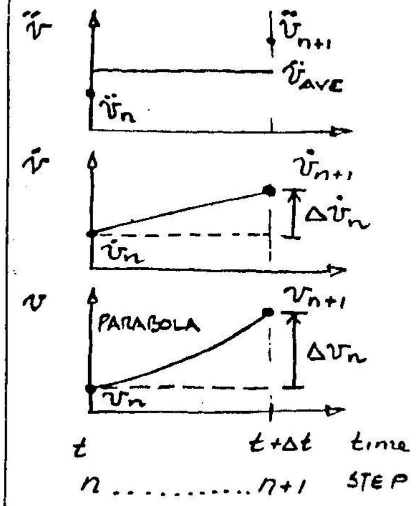

SINGLE STEP METHOD BASED ON CONSTANT "AVERAGE" ACCELERATION

EQUILIBRIUM COMPUTED AT $t=(n+1)\Delta t$

KNOWS: m; C; K; v_n; $\dot v_n$; $\ddot v_n$

$$\boxed{\Delta \ddot v_n = \ddot v_{n+1} - \ddot v_n} \qquad \text{I a}$$

$$\boxed{\ddot v_{AVE} = \ddot v_n + \frac{\Delta \ddot v_n}{2}} \qquad \text{I b}$$

FOR VELOCITY:

$$\boxed{\dot v_{n+1} = \dot v_n + \Delta \dot v_n} \qquad \text{II a}$$

$$\boxed{\Delta \dot v_n = \Delta t\, \ddot v_{AVE} = \ddot v_n \Delta t + \frac{\Delta t}{2}\Delta \ddot v_n} \qquad \text{II b}$$

FOR DISPLACEMENT:

$$\boxed{v_{n+1} = v_n + \Delta v_n} \qquad \text{III a}$$

$$\boxed{\Delta v_n = \frac{1}{2}\Delta t\, \Delta \dot v_n + \Delta t\, \dot v_n} \qquad \text{III b}$$

OR

$$\boxed{\Delta v_n = \frac{\Delta t^2}{2}\ddot v_n + \frac{\Delta t^2}{4}\Delta \ddot v_n + \Delta t\, \dot v_n} \qquad \text{III c}$$

TO GET $\ddot v_{n+1}$ IN TERMS OF v_{n+1} (SOLVING EQ. III c)

$$\boxed{\Delta \ddot v_n = \frac{4}{\Delta t^2}\left(\Delta v_n - \frac{1}{2}\ddot v_n \Delta t^2 - \dot v_n \Delta t\right) = \frac{4}{\Delta t^2}\Delta v_n - \frac{4}{\Delta t}\dot v_n - 2\ddot v_n} \qquad \text{IV (a)}$$

USING EQS I(a) and III(a)

$$\boxed{\ddot v_{n+1} = \ddot v_n + \Delta \ddot v_n = \frac{4}{\Delta t^2}\Delta v_n - \frac{4\dot v_n}{\Delta t} - \ddot v_n = \frac{4}{\Delta t^2}(v_{n+1} - v_n) - \frac{4}{\Delta t}\dot v_n - \ddot v_n} \qquad \text{IV (b)}$$

TO GET $\dot v_{n+1}$ IN TERMS OF v_{n+1} (SOLVING EQ III(b))

$$\boxed{\Delta \dot v_n = \frac{2}{\Delta t}(\Delta v_n - \dot v_n \Delta t) = \frac{2}{\Delta t}\Delta v_n - 2\dot v_n} \qquad \text{V(a)}$$

USING EQS. II a and III a

$$\boxed{\dot v_{n+1} = \dot v_n + \Delta \dot v_n = \frac{2}{\Delta t}\Delta v_n - \dot v_n = \frac{2}{\Delta t}(v_{n+1} - v_n) - \dot v_n} \qquad \text{V(b)}$$

SUBSTITUTING IV b and V b in EQUATIONS OF MOTION ($m\ddot v + c\dot v + Kv = P$)

$$m\left(\frac{4}{\Delta t^2}(v_{n+1} - v_n) - \frac{4}{\Delta t}\dot v_n - \ddot v_n\right) + C\left(\frac{2}{\Delta t}(v_{n+1} - v_n) - \dot v_n\right) + K v_{n+1} = P_{n+1}$$

COLLECTING TERMS RELATED TO v_{n+1}

$$\boxed{\left(\frac{4}{\Delta t^2}m + \frac{2}{\Delta t}C + K\right)v_{n+1} = P_{n+1} + m\left(\frac{4}{\Delta t^2}v_n + \frac{4}{\Delta t}\dot v_n + \ddot v_n\right) + C\left(\frac{2}{\Delta t}v_n + \dot v_n\right)} \qquad \text{VI}$$

SO $\boxed{v_{n+1} = \tilde K^{-1}\tilde P_{n+1}}$ WHERE: $\tilde K = \frac{4}{\Delta t^2}m + \frac{2}{\Delta t}C + K$

$$\tilde P_{n+1} = P_{n+1} + m\left(\frac{4}{\Delta t^2}v_n + \frac{4}{\Delta t}\dot v_n + \ddot v_n\right) + C\left(\frac{2}{\Delta t}v_n + \dot v_n\right)$$

PROCESS: GIVEN $n, v_n, \dot{v}_n, \ddot{v}_n, P_{n+1}, m, c, K$

1. COMPUTE $\tilde{K}$
2. COMPUTE $\tilde{P}$
3. SOLVE $v_{n+1} = \tilde{K}^{-1}\tilde{P}_{n+1}$
4. $\dot{v}_{n+1} = \frac{2}{\Delta t}(v_{n+1} - v_n) - \dot{v}_n$ (FROM EQ. IVb)
5. $\ddot{v}_{n+1} = \frac{4}{\Delta t^2}(v_{n+1} - v_n) - \frac{4}{\Delta t}\dot{v}_n - \ddot{v}_n$ (from EQ. IVb)

   OR

   $\ddot{v}_{n+1} = \dfrac{P_{n+1} - C\dot{v}_{n+1} - K v_{n+1}}{m}$ (by equilibrium).
6. $n = n+1$ GO TO STEP 1

ALTERNATIVE FORMULATION

FOR $m\Delta\ddot{v} + C\Delta\dot{v} + K\Delta v = \Delta P$ $\qquad \Delta P_n = P_{n+1} - P_n$

SUBSTITUTING EQS. IIIa and IIa

$$m\left(\frac{4}{\Delta t^2}\Delta v_n - \frac{4}{\Delta t}\dot{v}_n - 2\ddot{v}_n\right) + C\left(\frac{2}{\Delta t}\Delta v_n - 2\dot{v}_n\right) + K\,\Delta v_n = \Delta P_n$$

COLLECTING TERMS RELATED TO Δv_n

$$\boxed{\left(\frac{4}{\Delta t^2}m + \frac{2}{\Delta t}C + K\right)\Delta v_n = \Delta P + m\left(\frac{4}{\Delta t}\dot{v}_n + 2\ddot{v}_n\right) + 2C\dot{v}_n}$$

OR $\boxed{\Delta v_n = \hat{K}^{-1}\Delta\hat{P}_n}$ where $\hat{K} = \frac{4}{\Delta t^2}m + \frac{2}{\Delta t}C + K$ usar K tangente

$$\Delta\hat{P}_n = \Delta P_n + m\left(\frac{4}{\Delta t}\dot{v}_n + 2\ddot{v}_n\right) + 2C\dot{v}_n$$

PROCESS: GIVEN $n, v_n, \ddot{v}_n, \dot{v}_n; \Delta P_n = P_{n+1} - P_n; m, c, K$

1. COMPUTE $\hat{K}$
2. COMPUTE $\hat{P}$
3. SOLVE $\Delta v_n = \hat{K}^{-1}\Delta P_n$
4. $v_{n+1} = v_n + \Delta v_n$
5. $\Delta\dot{v}_n = \frac{2}{\Delta t}\Delta v_n - 2\dot{v}_n$

   $\dot{v}_{n+1} = \dot{v}_n + \Delta\dot{v}_n$
6. $\Delta\ddot{v}_n = \frac{4}{\Delta t^2}\Delta v_n - \frac{4}{\Delta t}\dot{v}_n - 2\ddot{v}_n$

   $\ddot{v}_{n+1} = \ddot{v}_n + \Delta\ddot{v}_n$ usa $F_E(T_{n+1})$

   [OR $\ddot{v}_{n+1} = \dfrac{P_{n+1} - C\dot{v}_{n+1} - K v_{n+1}}{m}$
7. GO TO 1.

NOTES:
- METHOD IS SELF STARTING.
- METHOD IS UNCONDITIONALLY STABLE
- FOR ACCURACY USE $\Delta t < \frac{T}{10}$
- FOR NONLINEAR SYSTEMS USE ALTERNATIVE METHOD AND SUBSTITUTE:
  - K_T for K in $K\Delta v$ terms
  - R for Kv in GLOBAL EQUILIBRIUM EQUATION