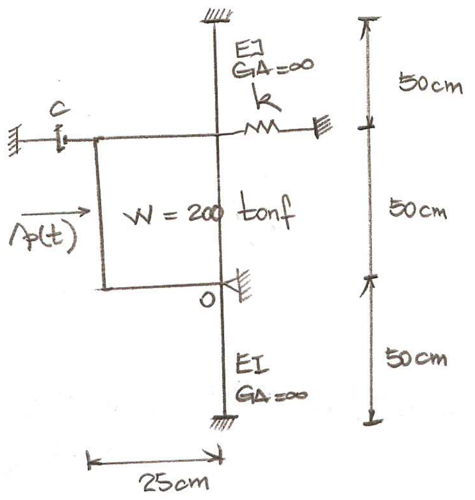
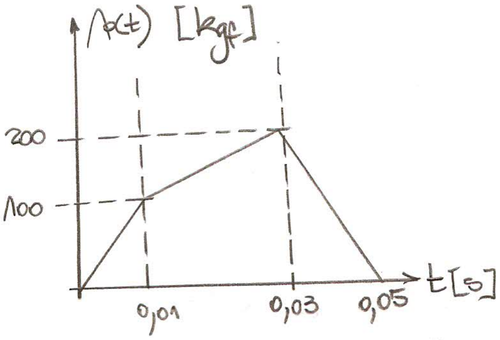
Problema 2 (Control 1, 2007)



* Constante de amortiguamiento (c) de modo que $u_{2_{max}} = 0{,}25\, u_{1_{max}}$.



Datos: $EI = 1000$ [kgf · m²]

$GA = \infty$

$W = 200$ [tonf]

$k = \dfrac{12\,EI}{(0{,}5)^3}$

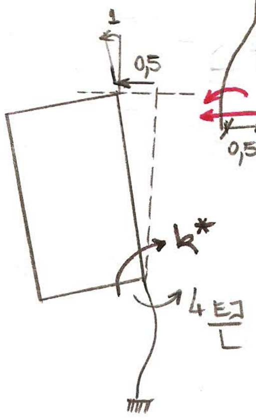
1. Calcular I_o:

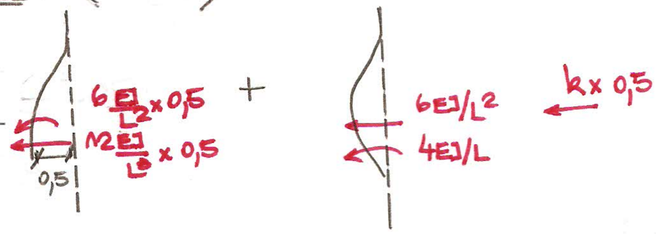
$$m = \frac{W}{g} = \frac{200\,000}{9{,}8} = 20408{,}2 \quad \left[\frac{kgf \cdot s^2}{m}\right]$$

$$I_o = \frac{m}{12}\left(0{,}25^2 + 0{,}5^2\right) + m\left(0{,}125^2 + 0{,}25^2\right)$$

$$= 2125{,}9 \quad [kgf \cdot m\, s^2]$$

2. Calcular Rigidez: $(\theta = 1)$





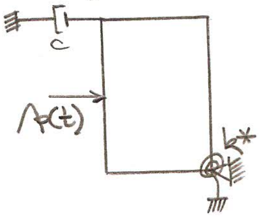
$$k^* = \frac{4EI}{(0{,}5)} + \frac{4EI}{(0{,}5)} + \frac{6EI}{(0{,}5)^{\cancel{2}}} \times \cancel{0{,}5} + \left\{\frac{12\,EI}{(0{,}5)^{\cancel{3}}} \times \cancel{0{,}5}\right\} \times \cancel{0{,}5}$$

$$+ \frac{6\,EI}{(0{,}5)^{\cancel{2}}} \times \cancel{0{,}5} + \left\{k \times 0{,}5\right\} \times 0{,}5$$

$$= \frac{4\,EI}{0{,}5} + \frac{4\,EI}{0{,}5} + \frac{6\,EI}{0{,}5} + \frac{6\,EI}{0{,}5} + \frac{12\,EI}{0{,}5} + \frac{12\,EI}{0{,}5}$$

ie, $k^* = 88\,000$ [kgf · m]

3. Período y frecuencia natural.



$$\omega_s = \sqrt{\frac{k^*}{I_o}} = 6{,}434 \quad \left[\frac{rad}{s}\right]$$

$$T_s = \frac{2\pi}{\omega_s} = 0{,}976 \quad [s]$$

Momento ejercido por F_D c/r a O

Ec. de movimiento

$$I_o\,\ddot{\theta} + c^*\dot{\theta} + k^*\theta = P^*(t)$$

$$\begin{cases} c^* = (0{,}5)^2\, c \\ P^*(t) = (0{,}25)\, p(t) \end{cases}$$

Como, $\dfrac{t_d}{T_o} = \dfrac{0{,}05}{0{,}976} = 0{,}051 < 0{,}25 \Rightarrow \hat{p}(t)$ es un impacto.

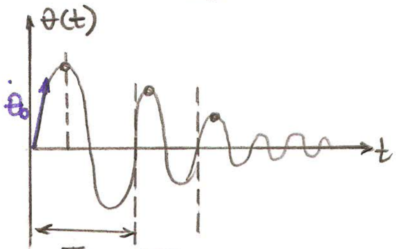
$$\dot{\theta}_o = \underbrace{\frac{1}{m^*}\int_0^{t_d} \hat{p}^*(t)\,dt}_{\text{Velocidad inicial equivalente}} = \frac{1}{I_o}\int_0^{t_d} 0{,}25\,\hat{p}(t)\,dt$$

$$= \frac{0{,}25}{2125{,}9}\left\{\frac{1}{2}\,0{,}01\times 100 + \frac{1}{2}(100+200)\,0{,}02 + \frac{1}{2}\,0{,}02\times 200\right\}$$

$$= 0{,}00065 \left[\frac{\text{RAD}}{\text{s}}\right]$$

4. <u>Amortiguamiento</u> (Impacto $\longleftrightarrow$ Velocidad Inicial) $*\ u(t) = e^{-\beta\omega_o t}\left\{ v_o\cos(\omega_d t) + \left(\dfrac{\dot{v}_o + \beta\omega_o v_o}{\omega_d}\right)\sin(\omega_d t)\right\}$

$$\theta(t) = \frac{\dot{\theta}_o}{\omega_D} e^{-\beta\omega_o t}\sin(\omega_d t)$$



$T_d = \dfrac{2\pi}{\omega_d}$

1er máximo: $t_1 = \frac{1}{4}T_d = \dfrac{\pi}{2\omega_d}$

3er máximo: $t_3 = t_1 + 2T_d$

$$= \frac{\pi}{2\omega_d} + \frac{4\pi}{\omega_d} = \frac{9}{2}\frac{\pi}{\omega_d}$$

$$\Rightarrow \theta_1 = \frac{\dot{\theta}_o}{\omega_d} e^{-\beta\omega_o t_1} \wedge \theta_3 = \frac{\dot{\theta}_o}{\omega_d} e^{-\beta\omega_o t_3}$$

Del enunciado,

$$\frac{\theta_3}{\theta_1} = 0{,}25 = \frac{e^{-\beta\omega_o t_3}}{e^{-\beta\omega_o t_1}} \Rightarrow e^{-\beta\omega_o\underbrace{(t_3 - t_1)}_{2T_d}} = 0{,}25$$

$$\Rightarrow e^{-\beta\cancel{\omega_o}\left(\frac{4\pi}{\cancel{\omega_o}\sqrt{1-\beta^2}}\right)} = 0{,}25$$

$$e^{-\frac{4\pi\beta}{\sqrt{1-\beta^2}}} = 0{,}25$$

Aplicando $\ln(\cdot)$,

$$\ln(0{,}25) = -\frac{4\pi\beta}{\sqrt{1-\beta^2}}$$

$$\Rightarrow \left(\frac{4\pi}{\ln(0{,}25)}\right)^2\beta^2 = 1-\beta^2 \longrightarrow \beta^2 = \frac{1}{\left(\frac{4\pi}{\ln(0{,}25)}\right)^2 + 1} = 0{,}012 \xrightarrow{+} \beta = 0{,}109$$

$(\beta = 11\%)$

Como $\beta = \dfrac{c^*}{2m^*\omega_o}$

$$\Rightarrow c^* = \beta\times 2m^*\omega_o = (0{,}11)\times 2\times(2125{,}9)\times(6{,}434)$$

$$= 3009{,}2$$

$$\longrightarrow c^* = (0{,}5)^2 c \longrightarrow c = 12037 \left[\frac{\text{kgf}}{\text{m}}\right]$$

5. <u>Respuesta máxima</u>

$$\theta(t_1) = \frac{\dot{\theta}_o}{\omega_d} e^{-\beta\omega_o t_1} = \frac{(0{,}00065)}{6{,}395} e^{-(0{,}11)(6{,}434)(0{,}245)} = 8{,}55\times 10^{-5}\ [\text{rad}]$$

$t_1 = \dfrac{\pi}{2\omega_d} = 0{,}245$ $\Rightarrow u_{max} = 50\,\theta_{max} = 0{,}0043$ [cm]

$\omega_d = 6{,}395$

6. <u>Duhamel - Fase II</u>: $(t \geq t_d)$

$$u(t) = \frac{1}{m\omega_d} \int_0^t p(\tau)\, e^{-\beta\omega_s(t-\tau)} \sin(\omega_d(t-\tau))\, d\tau .$$

$$* \ \sin(\omega_d(t-\tau)) = \sin(\omega_d t - \omega_d \tau) = \sin(\omega_d t)\cos(\omega_d \tau) - \cos(\omega_d t)\sin(\omega_d \tau)$$

Sigue que,

$$u(t) = \frac{1}{m\omega_d} e^{-\beta\omega_s t} \sin(\omega_d t) \left\{ \int_0^t p(\tau)\, e^{\beta\omega_s \tau} \cos(\omega_d \tau)\, d\tau \right\}$$

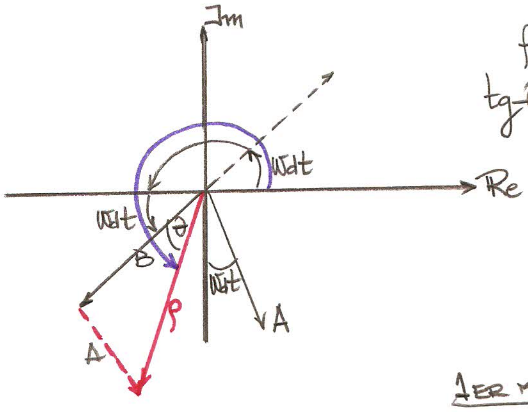
$$- \frac{1}{m\omega_d} e^{-\beta\omega_s t} \cos(\omega_d t) \left\{ \int_0^t p(\tau)\, e^{\beta\omega_s \tau} \sin(\omega_d \tau)\, d\tau \right\} \quad (1)$$

$u(t) \longrightarrow \theta(t)$
$m \longrightarrow I_o$
$p \longrightarrow 0{,}25\, p(t)$

Para $t > 0{,}05\,[s]$

$$\theta(t) = (1{,}839 \times 10^{-5})\, e^{-\beta\omega_s t} \left\{ A \sin(\omega_d t) - B \cos(\omega_d t) \right\} \quad (2)$$

$$* \ A = \int_0^{t_d} p(\tau)\, e^{\beta\omega_s \tau} \cos(\omega_d \tau)\, d\tau = 7{,}467 \quad \wedge \quad B = \int_0^{t_d} p(\tau)\, e^{\beta\omega_s \tau} \sin(\omega_d \tau)\, d\tau = 1{,}8214 .$$



$$\rho = \sqrt{A^2 + B^2} = 7{,}686$$

$$tg\,\theta = \frac{A}{B} = 4{,}1 \longrightarrow \theta = 1{,}33 < \pi/2 .$$

Reemplazando en (2),

$$\theta(t) = (1{,}413 \times 10^{-4})\, e^{-\beta\omega_s t} \cos(\omega_d t + \theta + \pi)$$

$$= -(1{,}413 \times 10^{-4})\, e^{-\beta\omega_s t} \cos(\omega_d t + \theta) \quad (3)$$

<u>1er máximo</u>: $\cos(\omega_d t_1 + \theta) = -1 \longrightarrow t_1 = \dfrac{\pi - \theta}{\omega_d} = 0{,}283\,[s] .$

Por lo tanto,

$$\theta_{max} = 1{,}413 \times 10^{-4}\, e^{-(0{,}11)(6{,}434)(0{,}283)}$$

$$= 1{,}16 \times 10^{-4} \ [rad]$$

$$u_{max} = 50 \times \theta_{max} = \underline{\underline{0{,}0057\ [cm]}} .$$