out that the Shields diagram was not validated for very small Reynolds number, say $\mathrm{Re}_* < 1$ (Julien 1995). This implies neither are (4), (9) and (12). In addition, several other empirical expressions, such as Brownlie's (Garcia 2000) and Michel's (2000), were proposed in the literature. The single purpose of this article is to remember Rouse's contributions to the Shields diagram by providing an/two equations under his name. For more Rouse's stories, readers are referred to two publications in Colorado State University (Albertson and Papadakis 1986) and the University of Iowa (Mutel 1998).

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