Engranajes

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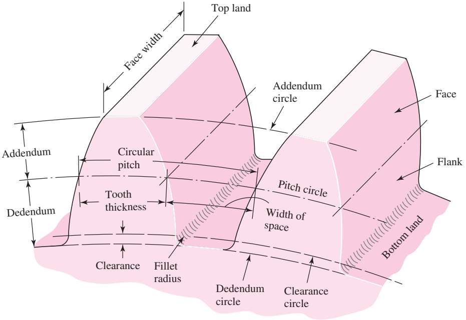
Otoño 2010

Engranaje

- Engranaje con menos dientes es denominado piñon (subíndice P).
- Engranaje con más dientes es denominado Corona (subíndice G).

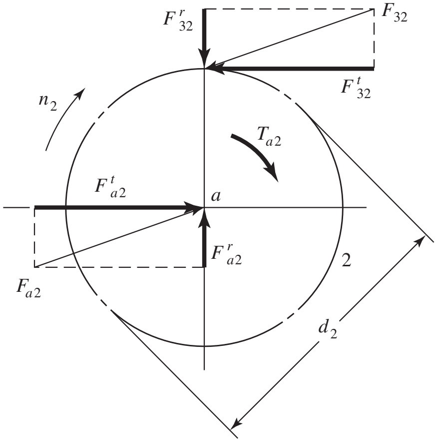


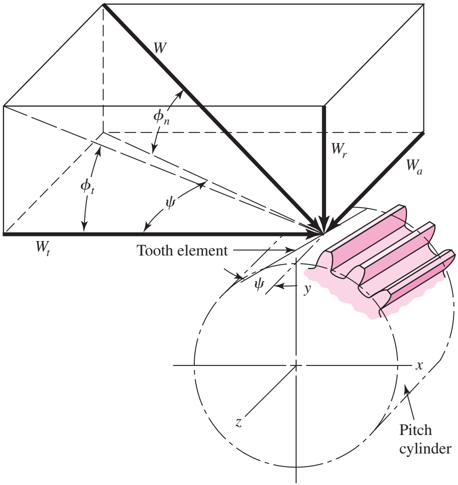
Nomenclatura General

- d: Diámetro primitivo.
- $P = \dfrac{N}{d}\left[\dfrac{dientes}{in}\right]$: Paso diametral.
- $m = \dfrac{d}{N}[mm]$: Módulo.
- $p = \dfrac{\pi \cdot d}{N} = \pi \cdot m$: Paso circular.
- $m_G = \dfrac{N_G}{N_P}$. Relación de velocidades.

- $W_t = P/V$
- $V = \pi dn$
- $T = W_t \dfrac{d}{2}$
- $P = T \cdot \omega$

Carga transmitida por engranajes rectos, $W_t = F_{32}^t$





W: Fuerza Total, sus componentes son

$$W_r = W \sin(\phi_n)$$

$$W_t = W \cos(\phi_n) \cdot \cos(\psi)$$

$$W_a = W \cos(\phi_n) \sin(\psi)$$

En función de W_t

$$W_r = W_t \tan(\phi_t)$$

$$W_a = W_t \tan(\psi)$$

$$W = \frac{W_t}{\cos(\phi_n) \cos(\psi)}$$

Carga Transmitida

En unidades Inglesas:

$$W_t = 33000\frac{H}{V}$$

- W_t $[lbf]$: Carga transmitida.
- H $[Hp]$: Potencia transmitida.
- V $\left[\frac{ft}{min}\right]$: Velocidad de linea de paso($V = \frac{\pi \cdot d \cdot n}{12}$).

En SI:

$$W_t = 60000\frac{H}{\pi \cdot d \cdot n}$$

- W_t $[kN]$: Carga transmitida.
- H $[kW]$: Potencia transmitida.
- d $[mm]$: Diámetro del engranaje.
- n $[rpm]$: Velocidad.

ANSI/AGMA 2001-D04, Flexión de Engranajes

Esfuerzo de flexión (unidades Inglesas):

$$\sigma = W_t K_O K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \quad \text{(ec.14-15)}$$

- K_O: Factor de sobrecarga (fig.14-17)
- K_v: Factor dinámico (depende de Q_v, nivel de exactitud en la transmisión, que va de 3 a 7 para engranajes comerciales y de 8 a 12 para engranajes de presición)(ec.14-27).
- K_s: Factor de tamaño (depende del factor de forma de Lewis Y, tab.14-2).
- P_d: Paso diametral.
- K_m: Factor de distribución de carga (ec.14-30).
- K_B: Factor de espesor de aro (ec.14-40).
- F: Ancho de Cara (tab.13-3).
- J: Factor geométrico (fig.14-6).

Esfuerzo de flexión permisible (unidades Inglesas):

$$\sigma_{perm} = \frac{S_t Y_N}{K_T K_R} \quad \text{(ec.14-17)}$$

- S_t: Esfuerzo de flexión permsible (tab.14-3, 14-4).
- Y_N: Factor de ciclos de esfuerzo del esfuerzo de flexión (fig.14-14).
- K_T: Factor de temperatura ($K_T = 1 \Leftrightarrow T^\circ < 120^\circ C(250^\circ F)$).
- K_R: Factor de confiabilidad (tab.14-10).
- S_F:Factor de seguridad

$$S_F = \frac{\sigma_{perm}}{\sigma}$$

Esfuerzo de contacto (unidades Inglesas):

$$\sigma_c = C_p \cdot \sqrt{W_t K_O K_V K_s \frac{K_m}{d_p F} \frac{C_f}{I}} \quad \text{(ec.14-16)}$$

- C_p: Coeficiente elástico (tab.14-8)
- C_f: Factor de la condición superficial ($C_f = 1$, no investigado).
- d_p: Diámetro de paso ($d_P = \frac{N_P}{P_d}$).
- I: Factor geométrico (depende de m_N, ec.14-21)(ec.14-23).

Esfuerzo de contacto permisible (unidades Inglesas):

$$\sigma_{c,perm} = \frac{S_c Z_N C_H}{K_T K_R} \quad \text{(ec.14-18)}$$

- S_c: Resistencia de contacto repetidamente aplicada (tab.14-6, 14-7).
- Z_N: Factor de ciclos de esfuerzo (fig-14-15).
- C_H: Factor de relación de la dureza (fig.14-12).
- S_H: Factor de seguridad de desgaste.

$$S_H = \frac{\sigma_{c,perm}}{\sigma_c}$$

Para determinar el modo crítico de falla, se compara S_F con S_H^2.

Factor Sobrecarga, figura 14-18

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Factor de Tamaño, tabla 14-2

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

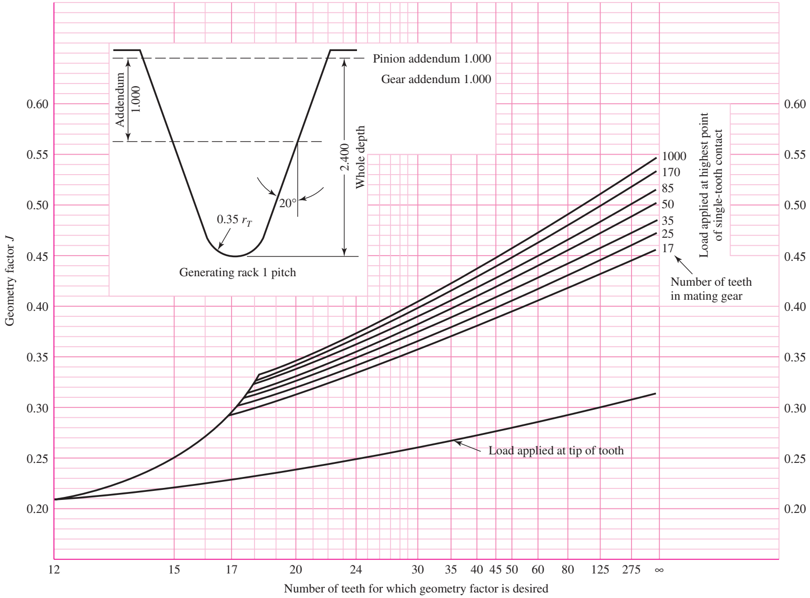
$$K_S = 1{,}192 \cdot \left(\frac{F\sqrt{Y}}{P}\right)^{0{,}0535}$$

Ancho de Cara, tabla 13-3

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90^\circ$
	$m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90^\circ$
Face width	$F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13; Gear: 16, 17, 20, 30

Pinion	16	15	14	13
Gear	16	17	20	30

Factor Geométrico, figura 14-6

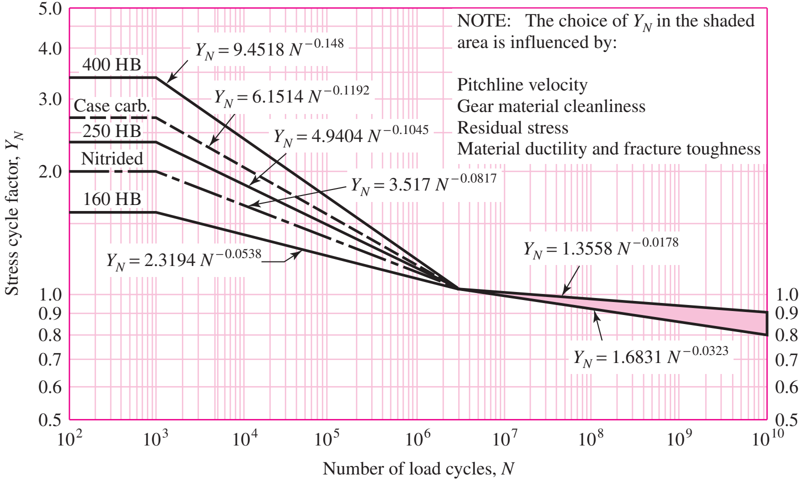


Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Esfuerzo de Flexión Permisible, tabla 14-4

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi
ASTM A48 gray cast iron	Class 20	As cast	—	5000
	Class 30	As cast	174 HB	8500
	Class 40	As cast	201 HB	13 000
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700
	ASTM B–148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600

Factor de Ciclos de Esfuerzo, figura 14-14



Factor de Confiabilidad, tabla 14-10

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Coeficiente Elástico, tabla 14-8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source:* AGMA 218.01

		Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*					
Pinion Material	**Pinion Modulus of Elasticity E_p psi (MPa)***	**Steel 30×10^6 (2×10^5)**	**Malleable Iron 25×10^6 (1.7×10^5)**	**Nodular Iron 24×10^6 (1.7×10^5)**	**Cast Iron 22×10^6 (1.5×10^5)**	**Aluminum Bronze 17.5×10^6 (1.2×10^5)**	**Tin Bronze 16×10^6 (1.1×10^5)**
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

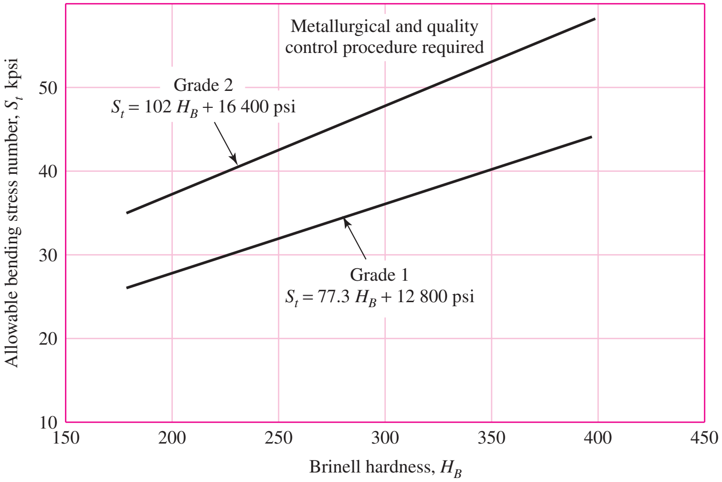
Resistencia de Contacto Repetidamente Aplicada, tabla 14-6

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi		
			Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

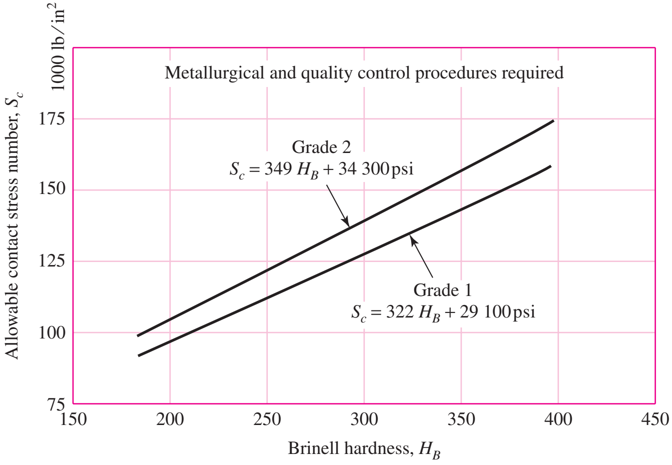
Resistencia de Contacto Repetidamente Aplicada, tabla 14-7

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Contact Stress Number,[3] S_c, psi
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000
	Class 30	As cast	174 HB	65 000–75 000
	Class 40	As cast	201 HB	75 000–85 000
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000
	Grade 80–55–06	Quenched and tempered	179 HB	77 000–92 000
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000

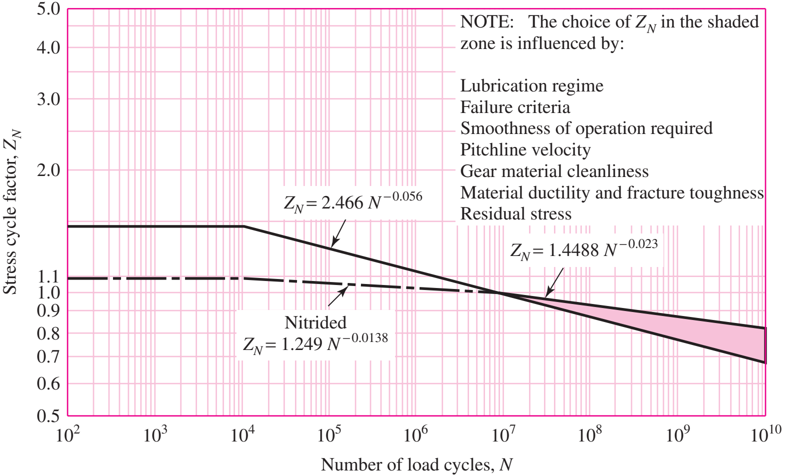
Esfuerzo de Flexión Permisible, figura 14-2

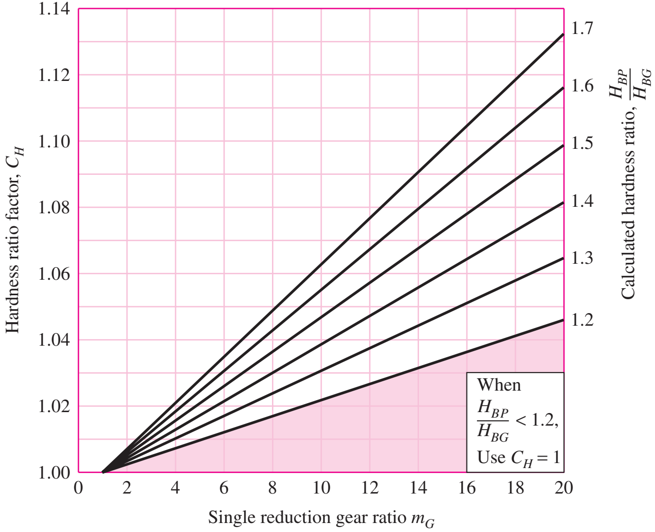


Resistencia de Contacto Repetidamente Aplicada, figura 14-5



Factor de Ciclos de Esfuerzo, figura 14-15




1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
Hardness ratio factor, C_H
0
2
4
6
8
10
12
14
16
18
20
Single reduction gear ratio m_G
1.7
1.6
1.5
1.4
1.3
1.2
Calculated hardness ratio, $\frac{H_{BP}}{H_{BG}}$
When
$\frac{H_{BP}}{H_{BG}} < 1.2$,
Use $C_H = 1$