Gestión de Recursos Pesqueros

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Curso MA45C Ecología Matemática

Outline

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Introduction

In the 50's, the economic theory of common-property fishery was developed by H.S. Gordon thought an equilibrium analysis of a simple dynamics (Gordon-Schaefer model):

$$\dot{x}(t) = F(x(t)) - h(t),$$

where $x(t)$ is the fish stock level at time t, $h(t)$ is the harvesting (tipically $h(t) = u(t)x(t)$ with $u(t)$ the fishing effort) and F is the species biological growth function.

F is usually assumed strictly concave and twice continuously diff. It is also assumed the existence of a saturation constant $K > 0$ satisfying $F(0) = F(K) = 0$ and $F(x) > 0$ for all $x \in]0, K[$.

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Study of Sustainable Equilibriums

We focus on sustainable equilibrium representing exploitation strategies:

$$0 = F(x^*) - h^*$$

So, we are interested in choosing x^* so that the benefit (harvesting) is the largest possible. This leads to chose:

$$x^* \text{ maximizing } F \quad (\text{that is } F'(x^*) = 0)$$

The respective $h^* = F(x^*)$ is called the maximum sustainable harvesting (or yield).

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Optimal Fishery Management

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Additionally, we assume the following:

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- We assume the sole owner is price taking, i.e. the price per unit of biomass p is constant (and known) over time.
- There are no costs; there is no rate of discount.
- There are no storage possibilities. Current sales and profits only depend on current harvesting.
- The price and all constants are known with certainty.
- The fishery is exploited in a given (fixed) period of time T, and initially it was not exploited.

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Optimal Fishery Management

So, our first example focus on the next optimal harvest policy problem:

$$\max_{u(\cdot)\in\mathcal{U}} \int_0^T pu(t)x(t)dt$$

subject to:

$$\begin{aligned} \dot{x}(t) &= F(x(t)) - u(t)x(t) \\ x(0) &= K \end{aligned}$$

where

$$\mathcal{U} := \{u : [0,T] \to [0,u_{\max}] \text{ medible, continua por pedazos, etc.}\}$$

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Gordon-Schaefer Model for Optimal Exploitation

Under the assumptions:

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- We assume the sole owner is price taking, i.e. the price per unit of biomass p is constant (and known) over time.
- There are no fixed costs. Total harvesting cost is equal to cu, where c is the cost average of a unit of fishing effort.
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Gordon-Schaefer Model for Optimal Exploitation

The analysis focus on the next optimal harvest policy problem:

$$\max_{u(\cdot)} \int_0^{\infty} e^{-rt}(pu(t)x(t) - cu(t))dt$$

subject to:

$$\begin{aligned} \dot{x}(t) &= F(x(t)) - u(t)x(t) \\ x(0) &= x_0 > 0 \end{aligned}$$

See Clark '73, Clark & Munro '75.

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- Schaefer (linear) technology of harvesting $H(t) = u(t)x(t)$ does not seem appropriate for small pelagic fisheries such as sardine, herring, Peruvian anchovy and Chilean Jack Mackerel (jurel):

- We have empiric reasons in order to propose the harvesting function $H(t) = u^{\alpha}(t)x^{\beta}(t)$ with $\alpha + \beta \geq 1$, $\alpha, \ \beta \geq 0$.
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The Model

- We consider N players (e.g. N types of fishing firms) in competition for the same fishery resource (say a single fish stock).
- The harvesting technology is given for each firm by a Cobb-Douglas function:

$$H_i(t) = u_i^{\alpha}(t) x^{\beta}(t),$$

where $u_i(t)$ is the firm i's fishing effort (normalized).
- We propose a differential game that explain the interaction between different firms exploiting a pelagic fishery and we study the social planner problems associated.
- We are interested in the behavior of the solutions of our problem for small values of β.

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Differential Game

Under the (additional) assumptions:

- Cooperative harvesting are not feasible because of high monitoring costs.
- We assume price taking firms, i.e. the price per unit of biomass p is constant (and known) over time and independent of industry harvesting.
- At every period t, each firm i choose its own fishing effort $u_i(t)$.
- There are no fixed costs. Total harvesting cost for i is equal to $c_i u_i$, where c_i is the constant average of fishing effort for the i^{th} firm.
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Differential Game

The dynamic and deterministic oligopoly harvesting problem[1] for each firm $i = 1, ..., N$ is

$$\max_{e_i(\cdot)} \int_0^\infty e^{-r_i t}(p u_i^\alpha(t) x^\beta(t) - c_i u_i(t))dt$$

subject to:

$$\begin{aligned} \dot{x}(t) &= F(x(t)) - \sum_{i=1}^{N} u_i^\alpha(t) x^\beta(t) \\ x(0) &= x_0 > 0 \end{aligned}$$

[1] Related works Clark 1980, Dockner et al. 1989, Plourde et al 1989

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Social Planner

We consider N symmetric player, that is, all firms have the same technology: $r_i = r$ and $c_i = c$, for all $i = 1, ..., N$.

We set $u(\cdot) \in [0, \bar{U}]$ as the control variable for the social planner.

The social planner optimization problem is the following

$$\max_{u(\cdot)} N \int_0^\infty e^{-rt}(pu^\alpha(t)x^\beta(t) - cu(t))dt \qquad (P_{SP})$$

subject to:

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Notice that in this case $u_i = u$.
From now on we suppose that $\alpha + \beta = 1$.

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Solution of the Social Planner Problem

Pontryaguin's Principle leads to the following system for the state x and adjoint state λ:

$$\begin{cases} \dot{x}(t) = \varphi_1(x(t), \lambda(t); \beta) \\ \dot{\lambda}(t) = \varphi_2(x(t), \lambda(t); \beta); \\ x(0) = x_0 \end{cases} \tag{PP}$$

where

$$\varphi_1(x, \lambda; \beta) := \begin{cases} F(x) & \text{if } \lambda \geq p \\ F(x) - N\phi^{1-\beta}(\lambda)x & \text{if } \lambda < p, \end{cases}$$

$$\varphi_2(x, \lambda; \beta) := \begin{cases} \lambda(r - F'(x)) & \text{if } \lambda \geq p \\ \lambda(r - F'(x)) - \beta N\phi^{1-\beta}(\lambda)(p - \lambda) & \text{if } \lambda < p, \end{cases}$$

and

$$\phi(\lambda) := \left(\frac{(1-\beta)(p-\lambda)}{c} \right)^{\frac{1}{\beta}}$$

Solution of the Social Planner Problem

For $\beta \in]0,1[$ small enough, system (PP) obtained from Pontryaguin's Principle is the following:

$$\begin{cases} \dot{x}(t) = \Phi_1(x,\lambda,\beta) := F(x) - N\phi^{1-\beta}(\lambda)x \\ \dot{\lambda}(t) = \Phi_2(x,\lambda,\beta) := \lambda(r - F'(x)) - \beta N\phi^{1-\beta}(\lambda)(p-\lambda); \\ x(0) = x_0, \end{cases} \tag{PP}$$

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Solution of the Social Planner Problem

Proposición

The optimal effort u, as a function of the adjoint state λ and the state x, is given by

$$u(x,\lambda) = \begin{cases} 0 & \text{if } \lambda \geq p \\ \left(\frac{(1-\beta)(p-\lambda)}{c}\right)^{\frac{1}{\beta}} x & \text{if } \lambda < p \end{cases}$$

Proposición

If $F'(0) \in]r, N((1-\beta)p/c)^{\frac{1-\beta}{\beta}}[$ then the Pontryaguin system (PP) has only one steady state (x^*, λ^*), which belongs to $]\bar{x}, K[\times]\bar{\lambda}_\beta, p[$, where

$$F'(\bar{x}) = r, \qquad \bar{\lambda}_\beta = p - \frac{c}{(1-\beta)}\left(\frac{F(\bar{x})}{N\bar{x}}\right)^{\frac{\beta}{1-\beta}}.$$

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$$u(x,\lambda) = \begin{cases} 0 & \text{if } \lambda \geq p \\ \left(\frac{(1-\beta)(p-\lambda)}{c}\right)^{\frac{1}{\beta}} x & \text{if } \lambda < p \end{cases}$$

Proposición

If $F'(0) \in]r, N((1-\beta)p/c)^{\frac{1-\beta}{\beta}}[$ then the Pontryaguin system (PP) has only one steady state (x^*, λ^*), which belongs to $]\bar{x}, K[\times]\bar{\lambda}_\beta, p[$, where

$$F'(\bar{x}) = r, \qquad \bar{\lambda}_\beta = p - \frac{c}{(1-\beta)}\left(\frac{F(\bar{x})}{N\bar{x}}\right)^{\frac{\beta}{1-\beta}}.$$

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- $\ln\left(\text{[illegible]}\right) + 1 > 0$ *implies* $\frac{d\lambda^*}{d\beta} < 0$, *i.e.* $\lambda^*(\beta) \uparrow$ *when* $\beta \downarrow$.
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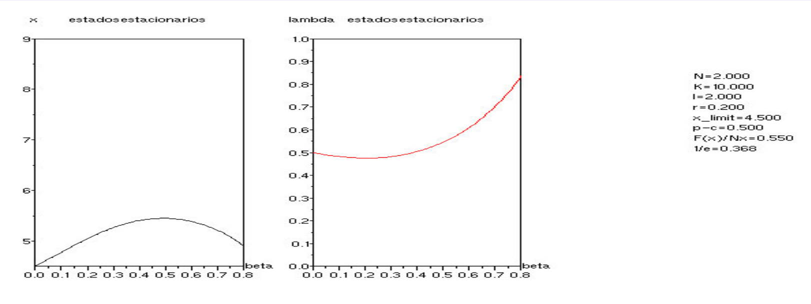

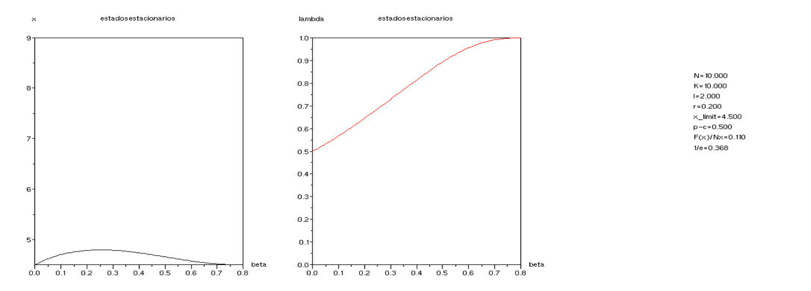

Some Comments

- We need to impose condition $F'(0) > r$ to ensure that the stationary solution $x^*(\beta)$ will be strictly positive.
- Otherwise, it would be optimal to fully deplete the resource x and thereby being able to invest the obtained harvesting profits at the market return $r > 0$.
- From the above theorem, relation $\lambda^*(\beta) \uparrow p - c$ when $\beta \downarrow 0$ (i.e. $\ln\left(\frac{F(\bar{x})}{N\bar{x}}\right) + 1 > 0$) is the solution which is consistent with economic intuition.
- This implies, on the one hand, an upper bound on the number of firms N, for given values of F and r,
- and, on the other hand, a lower bound on the discount rate r (its upper bound is given by the condition $F'(0) > r$), for given values of N and F.

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Outline

Gestión de Recursos Pesqueros

Héctor Ramírez

General Conclusions

- We have studied a Cobb-Douglas type harvest function based on empirical works in fishery management.
- The case $\alpha, \beta \geq 0$ and $\alpha + \beta = 1$ has been analyzed.
- In particular, we have established the behavior of the stationary couple of the Pontryaguin system when $\beta \to 0$.
- The possibility of approaching a fishing collapse outcome has been studied via the phase diagram analysis of the Pontryaguin system.

Open Problems

- It is interesting to study the case when $\alpha > 1$ and $\beta \geq 0$.
- To study the Nash equilibriums of our model[2].
- To study the sensitivity of these Nash equilibriums with respect to changes in the parameters α and β.
- An interesting but complex goal is the study Stackelberg's equilibriums and their dependence on β.

[2] Related work Clark 1980

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Thanks!!


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Grilled mackerel
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