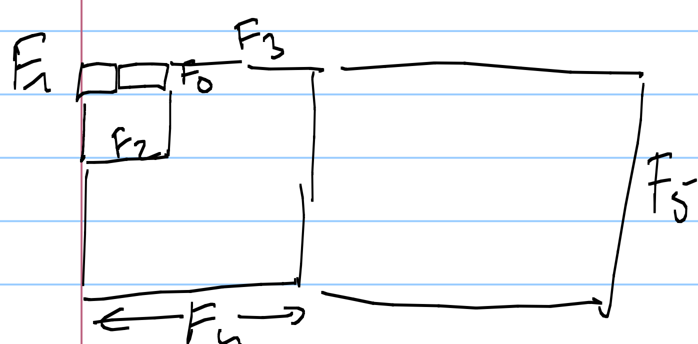
Recurrencia

Note Title 8/24/2010

Ejemplos

1 Fibonacci $\quad F_0 = F_1 = 1 \rightsquigarrow \dfrac{F_n}{F_{n-1}} \rightsquigarrow ?$



$$F_n = F_{n-2} + F_{n-1}$$

$\longrightarrow$ "Gold Ratio"

② Torre de Hanoi

$C_n = 2^n - 1$ ✓

③ $n! = n(n-1)\,\cdots\,3*2*1$ ✓

④ $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n} \longrightarrow 1$

⑤ $1 + 2 + 3 + 4 + \cdots + 100 = 101 * 50 = 5050$

$1 + \cdots + n = \frac{n(n+1)}{2}$

$\int_1^{\infty} \frac{1}{x}\,dx$

⑥ $X_n = X_{n-1} + a_n$

ⓐ $X_n = X_{n-1} + n$, $X_1 = 1$ — $1+2+3+\dots$

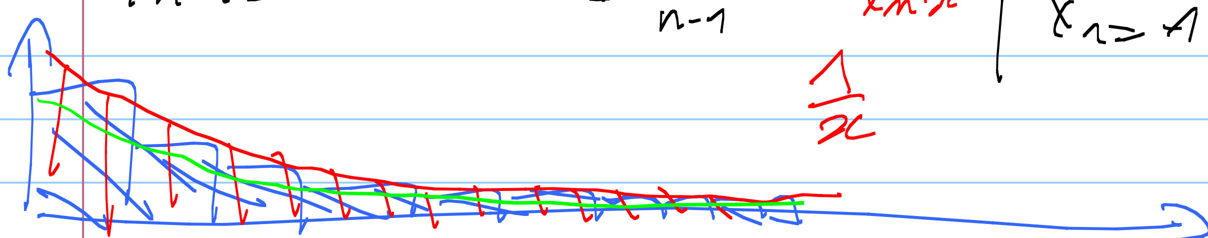
ⓑ $X_n = X_{n-1} + \frac{1}{n}$, $X_1 = 1$

ⓒ $X_n = X_{n-1} + \frac{1}{2^n}$, $X_1 = 1$

ⓑ $X_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \leq \int_1^{\infty} \frac{1}{x}\,dx = \ln x$

$X_{n-1} = 1 + \dots + \frac{1}{n-1}$

$\frac{1}{x}$



back to 1: Fibonacci $\qquad F_n \to \infty$

$$F_0 = F_1 = 1, \qquad F_n = F_{n-2} + F_{n-1}$$

$$\begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}}_{= M} \begin{pmatrix} F_{n-1} \\ F_{n-2} \end{pmatrix}$$

$$\begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = M^n \begin{pmatrix} F_0 \\ F_1 \end{pmatrix}$$

$$(PDP^{-1})^n = PD\cancel{P^{-1}P}DP\cancel{^{-1} - - -P}DP^{-1}$$

$$\underset{Id}{}$$

$$= PD^nP^{-1}$$

$\rightsquigarrow$ valores proprio de polynomio caracteristico de M

$$p(\lambda) = \det(M - \lambda I)$$

$$= \det \begin{pmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = -\lambda(1-\lambda) - 1$$

$$= \lambda^2 - \lambda - 1$$

$$= \left(\lambda - \frac{\sqrt{\Delta} - b}{2a}\right)\left(\lambda - \frac{\sqrt{\Delta} + b}{2a}\right)$$

$$\Delta = b^2 - 4ac$$
$$= 1 + 4 = 5$$

$$= \left(\lambda - \frac{\sqrt{5} - 1}{2}\right)\left(\lambda - \frac{\sqrt{5} + 1}{2}\right)$$

A TERMINAR EN CASA

- TUTORIAL
- Revisar Teorema Maestro.

Complejidad de Calculo?

(1) $F_n = F_{n-2} + F_{n-1}$ $\rightarrow$ Tiempo $O(2^n)$

$\rightarrow$ Tiempo $O(n)$

$F_{n-4} + F_{n-3}$ $\qquad$ $F_{n-3} + F_{n-2}$

$F_{n-6} + F_{n-5}$ $\qquad$ $F_{n-5} + F_{n-4}$ $\qquad$ — — — —

(2) con $\left(\frac{\sqrt{5}-1}{2}\right)^n \to O(1)$ por n "razonable"

$\to O(\log n)$ por n muy grande, usando la multiplicacion egiptiana.