$$\mathbb{E}(s^2(n)) = \frac{1}{(n-1)} \sum \mathbb{E}((x_i - \bar{x})^2)$$

$$= \frac{1}{(n-1)} \sum \operatorname{var}(x_i - \bar{x}) + \mathbb{E}(\cancel{x_i - \bar{x}})^2 \xrightarrow{} 0$$

$$= \frac{1}{(n-1)} \sum \operatorname{var}(x_i) + \operatorname{var}(\bar{x}) - 2\operatorname{cov}(x_i, \bar{x})$$

$$= \frac{1}{(n-1)} \sum_i \operatorname{var}(x_i) + \operatorname{var}\left(\frac{1}{n}\sum_i x_i\right) - 2\operatorname{cov}\left(x_i, \frac{1}{n}\sum_i x_i\right)$$

$$= \frac{1}{n-1} \sum_i \left[\operatorname{var}(x_i) + \frac{1}{n^2}\sum_i \operatorname{var}(x_i) - \frac{2}{n}\operatorname{cov}(x_i, x_i)\right]$$

$$= \frac{1}{n-1} \sum_i \left[\operatorname{var}(x_i) + \frac{1}{n^2}\sum_i \operatorname{var}(x_i) - \frac{2}{n}\operatorname{var}(x_i)\right]$$

$$= \frac{1}{n-1} \sum_i \left[\operatorname{var}(x_i) + \frac{1}{n}\operatorname{var}(x_i) - \frac{2}{n}\operatorname{var}(x_i)\right]$$

$$= \frac{1}{n-1} \sum \frac{n-1}{n} \operatorname{var}(x_i)$$

$$= \operatorname{var}(x_i) = \sigma^2$$

1. $\operatorname{cov}(x, \alpha y) = \alpha \operatorname{Cov}(x, y)$

2. $\operatorname{Cov}(x, y+z) = \operatorname{Cov}(x, y) + \operatorname{cov}(x, z)$

3. $V(\bar{x}) = \frac{1}{n^2} \sum \operatorname{var}(x_i)$