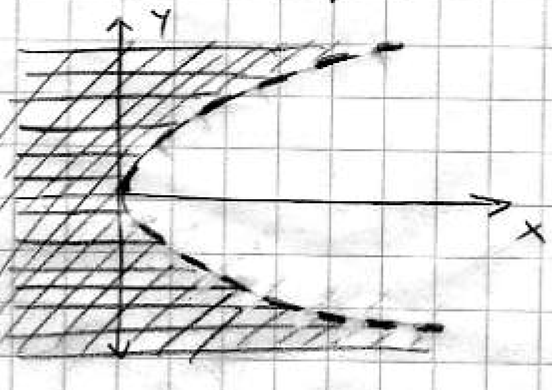
P2) a) i) $f(x,y)=\dfrac{y}{\sqrt{y^2-x}}$

$\mathrm{Dom}(f)=\{(x,y)\in\mathbb{R}^2 \mid \sqrt{y^2-x}\neq 0 \wedge y^2-x>0\}$

$=\{(x,y)\in\mathbb{R}^2 \mid y^2\neq x \wedge y^2>x\}$ 0,5 ptos



0,5 ptos

ii) Sea $c\in\mathbb{R}$.

$N_c(f)=\{(x,y)\in \mathrm{Dom}(f) \mid f(x,y)=c\}$

$=\{(x,y)\in \mathrm{Dom}(f) \mid \dfrac{y}{\sqrt{y^2-x}}=c\}$

$\rightarrow\ y=c\cdot\sqrt{y^2-x} \qquad /(\ )^2$

$\Longrightarrow y^2=c^2(y^2-x)$

$\Longleftrightarrow y^2(1-c^2)=-c^2x$

$\Longrightarrow x=\left(\dfrac{c^2-1}{c^2}\right)y^2 \qquad \left(\Rightarrow \dfrac{c^2-1}{c^2}<1\right)$

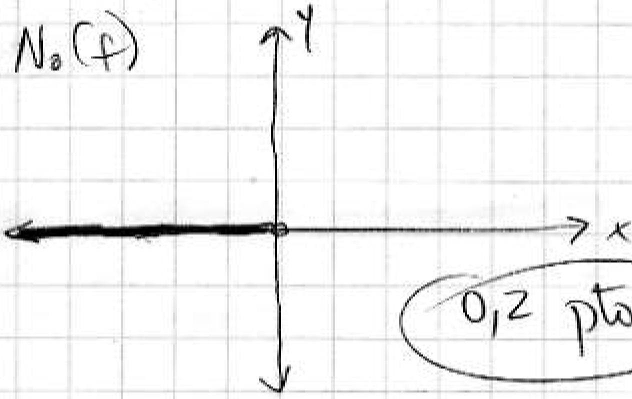
$\Longrightarrow N_c(f)=\{(x,y)\in \mathrm{Dom}(f) \mid x=\dfrac{c^2-1}{c^2}y^2\}$ 0,2 ptos

$c=0 \Longrightarrow \dfrac{y}{\sqrt{y^2-x}}=0 \Longrightarrow y=0 \wedge -x>0$

$\Longrightarrow y=0 \wedge x<0$

$N_0(f)=\{(x,y)\in \mathrm{Dom}(f) \mid (x,y)=(x,0),\ x<0\}$.

$N_0(f)$



0,2 ptos

Veamos cuales son los c que se pueden alcanzar por f:

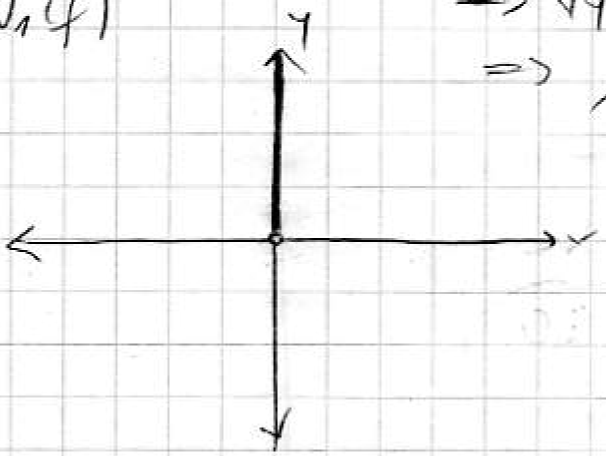
$$\frac{c^2-1}{c^2} < 1 \Rightarrow \cancel{c^2}-1<\cancel{c^2} \Rightarrow -1<0 \Leftrightarrow V$$

Luego $\forall\, c \in \mathbb{R},\ \exists\, N_c(f)$.

$\boxed{c=1} \Rightarrow 1 = \frac{y}{\sqrt{y^2-x}} \quad (y>0)$

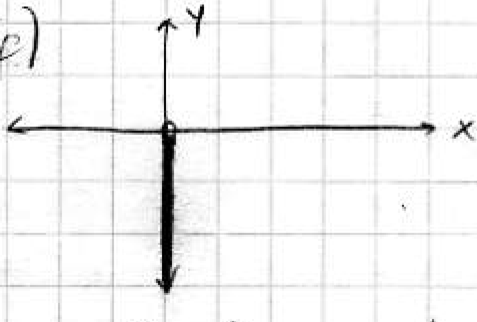
$N_1(f)$

$$\Rightarrow \sqrt{y^2-x} = y \quad /(\ )^2$$
$$\Rightarrow \cancel{y^2}-x=\cancel{y^2}$$
$$\Rightarrow x=0 \wedge y>0.$$



$\boxed{c=-1}$ Análogamente $\Rightarrow x=0 \wedge y<0$

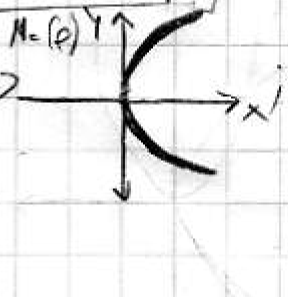
$N_{-1}(f)$



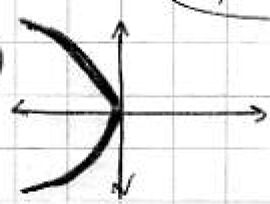
$N_1(f)$ y $N_{-1}(f)$ 0,3 ptos

$\boxed{c \in \mathbb{R} \setminus \{-1,0,1\}}$ $\quad x=\left(\frac{c^2-1}{c^2}\right)y^2$ 0,3 ptos

si $\boxed{|c|>1} \Rightarrow$



$|c|<1 \Rightarrow$ $N_c(f)$



(iii) Para (x,y) tq $x<y^2$, f es continua por álgebra y composición de fnes continuas. (0,2 ptos)

ahora, veamos si se puede tener continuidad en $(0,0)$.

Sea $(x_n,y_n) \xrightarrow[n\to\infty]{} (0,0)$, con $y_n \to 0$, $x_n = y_n^2 - \frac{1}{n}$ (0,4 ptos)

$$f(x_n,y_n) = \frac{y_n}{\sqrt{\cancel{y_n^2} - \cancel{y_n^2} + \frac{1}{n}}} = \frac{y_n}{\frac{1}{\sqrt{n}}} = \sqrt{n}\cdot y_n.$$

tomando $y_n = \frac{1}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0 \Rightarrow f(x_n,y_n) = 1 \xrightarrow[n\to\infty]{} 1$

y $y_n = \frac{1}{n} \xrightarrow[n\to\infty]{} 0 \Rightarrow f(x_n,y_n) = \frac{1}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$

Se tiene que los límites son distintos y por lo tanto, f no puede ser continua en $(0,0)$. (0,4 ptos)

b)

(i) $$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y)\right)$$
$$= \left(-2\left(\sqrt{x^2+y^2}-2\right)\frac{x}{\sqrt{x^2+y^2}}, -2\left(\sqrt{x^2+y^2}-2\right)\frac{y}{\sqrt{x^2+y^2}}\right)$$ (0,4 ptos)

$f(0,1) = 1-(1-2)^2 = 0$

$f(3,4) = 1-(5-2)^2 = -8$

$\nabla f(0,1) = (0,2)$; $\nabla f(3,4) = \left(\frac{-18}{5}, \frac{-24}{5}\right)$

$\Pi_f(0,1)$: $z = 0 + 0\cdot(x-0) + 2(y-1)$
$= 2y-2$ (0,8 ptos)

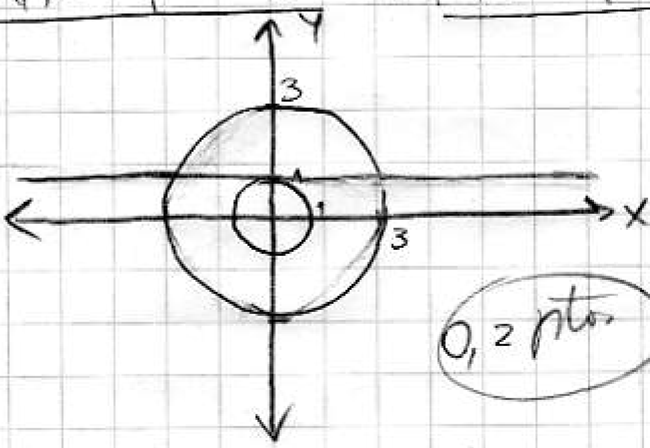
$\Pi_f(3,4): z = -8 - \frac{18}{5}(x-3) - \frac{24}{5}(y-4) = -\frac{18}{5}x - \frac{24}{5}y + 22$ 0,8 ptos

ii) $N_0(f) = \{(x,y) \in \mathbb{R}^2 \mid 0 = 1 - (\sqrt{x^2+y^2} - 2)^2\}$

$1 = (\sqrt{x^2+y^2} - 2)^2 \quad / \sqrt{(\;)}$

$\Rightarrow \sqrt{x^2+y^2} - 2 = 1 \lor \sqrt{x^2+y^2} - 2 = -1$

$\Rightarrow \boxed{\sqrt{x^2+y^2} = 3} \lor \boxed{\sqrt{x^2+y^2} = 1}$ 0,6 ptos



0,2 ptos

$\Pi_f(0,1): z = 2y - 2$

$z = 0 \Rightarrow \boxed{y = 1}$ 0,2 ptos