Subgrade reaction and load-settlement characteristics of gravelly cobble deposits by plate-load tests

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Abstract: This paper presents the result of plate-load tests conducted on a gravelly cobble deposit in Taichung Basin, Taiwan. The geologic formation of the gravelly cobble deposit makes it very difficult to obtain large undisturbed samples for laboratory testing. These field tests provide an opportunity to examine the applicability of existing theories on bearing capacity and subgrade reaction in this geologic formation. The modulus of subgrade reaction is of particular importance in the local practice of designing high-rise buildings on mat foundations. The results of the plate-load tests on this soil deposit are analyzed and discussed.

Key words: plate-load test, gravelly cobble deposit, modulus of subgrade reaction, bearing capacity.

Résumé : Cet article présente les résultats d'essais de chargement à la plaque effectués sur un dépôt de galets et graviers dans le bassin de Taichung, à Taiwan. La disposition géologique de ce dépôt rend très difficile la collecte d'échantillons non remaniés de grande dimension pour les essais de laboratoire. Les essais en place présentés offrent l'occasion de se demander si l'on peut appliquer les théories existantes sur la capacité portante et sur la réaction du sol à cette formation géologique. Le module de réaction revêt une importance particulière en ce qui concerne la pratique locale de conception des immeubles de grande hauteur sur radier. Les résultats des essais de chargement à la plaque sont analysés et discutés.

Mots clés : essais de chargement à la plaque, dépôt de galets et graviers, module de réaction du sol, capacité portante.

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Introduction

Most high-rise buildings in Taiwan use mat foundations as their main support system (Hsieh and Cherng 1996). A mat foundation usually consists of three parts: a base plate with a thickness of about 50–90 cm, a beam with a depth of about 150–300 cm, and a top plate with a thickness of about 15–20 cm. Each part of the mat is constructed separately. The space between the top and base plates is backfilled with selected soils. Structural loads (column loads) are transmitted through the beam and base plate to the soil mass beneath the base plate. In local practice for structural design of mat foundations, structural engineers prefer to model the soil mass as a series of elastic springs, known as the Winkler foundation. The elastic constant of the assumed springs is referred to as the modulus of subgrade reaction, also known as the coefficient of subgrade reaction. Conceptually, the modulus of subgrade reaction (K_S) is defined as (Terzaghi 1955; Burmister 1962; Sowers and Sowers 1962; Teng 1962)

$$[1] \qquad K_S = \frac{q}{\delta}$$

where q is the load per unit area (or contact pressure) applied to the mat of width B, and δ is the settlement of the mat foundation.

Theoretically, if the contact pressure q and the settlement δ can be estimated, the modulus of subgrade reaction K_S can be determined. In reality, q and δ interact with each other and are difficult to estimate. Settlement δ is caused by the contact pressure q. However, once the foundation settles, the contact pressure redistributes. Thus, the modulus of subgrade reaction K_S is not just a soil parameter, it is also affected by the structural stiffness. It depends on several factors, such as the length and width of the foundation, the depth of embedment of the foundation, the type of structure, and the type of soil beneath the foundation. It can also be time dependent, since much of the settlement of mats on deep compressible soils is due to consolidation.

According to the theory of elasticity, the response of an elastic body to a load may be characterized by at least two parameters, such as modulus of elasticity and Poisson's ratio. Soil is an inelastic, nonhomogeneous material, and thus a precise characterization of its mechanical behavior usually requires more than two parameters. Use of a single parameter such as the modulus of subgrade reaction to characterize the response of a soil to an applied load is thus an oversimplified concept. However, such simplification is generally needed for a practical structural design of mat foundations and represents the current state of practice in Taiwan. In this regard, it is an important issue to select an appropriate equivalent K_S value in the structural design of mat foundations. To improve the accuracy of the mat foundation analy-

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sis, the discrete-area method (Ulrich 1991) may be used, which requires use of varying moduli of subgrade reaction at different sections of the mat. However, the simplest approach of doubling the K_S value along the edges of the mat may not yield a satisfactory result (Horvath 1995). Careful evaluation of the magnitude and variation of the K_S value across a mat for a given project, using the bearing-capacity theories and the discrete-area method, is warranted in this regard. A sophisticated subgrade reaction model for mat foundations has also been proposed by Horvath (1983, 1995).

The K_S value may be determined by semiempirical methods such as that proposed by Vesic (1961). These methods, however, require knowledge of other soil parameters that are also difficult to estimate, such as the modulus of elasticity and Poisson's ratio. A simpler, empirical equation has been proposed by Scott (1981) for sandy soils, which relates K_S to standard penetration resistance N. Wrench and Nowatzki (1986) also developed a relationship between the deformation modulus and N value. For a preliminary analysis, ranges of K_S values suggested by Terzaghi (1955) for different soils might be used as a basis to select an equivalent K_S value. However, Terzaghi's values were established primarily from the viewpoint of soil mechanics and did not consider the effect of structure stiffness. Thus, they are generally too conservative for the design of mat foundations. Various ranges of K_S values have also been proposed by other engineers (Bowles 1996; Das 1996). These values should be used with caution. When in doubt, a parametric study should be conducted to evaluate the effect of the uncertainty of the K_S values on the mat design.

The plate-load test has been a traditional in situ method for estimating soil modulus for the purposes of estimating the settlement of spread footings. Experience has shown that the plate-load test can provide reliable estimates of vertical modulus for settlement calculations (Canadian Geotechnical Society 1985). However, the test results must be adjusted to compensate for differences in width, shape, and depth of the plate and the mat (Coduto 1994). Although the extrapolation from a small plate to a mat may induce a significant amount of uncertainty, it may be the only feasible choice when dealing with an unusual soil formation without prior experience. In the present study a series of plate-load tests is performed to investigate the load–settlement characteristics of a gravelly cobble deposit in Taiwan. The results of these plate-load tests are presented in this paper.

Note that the characteristics of coarse granular materials might be better estimated by several other methods, such as shear- and compression-wave measurements, Becker Hammer Drill penetration tests, and the photo-sieving method. However, these topics are beyond the scope of the present paper. Note also that in the characterization of gravels for important projects, a thorough assessment of the engineering geology is perhaps more important than one in situ test. While the importance of engineering geology is well recognized, the results of plate-load tests in the present study contribute to the understanding of the load-bearing characteristics of gravelly cobble deposits.

Geology of test site and soil profile

The test site is located in the City of Taichung, Taiwan, which is in the Taichung Basin, a concave Neotectonic basin located in midwest Taiwan. The length of the basin from north to south is about 48 km and the maximum width from east to west is about 14 km. The west boundary of the basin is the Tatu Terrace, and the east boundary is the Taiwan Western Foothills. The Taichung Basin is mainly composed of alluvial fan deposits and there is no evidence of uplift. The Tatu Terrace is mainly composed of Pleistocene sandstone and mudstone. The Western Foothills are mainly composed of Miocene to Pleistocene sandstone and shale strata.

The surface of the Taichung Basin is generally covered by 2–5 m of silty soils. Beneath the surface is a deep alluvial fan deposit of normally consolidated quartzitic gravel (>70%) and cobbles (up to 1 m in diameter). Sand- and silt-size particles fill the space not occupied by the gravel and cobbles. Little or no cementation is observed. Beneath the alluvial fan gravel and cobble deposits is the Late Pleistocene conglomerate formation, which extends as deep as 300 m. Figure 1 shows a geologic map and profile of the Taichung Basin.

The soil profile of the test site is shown in Fig. 2 to a depth of 18 m. The groundwater table was at a depth of about 5.2 m during the plate-load tests. The top layer is a fill of about 0.5 m thick. The second layer, from 0.5 to 2 m, is a yellowish brown silty clay. The third layer, from 2 to 2.5 m, is a gray silty sand. The bottom layer is a gravelly cobble deposit, occasionally with thin layers of sand. Figure 3 shows an exposure of the gravelly cobble deposit at the test site during excavation and Fig. 4 shows a close-up of this gravelly cobble layer.

Experimental program

The experimental program consisted of laboratory and field tests. The field portion of the study included a test pit exploration and plate-load tests. The field test pit exploration included in-place unit weight and moisture content tests, conducted according to American Society for Testing and Materials (ASTM) Standard D2167-94 (ASTM 1995*b*). Soil samples taken from the test pit were sieved through a set of sieves ranging in size from 30.5 to 0.5 cm (No. 4 sieve). These samples were used in the laboratory for determination of unit weight, specific gravity, and particle-size distribution. Standard testing methods as prescribed in ASTM (1995*b*) were followed.

Plate-load tests were the focus of the experimental program. The tests were conducted according to the method prescribed in ASTM Standard D1194-94 (ASTM 1995*a*). Various aspects of the plate-load test, including design of reaction anchors and reaction beams, test setup, and test procedure, are described below.

Design of reaction anchors

The first step in the design of reaction anchors was to estimate the ultimate bearing capacity of the plate. Chu et al. (1989) conducted a series of field direct shear tests in a similar gravelly cobble deposit and reported a friction angle (ϕ) of 54.3° and a cohesion (c) of 14.7 kN/m^2. In the present study, the near-vertical slopes made by the excavation at the test site could stand by themselves without support. In addition, the angle of repose of the excavated gravelly cobble deposit could reach as high as about 55°. This led to the assumption of a friction angle of 55–60° for the in-place

Fig. 1. Geologic map of Taichung Basin.

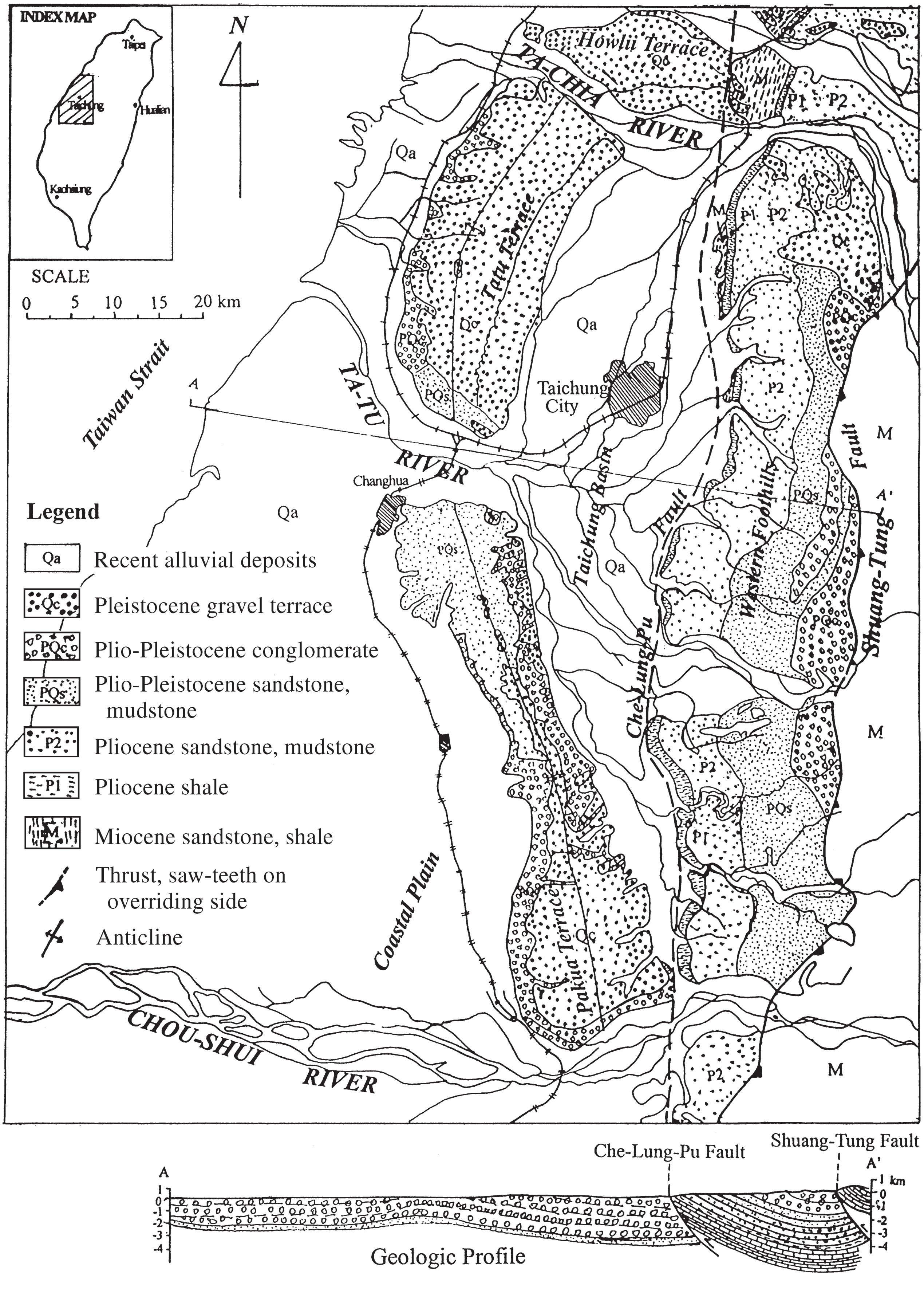
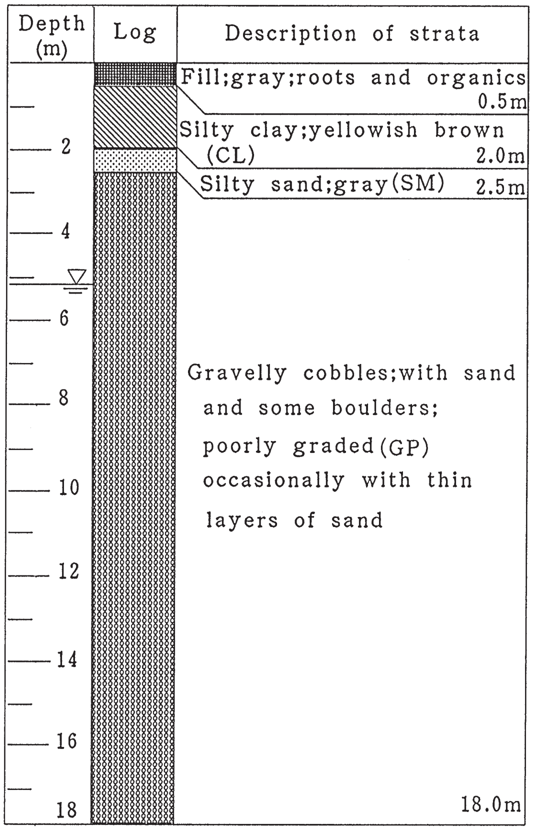


Fig. 2. Soil profile of the test site.



gravelly cobble deposit. Since the voids within the gravelly cobble deposit at the test site are filled with sandy soils, the cohesion was assumed to be zero. These shear strength parameters along with a unit weight of 19.6 kN/m^3 (2.0 t/m^3) were used to estimate the ultimate bearing capacity of the 0.75 m diameter plate in the plate-load test. Based on Hansen's (1970) formula, the ultimate bearing capacity was about 8430 kN (860 t).

To accommodate larger size plates, it was decided to use eight reaction anchors, each carrying an allowable load of up to 1176 kN (120 t). This gave a total allowable load of 9415 kN (960 t). The individual reaction anchor was designed according to Littlejohn's (1970) formula. The frictional resistance was estimated at 343 kN/m^2 (35 t/m^2), and thus the length of the anchor with a diameter of 0.15 m was determined to be 11.6 m. A length of 12 m was used. Using a factor of safety of 1.6, the allowable pull-out resistance of this reaction anchor was 1216 kN (124 t). Each reaction anchor consisted of 12 steel bars. The ultimate strength of each steel bar was 183 kN (18.7 t), and thus the total allowable tensile resistance of a reaction anchor was 1755 kN (179 t). In this calculation, a reduction factor of 0.8 was used to account for the bonding between steel and concrete. Thus, it was concluded that the total pull-out resistance of the eight reaction anchors would be 14 043 kN (1432 t), much greater than the required total allowable load of 9415 kN (960 t).

Plate-load test setup and procedure

The test site was excavated to a depth of 6.2 m, well into the layer of gravelly cobble deposits. The area at the bottom of the excavation was about 4 m^2. The main reaction beam, measured at 13 m by 1.2 m, was placed at a depth of 4.75 m. The secondary reaction beam, measured at 9 m by 1.2 m, was placed at a depth of 3 m. Figures 5 and 6 show schematic diagrams of the setup of the plate-load test, Fig. 5 for the plan view and Fig. 6 for the side view. The main reaction beam was capable of supporting a maximum load of 11 768 kN (1200 t).

The groundwater table was initially at a depth of 5.2 m before the tests. It was decided to lower the water table, by pumping, to below the depth of the plate during the test to provide a desirable working environment. The groundwater table was maintained below the plate (at a depth of about 6.2 m) throughout the tests. The plate-load tests conducted in the present study followed the procedures described in ASTM Standard D1194-94 (ASTM 1995*a*). The test procedure and placement of other devices are summarized as follows:

(1) Clean up, smooth the bottom of the excavation, and then place a thin layer of sand before placing the plate on it. Adjust the plate so that it has good contact with the ground at the bottom of the excavation.

(2) Place a high-capacity jack on the top of the loading plate.

(3) Assemble the reference beams and mount four dial gauges on the reference beams to measure the settlement of the plate at the four corners.

(4) Adjust the jack so that it properly contacts the reaction beam. Figure 7 shows the complete setup for the plate-load test.

(5) For each test, load is applied in two stages. In the first stage, where the load is less than 1471 kN (150 t), the load is measured by a load gauge which has a maximum capacity of only 1961 kN (200 t) but is more accurate. In the second stage, the load increment is measured by the pressure gauge of the high-capacity jack system.

(6) At each load increment (215–245 kN/m^2), the settlement of the plate is recorded for at least 15 min, and until a settlement rate of less than 0.0025 cm/min is reached.

(7) Continue to increase the loading until near failure.

To reduce the potential effect of loading–unloading–reloading on the stiffness of the soil, ASTM Standard D1194-94 (ASTM 1995*a*) recommends that "…the distance between test locations shall not be less than five times the diameter of the largest plate used in the tests." However, because of the cost of constructing the reaction beam – reaction anchor system, the three plate-load tests were performed side by side using the same reaction system. Thus, while the plates were seated at different locations beneath the main reaction beam, the ASTM distance specification was not followed. Nevertheless, the tests were conducted from small plate to large plate, with a 10 day time interval between the tests, to minimize the effect.

Note that reseating of each plate before loading was necessary, as the three plates were placed at different locations. For each plate-load test, a contact pressure of about 19.6 kN/m^2 (2 t/m^2) was observed during the seating, which caused a settlement of about 0.02 cm. Before applying the loads, the pressure gauge and the dial gauge were set to zero. Thus, results from the three plate-load tests were readily comparable.

Fig. 3. Exposure of gravelly cobble deposits from an excavation at the test site. Shovel for scale.

Fig. 4. Close-up of gravelly cobble deposits. Ruler is approximately 30 cm long.

Results of plate-load tests

Physical properties of soils

The in-place unit weight of the gravelly cobble deposit is 21.5 kN/m^3 (2.19 t/m^3) and the moisture content is about 2%. The gravelly cobble deposit at this site consists of 80% gravelly cobble and 20% sand matrix. The apparent specific gravity of the gravelly cobble deposit is 2.68 and its absorption is about 1%. The unified soil classification of the gravelly cobble deposit is GP. The particle-size distribution of the composite gravelly cobble deposit (with sand matrix) is shown in Fig. 8. The tail portion of this composite curve (the portion with particle sizes smaller than 5 mm, or in this case, at a percent passing of less than 20%) is the distribution of sand matrix. Note that the particle-size distribution of this sand matrix alone, shown separately in Fig. 8, is essentially a "normalized" curve of the tail portion of the composite distribution curve. Because the sand matrix accounts for 20% of the gravelly cobble composite, the values of percent passing in the sand matrix curve are five times those shown in the tail portion of the composite distribution curve.

Plate-load test results and analysis

Three different sizes of plates, 0.75, 0.90, and 1.05 m in diameter, were used in the plate-load tests. Load–settlement curves obtained by using the three plates are shown in

Fig. 5. Schematic diagram of the plate-load test setup: plan view.

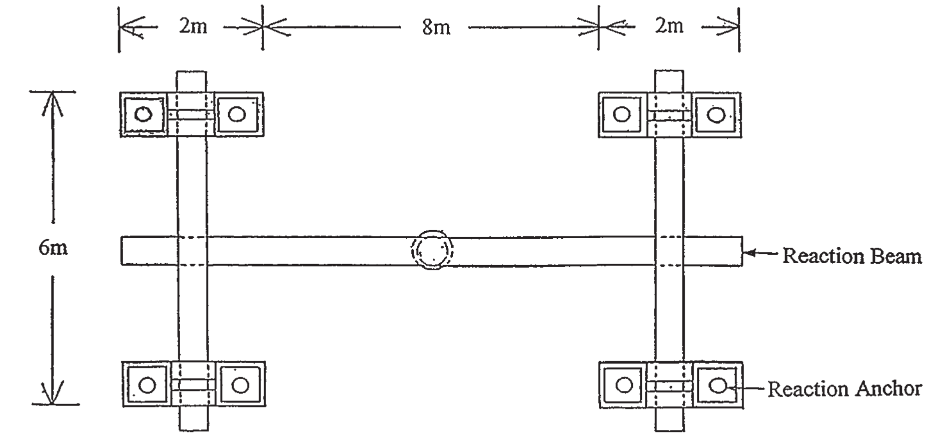


Fig. 6. Schematic diagram of the plate-load test setup: side view.

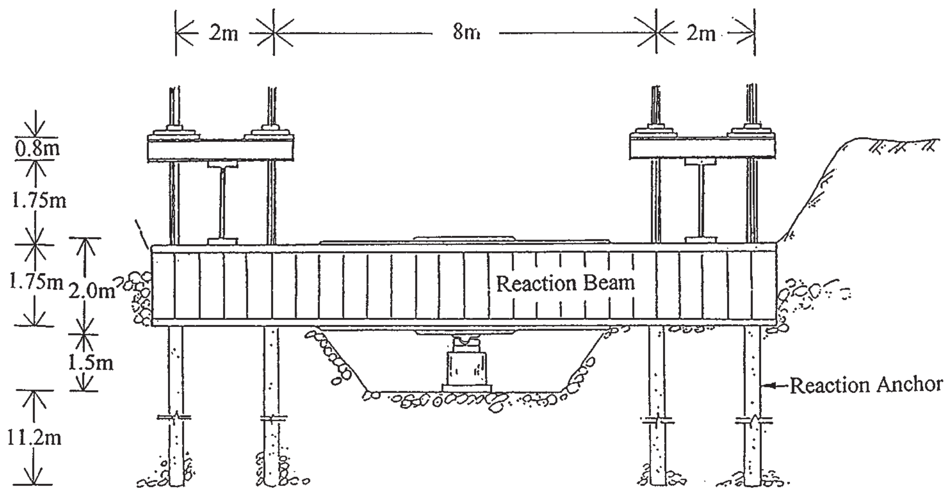


Fig. 7. Complete setup for the plate-load test.

Fig. 8. Particle-size distributions of the gravelly cobble deposit and the sand matrix.

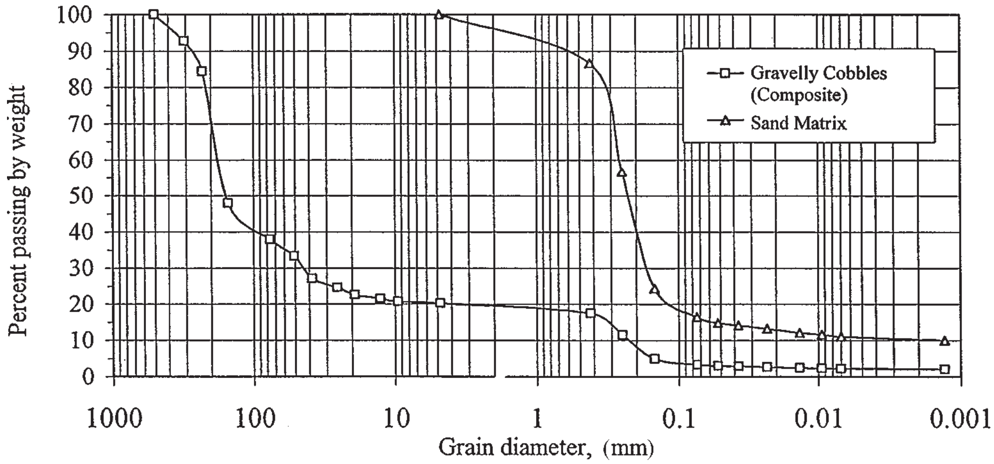


Fig. 9. Load–settlement curves for the plate-load tests for varying plate diameters *d*.

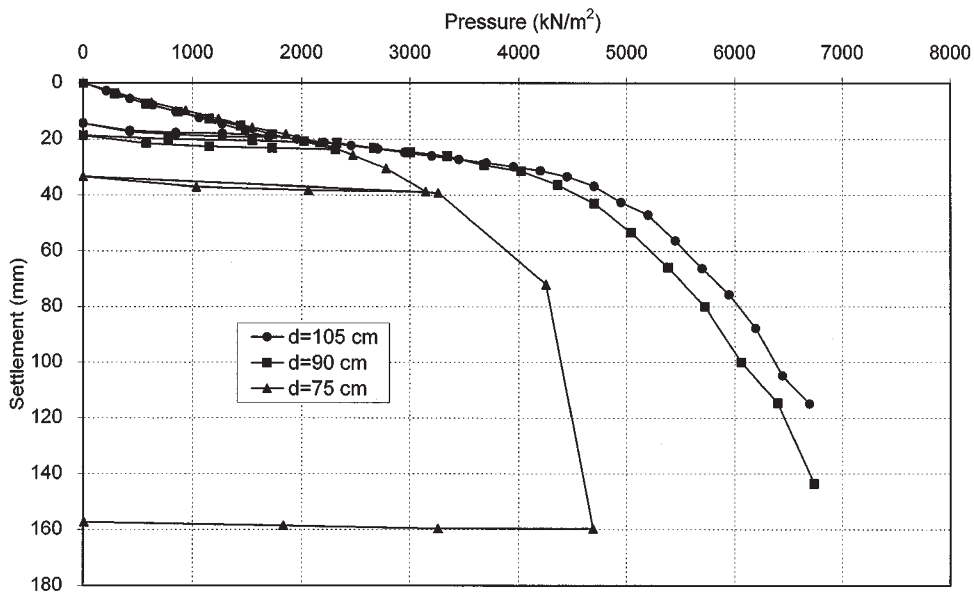


Fig. 9. To interpret the results, the ultimate bearing capacity (q_u) is defined herein as the load (or most precisely, the pressure) at the intersection of the tangent to the initial portion of the curve and the tangent to the last portion of the curve. The q_u values of the 0.75 m plate, the 0.90 m plate, and the 1.05 m plate are interpreted to be 4100, 5000, and 5400 kN/m^2, respectively.

Figure 10 shows a plot of the measured ultimate bearing capacity versus the size of the plate used in the load test. According to commonly used bearing-capacity equations such as those of Terzaghi (1943), Meyerhof (1965), and Hansen (1970), for a footing where cohesion $c = 0$ and overburden pressure $q = 0$, the bearing capacity q_u should increase, linearly, with the size of the footing in a homogeneous soil deposit. Figure 10 shows a similar trend, although the relation is not exactly linear. Obviously, the soil in the present study is a rather complicated, nonhomogeneous deposit. In addition, the load test with the 1.05 m plate was not carried out to failure, because the test had to be stopped due to a slight movement of the reaction beam at the end. However, the load tests did result in an important finding: the measured allowable bearing capacity with a factor of safety of 3 is at least two times greater than that generally adopted in the local practice, which limits it to 588 kN/m^2 (60 t/m^2). A more economic design can be prescribed according to the results of the present study.

Figure 11 shows the relationship between the modulus of subgrade reaction and the size of the plate. Here, the modulus K_S is calculated based on the measured settlement (δ_a) at the allowable bearing capacity (q_a), defined in Fig. 12 as the measured (or interpreted) ultimate bearing capacity divided by a factor of safety of 3.

The modulus K_S decreases as the size of the plate increases. This relationship had been investigated by Terzaghi (1955) and it is generally accepted that for foundations on sandy soils

[2] $$K_B = K_{0.3}\left(\frac{B+0.3}{2B}\right)^2$$

Fig. 10. Relationship between ultimate bearing capacity and plate diameter.

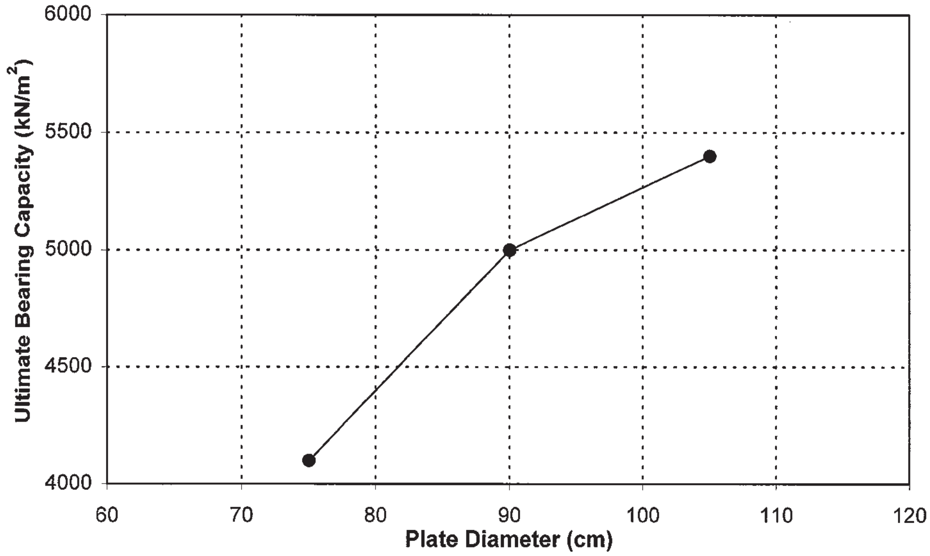
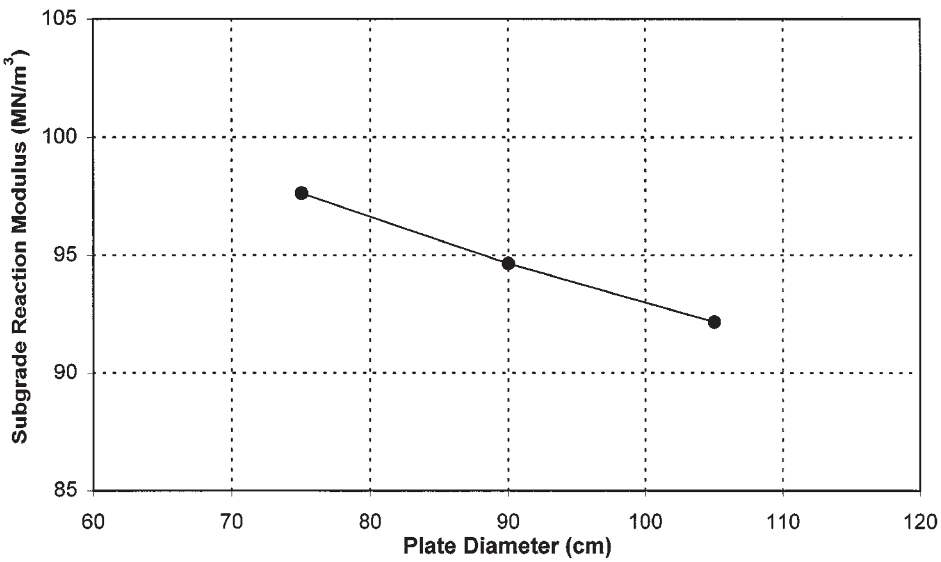


Fig. 11. Relationship between modulus of subgrade reaction and plate diameter.



where $K_{0.3}$ is the modulus of subgrade reaction determined with a 0.3 m plate, and K_B is the modulus of subgrade reaction of a $B \times B$ footing (width B is in metres). For foundations on clays, the following relationship is generally accepted:

$$[3] \qquad K_B = K_{0.3}\left(\frac{0.3}{B}\right)$$

Although a decreasing trend in K_S is also observed in the present study of the plate-load tests on the gravelly cobble deposit, the reduction is not as sharp as suggested by eqs. [2] and [3]. This may be because the gravelly cobble deposit is much stiffer than the soils considered by Terzaghi. Figure 13 shows a comparison of these decreasing trends (i.e., effects of plate size and soil type), using $K_{0.75}$ as a reference, where $K_{0.75}$ is the modulus of subgrade reaction with a 0.75 m plate.

Note that extrapolating the K_S trend to B = 0.3 m would yield a subgrade modulus ($K_{0.3}$) of about 108 MN/m^3 (11 000 t/m^3) for the gravelly cobble deposit investigated. This inferred value (or the measured values of 88–98 MN/m^3 for B = 0.75–1.05 m) is deemed consistent with typical values suggested in the literature. For example, the range of K_S values suggested by Bowles (1996) for dense sands is from 63 to 127 MN/m^3. The range of K_S values for bearing strata (those with standard penetration blow counts of greater than 50) is from 78 to 98 MN/m^3 (Hseih and Cherng 1996). Note that in the first reference cited above, no specific reasoning or analysis was given as to how these typical $K_S$values were determined. On the other hand, Hseih and Cherng's K_S values were established mostly based on finite-element analyses and field measurements of a number of high-rise buildings supported by mat foundations.

The angle of internal friction ϕ may be back-calculated from the measured ultimate bearing capacity. Using the

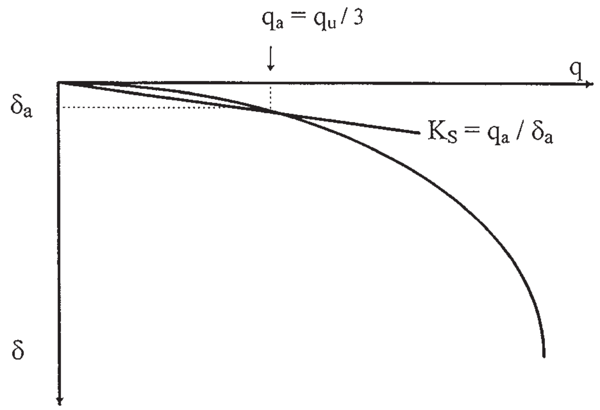


Fig. 12. Schematic diagram for determination of the modulus of subgrade reaction.

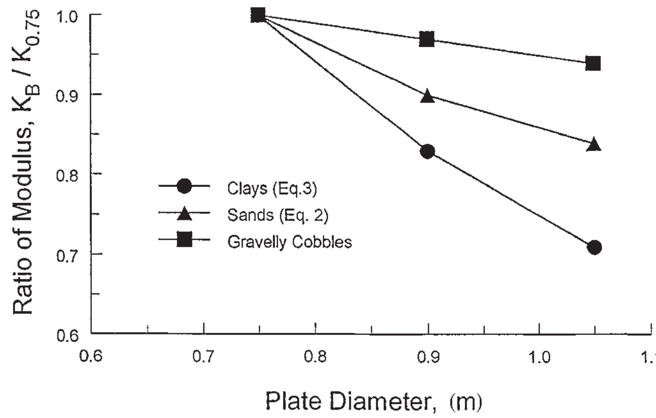


Fig. 13. Effect of plate diameter and soil type on the modulus of subgrade reaction.

Table 1. Summary of plate-load tests.

Plate size (m)	Ultimate bearing capacity (kN/m^2)	Allowable bearing capacity (kN/m^2)	Settlement at allowable bearing capacity (mm)	Modulus of subgrade reaction (MN/m^3)	Friction angle (°) back-calculated by bearing-capacity theory		
					Terzaghi	Meyerhof	Hansen
0.75	4100	1367	14.1	97.6	51	52	54
0.90	5000	1667	17.6	94.7	51	52	54
1.05	5400	1800	19.5	92.2	51	52	54

bearing-capacity factor N_γ of Terzaghi (1943), Meyerhof (1965), and Hansen (1970), the friction angle ϕ is back-calculated and the result is shown in Table 1, along with other results of the plate-load tests. The values of ϕ in Table 1 are practically constant regardless of the size of the plate in the present study. This reinforces the observation of the K_S–B relationship shown in Fig. 13 for the case of the gravelly cobble deposit that the reduction in K_S is not as sharp as that suggested by eqs. [2] and [3].

Note that in all bearing-capacity calculations reported herein, the groundwater table was assumed to be at a depth of 6.2 m, immediately below the plates during the test. As shown in Table 1, the ϕ values back-calculated with Terzaghi's (1943) formula are lowest. This simply confirms the well-recognized fact that Terzaghi's bearing-capacity theory is usually the most conservative. All ϕ values that are back-calculated from these bearing-capacity theories are less than 55°. As the actual friction angle ϕ is believed to be within the range of 55–60°, these bearing-capacity theories are considered to be conservative and appropriate for design. Hansen's (1970) formula appears to be most accurate in the present study.

Conclusions

The following conclusions may be drawn based on the results of the present study of a gravelly cobble deposit:

(1) The modulus of subgrade reaction of the gravelly cobble deposit decreases as the size of the plate increases. However, the decreasing trend is not as sharp as those suggested by Terzaghi (1955) (eqs. [2] and [3]) for sands and clays. This may be because the gravelly cobble deposit is much stiffer than clayey and sandy soils.

(2) The angle of internal friction ϕ back-calculated with commonly used bearing-capacity theories is practically a constant regardless of the size of the plate. This reinforces the observation about the relationship between the modulus of subgrade reaction and the size of the plate in the case of the gravelly cobble deposit. The ϕ values back-calculated with Terzaghi's (1955) formula are the lowest. This confirms the well-recognized fact that Terzaghi's bearing-capacity theory is usually the most conservative one. Hansen's (1970) bearing-capacity theory is the most accurate one based on the limited plate-load tests conducted in the present study.

(3) The ultimate bearing capacity increases as the size of the plate increases. However, in the present study of the gravelly cobble deposit, the relationship between the ultimate bearing capacity and the plate size is not exactly linear.

(4) The measured allowable bearing capacity with a factor of safety of 3 obtained from the plate-load tests is at least two times greater than the limit value 588 kN/m^2 (60 t/m^2) used in the current practice in Taiwan for a gravelly cobble deposit. A more economic design of the mat foundation for high-rise buildings may be prescribed.

(5) The results of the present study are believed to be valid in the gravelly cobble deposit investigated and may be applicable to similar geologic settings. Application of the results to other geologic settings, however, must be exercised with caution.

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References

ASTM. 1995*a*. Test method for bearing capacity of soil for static load on spread footings (D1194-94). *In* 1995 Annual Book of ASTM Standards, Vol. 04.08. American Society for Testing and Materials, Philadelphia, Pa., pp. 98–100.

ASTM. 1995*b*. Test method for density and unit weight of soil in place by the Rubber Balloon Method (D2167-94). *In* 1995 Annual Book of ASTM Standards, Vol. 04.08. American Society for Testing and Materials, Philadelphia, Pa., pp. 167–170.

Bowles, J.E. 1996. Foundation analysis and design. 5th ed. McGraw-Hill, New York.

Burmister, D.M. 1962. Prototype load-bearing tests for foundations and pavements. American Society for Testing and Materials, Special Technical Publication STP 322.

Canadian Geotechnical Society. 1985. Canadian foundation engineering manual. 2nd ed. BiTech Publishers, Vancouver, B.C.

Chu, P.L., Hunag, C.L., Jeng, S.Y., and Pan, J.M. 1989. Study of field direct shear tests in terrace deposits and Toukoshan conglomerate formation. *In* Proceedings of the 3rd Conference on Current Research in Geotechnical Engineering in Taiwan, Kenting, Taiwan, May 5–7, pp. 695–706.

Coduto, D.P. 1994. Foundation design — principles and practices. Prentice Hall Inc., Englewood Cliffs, N.J.

Das, B.M. 1996. Principles of foundation engineering. 3rd ed. PWS Publishing Co., Boston.

Hansen, J.B. 1970. A revisited and extended formula for bearing capacity. Danish Geotechnical Institute, Bulletin 28.

Horvath, J.S. 1983. Modulus of subgrade reaction: new perspective. Journal of Geotechnical Engineering, ASCE, **109**(12): 1591–1596.

Horvath, J.S. 1995. Subgrade modeling for mat foundations: a review and critique of analytical methods. *In* Design and performance of mat foundations. *Edited by* E.J. Ulrich. American Concrete Institute, Publication SP-152, pp. 117–160.

Hsieh, H.S., and Cherng, J.C. 1996. A note on the modulus of subgrade reaction for raft foundation design. Sino-Geotechnics, No. 53, February 1996, pp. 45–54.

Littlejohn, G.S. 1970. Soil anchors. Ground Engineering, Institute of Civil Engineers, London, pp. 36–40.

Meyerhof, G.G. 1965. Shallow foundations. Journal of the Soil Mechanics and Foundations Division, ASCE, **91**(SM2): 21–31.

Scott, R.F. 1981. Foundation analysis. Prentice Hall Inc., Englewood Cliffs, N.J.

Sowers, G.B., and Sowers, G.F. 1962. Introductory soil mechanics and foundations, The McMillan Co., New York.

Teng, W.C. 1962. Foundation design. Prentice Hall Inc., Englewood Cliffs, N.J.

Terzaghi, K. 1943. Theoretical soil mechanics. John Wiley and Sons, New York.

Terzaghi, K. 1955. Evaluation of coefficients of subgrade reaction. Géotechnique, **5**(4): 297–326.

Ulrich, E.J. 1991. Subgrade reaction in mat foundation design. Concrete International, **13**(4): 41–50.

Vesic, A.S. 1961. Bending of beams resting on isotropic solid. Journal of the Engineering Mechanics Division, ASCE, **87**(EM2): 35–53.

Wrench, B.P., and Nowatzki, E.A. 1986. A relationship between deformation modulus and SPT N for gravels. *In* Proceedings, use of in situ tests in geotechnical engineering. *Edited by* S. Clemence. American Society of Civil Engineers, New York, pp. 1163–1177.